

1

- (a)
- $\vec{AB} = \begin{pmatrix} -4 \\ 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ 3 \\ 7 \end{pmatrix}$ (M1)
 - $\vec{AB} = -7\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ (A1)
- (b)
- $|\vec{AB}| = \sqrt{(-7)^2 + 3^2 + 7^2}$ (M1)
 - $= \sqrt{49 + 9 + 49} = \sqrt{107}$ (A1)

(Total for question 1 is 4 marks)

2

- $\sqrt{3^2 + k^2 + 2^2} = 7$ (M1)
- $9 + k^2 + 4 = 49 \Rightarrow k^2 = 36$ (M1)
- $k = \pm 6$ (A1)

(Total for question 2 is 3 marks)

3

- (a)
- $\vec{AB} = \begin{pmatrix} -8 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \\ 2 \end{pmatrix}$ (M1)
 - $\vec{AB} = -3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ (A1)
- (b)
- $|\vec{AB}| = \sqrt{(-3)^2 + (-5)^2 + 2^2}$ (M1)
 - $= \sqrt{9 + 25 + 4} = \sqrt{38}$ (A1)

(Total for question 3 is 4 marks)

4

- $\begin{pmatrix} -5 \\ 7 \\ 2 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$ (M1)
- $-5 + x = -2 \Rightarrow x = 3$ (A1)
- $7 + y = -3 \Rightarrow y = -10$ and $2 + z = 6 \Rightarrow z = 4$ (A1)

(Total for question 4 is 3 marks)

5

- (a)
- $\vec{BC} = \vec{BA} + \vec{AC} = -\vec{AB} + \vec{AC}$ (M1)
 - $\vec{BC} = \begin{pmatrix} -6 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$ (M1)
 - $\vec{BC} = 2\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$ (A1)

- (b)
- $|AB| = \sqrt{6^2 + 2^2 + (-1)^2}$ (M1)
 - $= \sqrt{36 + 4 + 1} = \sqrt{41}$ (A1)

(Total for question 5 is 5 marks)

- 6
- Resultant $= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ (M1)
 - Resultant $= \begin{pmatrix} -5 + 4 + 3 \\ 7 + 6 - 5 \\ 4 - 2 + 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$ (M1)
 - Resultant $= 2\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$ (A1)

(Total for question 6 is 3 marks)

- 7
- $PQ = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix} = 2(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ (M1)
 - $RS = \mathbf{s} - \mathbf{r} = \begin{pmatrix} 12 \\ -9 \\ 15 \end{pmatrix} = 3(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ (M1)
 - $PQ = \frac{2}{3}RS$ (M1)
 - Since PQ is a scalar multiple of RS , they are parallel. (A1)

(Total for question 7 is 4 marks)

- 8
- (a)
- $AB = \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -9 \\ 5 \end{pmatrix}$ (M1)
 - $OD = OB + BD = \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -9 \\ 5 \end{pmatrix} = \begin{pmatrix} 11 \\ -14 \\ 8 \end{pmatrix}$ (A1)
- (b)
- $AC = \begin{pmatrix} a - 3 \\ -1 \\ -2 \end{pmatrix}$ (M1)
 - $(a - 3)^2 + (-1)^2 + (-2)^2 = 5^2$ (M1)
 - $(a - 3)^2 = 20 \Rightarrow a - 3 = \sqrt{20} = 2\sqrt{5}$ (since $a > 0$)
 - $a = 3 + 2\sqrt{5}$ (A1)

(Total for question 8 is 5 marks)

9

- (a)
- $BC = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 3 \\ -7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -10 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$ (M1)
 - $BC = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (A1)
- (b)
- $OA = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ (B1)
 - $CB = -BC = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ (M1)
 - Since $OA = CB$, the opposite sides are parallel and equal in length, therefore $OABC$ is a parallelogram. (A1)

(Total for question 9 is 5 marks)

10

- (a)
- $AB = \begin{pmatrix} 6 \\ 1 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$ (M1)
 - $AB = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (A1)
- (b)
- $QC = 4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k} = 2(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ (M1)
 - $QC = 2AB$, so QC and AB are parallel but of different lengths. Therefore $OABC$ is a trapezium. (A1)

(Total for question 10 is 4 marks)

11

- (a)
- $AB = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$ (M1)
 - $|AB| = \sqrt{2^2 + (-3)^2 + (-2)^2} = \sqrt{17}$ (A1)
- (b)
- Midpoint = $\left(\frac{1+3}{2}, \frac{-2-5}{2}, \frac{4+2}{2}\right) = (2, -3.5, 3)$ (M1)
 - Midpoint vector = $2\mathbf{i} - \frac{7}{2}\mathbf{j} + 3\mathbf{k}$ (A1)
- (c)
- $QP = \mathbf{p} - \mathbf{q} = \begin{pmatrix} 4-2 \\ 2-5 \\ 1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$ (M1)
 - $AB = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$ (M1)
 - Since $AB = QP$, the sides AB and QP are parallel and equal in length, so $ABPQ$ is a parallelogram. (A1)

(Total for question 11 is 7 marks)

12

(a)

- $AB = \begin{pmatrix} 2-a \\ 3-b \\ 1 \end{pmatrix}$ and $AC = \begin{pmatrix} -a \\ 1-b \\ -1 \end{pmatrix}$ (M1)
- $(2-a)^2 + (3-b)^2 + 1^2 = (-a)^2 + (1-b)^2 + (-1)^2$ (M1)
- $4 - 4a + a^2 + 9 - 6b + b^2 + 1 = a^2 + 1 - 2b + b^2 + 1$ (M1)
- $14 - 4a - 6b = 2 - 2b \Rightarrow 12 = 4a + 4b \Rightarrow a + b = 3$ (A1)

(b)

- Base $BC = \sqrt{(-2)^2 + (-2)^2 + (-2)^2} = \sqrt{12}$ (M1)
- Midpoint M of $BC = (1, 2, 3)$ (M1)
- Height $h = |AM| = \sqrt{(1-a)^2 + (2-b)^2 + (3-3)^2}$ (M1)
- Substitute $b = 3 - a$: $h = \sqrt{(1-a)^2 + (2-(3-a))^2} = \sqrt{2(a-1)^2}$ (M1)
- Area $= \frac{1}{2} \times \sqrt{12} \times \sqrt{2(a-1)^2} = \sqrt{8} \Rightarrow 3(2(a-1)^2) = 8 \Rightarrow 6(a-1)^2 = 8$ (M1)
- $(a-1)^2 = \frac{4}{3} \Rightarrow a = 1 + \frac{2}{\sqrt{3}} = \frac{3+2\sqrt{3}}{3}$ and $b = \frac{6-2\sqrt{3}}{3}$ (A1)

(Total for question 12 is 10 marks)

13

- $AB = \begin{pmatrix} 1 - (-3) \\ 4 - 5 \\ 7 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 6 \end{pmatrix}$ (M1)
- $|AB| = \sqrt{4^2 + (-1)^2 + 6^2} = \sqrt{16 + 1 + 36} = \sqrt{53}$ (A1)

(Total for question 13 is 2 marks)