

- 1
- Sets up the identity $x + 5 = A(x + 3) + B(x + 2)$ (M1)
 - Finds $A = 3$ and $B = -2$ by substituting $x = -2$ and $x = -3$ (A1)
 - Final answer: $\frac{3}{(x+2)} - \frac{2}{(x+3)}$ (A1)

(Total for question 1 is 3 marks)

- 2
- Sets up the identity $\frac{9x-5}{2} = A(x-1) + B(2x-1)$ (M1)
 - Finds $A = \frac{1}{2}$ and $B = 2$ (A1)
 - Final answer: $\frac{1}{2(2x-1)} + \frac{2}{x-1}$ (A1)

(Total for question 2 is 3 marks)

- 3
- Sets up the identity $5x - 13 = A(x-3)^2 + B(x-1)(x-3) + C(x-1)$ (M1)
 - Finds $A = -2$ and $C = 1$ (A1)
 - Finds $B = 2$ (A1)
 - Final answer: $\frac{2}{x-3} + \frac{1}{(x-3)^2} - \frac{2}{x-1}$ (A1)

(Total for question 3 is 4 marks)

- 4
- Sets up the identity $5x^2 - 12x - 1 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)$ (M1)
 - Finds $C = -2$ and $A = 5$ (A1)
 - Finds $B = 0$ (A1)
 - Final answer: $\frac{5}{x+3} - \frac{2}{(x-1)^2}$ (A1)

(Total for question 4 is 4 marks)

- 5
- Sets up the identity $6x^2 - x + 1 = A(3x-1)(x+1) + B(x+1) + C(3x-1)$ (M1)
 - Finds $C = -2$ (A1)
 - Finds $B = 1$ (A1)
 - Finds $A = 2$ (A1)
 - Final answer: $2 + \frac{1}{3x-1} - \frac{2}{x+1}$ (A1)

(Total for question 5 is 5 marks)

- 6
- Sets up the identity $2x^3 + 7x^2 + 8x + 2 = Ax(x+1)(x+2) + B(x+1)(x+2) + C(x+2) + D(x+1)$ (M1)
 - Finds $C = -1$ and $D = 2$ (A1)
 - Finds $B = 1$ (A1)
 - Finds $A = 2$ (A1)
 - Final answer: $2x + 1 - \frac{1}{x+1} + \frac{2}{x+2}$ (A1)

(Total for question 6 is 5 marks)

- 7
- Sets up the identity $6(x^2 + 4x + 1) = A(2x+3)(3x-1) + B(3x-1) + C(2x+3)$ (M1)
 - Finds $C = 4$ and $B = 3$ (A1)
 - Finds $A = 1$ (A1)
 - Final answer: $1 + \frac{3}{2x+3} + \frac{4}{3x-1}$ (A1)

(Total for question 7 is 4 marks)

- 8
- Sets up the identity $9 - 4x^2 - x = A(2x-1)(2-x) + B(2-x) + C(2x-1)$ (M1)
 - Finds $C = -3$ and $B = 5$ (A1)
 - Finds $A = 2$ (A1)
 - Final answer: $2 + \frac{5}{2x-1} - \frac{3}{2-x}$ (A1)

(Total for question 8 is 4 marks)

- 9
- Sets up the identity $3x^2 + 2x - 23 = Ax(x+3) + B(x+3) + C$ (M1)
 - Finds $C = -2$ and $B = -7$ (A1)
 - Finds $A = 3$ (A1)
 - Final answer: $3x - 7 - \frac{2}{x+3}$ (A1)

(Total for question 9 is 4 marks)

- 10
- Sets up the identity $5x - 7 = A(x-1) + B$ (M1)
 - Finds $B = -2$ and $A = 5$ (A1)
 - Final answer: $\frac{5}{x-1} - \frac{2}{(x-1)^2}$ (A1)

(Total for question 10 is 3 marks)

- 11
- Identifies that the partial fraction decomposition for a repeated factor $(x-2)^2$ must include a term for $\frac{B}{x-2}$ as well as $\frac{C}{(x-2)^2}$. (B1)

