

- 1
- Use factor theorem to find a root: $f(-2) = 2(-2)^3 - 7(-2)^2 - 17(-2) + 10 = 0$, so $(x + 2)$ is a factor. (M1)
 - Perform algebraic division to find the quadratic factor: $(2x^3 - 7x^2 - 17x + 10) \div (x + 2) = 2x^2 - 11x + 5$. (M1) (A1)
 - Factorise the quadratic: $2x^2 - 11x + 5 = (2x - 1)(x - 5)$. (M1) (A1)
 - State final factorised form: $(x + 2)(2x - 1)(x - 5)$. (A1)

(Total for question 1 is 6 marks)

- 2
- (a)
- Substitute $x = -3$ into $g(x)$: $g(-3) = 4(-3)^3 - 8(-3)^2 - 35(-3) + 75$. (M1)
 - Evaluate to show $g(-3) = -108 - 72 + 105 + 75 = 0$ and conclude $(x + 3)$ is a factor. (A1)
- (b)
- Divide $g(x)$ by $(x + 3)$ to get $4x^2 - 20x + 25$. (M1) (A1)
 - Recognise $4x^2 - 20x + 25$ as a perfect square: $(2x - 5)^2$. (M1)
 - State $a = 2$ and $b = -5$. (A1)

(Total for question 2 is 6 marks)

- 3
- (a)
- Set up equation for $f(4) = 0$: $4^3 + 6(4)^2 + 4p + q = 0 \Rightarrow 4p + q = -160$. (M1)
 - Set up equation for $f(-5) = 36$: $(-5)^3 + 6(-5)^2 - 5p + q = 36 \Rightarrow -5p + q = 11$. (M1)
 - Solve simultaneous equations: $9p = -171 \Rightarrow p = -19$. (A1)
 - Find q : $q = -160 - 4(-19) = -84$. (A1)
- (b)
- Use $(x - 4)$ as a factor and divide $x^3 + 6x^2 - 19x - 84$ by $(x - 4)$. (M1)
 - Obtain quadratic factor $x^2 + 10x + 21$. (A1)
 - Factorise quadratic: $(x + 7)(x + 3)$. (M1)
 - Final answer: $(x - 4)(x + 7)(x + 3)$. (A1)

(Total for question 3 is 8 marks)

4

- (a)
- Evaluate $f(2) = 2(2)^3 - (2)^2 - 13(2) + 14$. (M1)
 - Show $f(2) = 16 - 4 - 26 + 14 = 0$ and conclude $(x - 2)$ is a factor. (A1)
- (b)
- Divide $f(x)$ by $(x - 2)$ to get $2x^2 + 3x - 7$. (M1) (A1)
 - Solve $2x^2 + 3x - 7 = 0$ using the quadratic formula: $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$. (M1)
 - Calculate roots: $x = \frac{-3 \pm \sqrt{65}}{4}$. (A1)
 - Final answers: $x = 2, x = 1.27, x = -2.77$. (A1)

(Total for question 4 is 7 marks)

5

- (a)
- Use factor theorem: $f(2) = 0 \Rightarrow 2^3 + 2k - 2 = 0$. (M1)
 - Solve for k : $8 + 2k - 2 = 0 \Rightarrow 2k = -6 \Rightarrow k = -3$. (A1)
- (b)
- Divide $x^3 - 3x - 2$ by $(x - 2)$ to get $x^2 + 2x + 1$. (M1) (A1)
 - Factorise quadratic: $(x + 1)^2$. (M1)
 - State solutions: $x = 2$ and $x = -1$. (A1)

(Total for question 5 is 6 marks)

6

- (a)
- Evaluate $f(-3) = (-3)^3 + 6(-3)^2 + 4(-3) - 15$. (M1)
 - Show $f(-3) = -27 + 54 - 12 - 15 = 0$ and conclude $x = -3$ is a solution. (A1)
- (b)
- Divide $f(x)$ by $(x + 3)$ to get $x^2 + 3x - 5$. (M1) (A1)
 - Solve $x^2 + 3x - 5 = 0$ using the quadratic formula: $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$. (M1)
 - Calculate roots: $x = \frac{-3 \pm \sqrt{29}}{2}$. (A1)
 - Final answers: $x = 1.19$ and $x = -4.19$. (A1)