(Total for question 11 is 1 mark)

- 12
- Sets up the identity $4x^2 + 12x + 11 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1)$ (M1)
 - Finds $A = 3$ and $C = -1$ (A1)
 - Finds $B = 1$ (A1)
 - Final answer: $\frac{3}{x-1} + \frac{1}{x+2} - \frac{1}{(x+2)^2}$ (A1)
 - Correct use of partial fraction form (A1)

(Total for question 12 is 5 marks)

- 13
- Assume there are integers x and y for which $6x + 9y = 1$ (B1)
 - Factorise the LHS: $3(2x + 3y) = 1$ (M1)
 - Deduce $2x + 3y = 1/3$ (A1)
 - State that if x and y are integers, $2x + 3y$ must be an integer, which contradicts $1/3$ being an integer. (A1)

(Total for question 13 is 4 marks)

- 14
- Assume there are integers x and y for which $30x + 20y = 7$ (B1)
 - Factorise the LHS: $10(3x + 2y) = 7$ (M1)
 - Deduce $3x + 2y = 7/10$ (A1)
 - State that if x and y are integers, $3x + 2y$ must be an integer, which contradicts $7/10$ being an integer. (A1)

(Total for question 14 is 4 marks)

- 15
- Assume $\sqrt{3}$ is rational, so $\sqrt{3} = a/b$ where a, b are integers with no common factors. (B1)
 - Square both sides: $3 = a^2/b^2 \Rightarrow 3b^2 = a^2$ (M1)
 - Deduce a^2 is a multiple of 3, so a is a multiple of 3. (A1)
 - Let $a = 3n$, then $3b^2 = (3n)^2 = 9n^2 \Rightarrow b^2 = 3n^2$ (M1)
 - Deduce b^2 is a multiple of 3, so b is a multiple of 3. (A1)
 - Conclude that a and b have a common factor of 3, which contradicts the initial assumption. (A1)

(Total for question 15 is 6 marks)

- 16
- Assume $\sqrt{2}$ is rational, so $\sqrt{2} = a/b$ where a, b are integers with no common factors. (B1)
 - Square both sides: $2 = a^2/b^2 \Rightarrow 2b^2 = a^2$ (M1)
 - Deduce a^2 is a multiple of 2, so a is a multiple of 2. (A1)
 - Let $a = 2n$, then $2b^2 = (2n)^2 = 4n^2 \Rightarrow b^2 = 2n^2$ (M1)
 - Deduce b^2 is a multiple of 2, so b is a multiple of 2. (A1)
 - Conclude that a and b have a common factor of 2, which contradicts the initial assumption. (A1)

(Total for question 16 is 6 marks)

- 17
- Assume the sum of a rational number and an irrational number is rational: $\frac{a}{b} + i = \frac{c}{d}$ where a, b, c, d are integers. (B1)
 - Rearrange for the irrational number: $i = \frac{c}{d} - \frac{a}{b}$ (M1)
 - Combine into a single fraction: $i = \frac{cb-ad}{bd}$ (M1)
 - State that since a, b, c, d are integers, $cb - ad$ and bd are integers. (A1)
 - Conclude that i must be rational, which is a contradiction. (A1)
 - Final conclusion that the sum must be irrational. (A1)

(Total for question 17 is 6 marks)

- 18
- Assume there is a finite number of prime numbers: $p_1, p_2, p_3, \dots, p_n$ (B1)
 - Construct a new number $N = (p_1 \times p_2 \times p_3 \times \dots \times p_n) + 1$ (M1)
 - Explain that dividing N by any of the primes $p_1, \dots, p_n$ leaves a remainder of 1. (M1)
 - Deduce that N must either be prime itself or have a prime factor not in the original list. (A1)
 - State that this contradicts the assumption that the list contained all prime numbers. (A1)
 - Conclude that there must be infinitely many prime numbers. (A1)

(Total for question 18 is 6 marks)