(Total for question 6 is 7 marks)

7

- Complete the square: $(x - 2)^2 - 4 + 7$. (M1)
- Simplify to $(x - 2)^2 + 3$. (A1)
- Explain that $(x - 2)^2 \geq 0$ for all x , so the minimum value is 3, which is > 0 . (A1)

(Total for question 7 is 3 marks)

- 8
- Find a counterexample, e.g., let $n = 10$. (M1)
 - Evaluate: $10^2 - 10 + 3 = 93$. (A1)
 - Show $93 = 3 \times 31$, so it is not prime. (A1)

(Total for question 8 is 3 marks)

- 9
- Define two consecutive odd numbers: $(2n + 1)$ and $(2n + 3)$. (M1)
 - Sum them: $(2n + 1) + (2n + 3) = 4n + 4$. (M1)
 - Factorise: $4(n + 1)$, which is a multiple of 4. (A1)

(Total for question 9 is 3 marks)

- 10
- Use a counterexample: let $x = 2$ and $y = 3$. (M1)
 - Evaluate LHS: $(2 + 3)^2 = 25$. (A1)
 - Evaluate RHS: $2^2 + 3^2 = 13$. (A1)
 - Conclude $25 \neq 13$.

(Total for question 10 is 3 marks)

- 11
- (a)
- Test $n = 1$: $1^2 + 1 + 11 = 13$ (prime). (M1)
 - Test $n = 2$: $2^2 + 2 + 11 = 17$ (prime). (M1)
 - Test $n = 3, 4, 5$: 23, 31, 41 (all prime). (A1)
- (b)
- Find a counterexample, e.g., $n = 10$ or $n = 11$. (M1)
 - Evaluate: $10^2 + 10 + 11 = 121$. (A1)
 - Show $121 = 11 \times 11$, so it is not prime.

(Total for question 11 is 5 marks)

- 12
- List even positive integers less than 10: 2, 4, 6, 8. (M1)
 - Show sums: $2 + 2 = 4, 2 + 4 = 6, 2 + 6 = 8, 2 + 8 = 10, 4 + 4 = 8, 4 + 6 = 10, 4 + 8 = 12, 6 + 6 = 12, 6 + 8 = 14, 8 + 8 = 16$. (M1)
 - Conclude all results are even. (A1)

(Total for question 12 is 3 marks)

- 13
- Test a positive number: $n = 5, 2(5) + 5 = 15, 15 > 5$ (True). (M1)
 - Test a negative number: $n = -5, 2(-5) + 5 = -5, -5$ is not > -5 (False). (A1)
 - Conclude: Sometimes true.

(Total for question 13 is 2 marks)

- 14
- Expand brackets: $(4n^2 + 12n + 9) - (4n^2 - 12n + 9)$. (M1)
 - Simplify: $24n$. (A1)
 - Show as a multiple of 6: $6(4n)$. (A1)
 - Conclude it is a multiple of 6. (A1)

(Total for question 14 is 4 marks)

- 15
- Define consecutive odd integers: $(2n + 1)$ and $(2n + 3)$. (M1)
 - Sum of squares: $(2n + 1)^2 + (2n + 3)^2 = (4n^2 + 4n + 1) + (4n^2 + 12n + 9)$. (M1)
 - Simplify: $8n^2 + 16n + 10$. (A1)
 - Rewrite as $8(n^2 + 2n + 1) + 2$, which is 2 more than a multiple of 8. (A1)

(Total for question 15 is 4 marks)

- 16
- Rearrange to $n^2 + 6n + 12 > 0$. (M1)
 - Complete the square: $(n + 3)^2 - 9 + 12 > 0 \Rightarrow (n + 3)^2 + 3 > 0$. (M1)
 - Explain that $(n + 3)^2 \geq 0$, so the minimum value is 3. (A1)
 - Conclude the inequality holds for all n . (A1)

(Total for question 16 is 4 marks)

- 17
- (a)
- Multiply by ab (since $a, b > 0$): $9a^2 + 4b^2 \geq 12ab$. (M1)
 - Rearrange: $9a^2 - 12ab + 4b^2 \geq 0$. (M1)
 - Factorise: $(3a - 2b)^2 \geq 0$. (A1)
 - Conclude that since any real number squared is ≥ 0 , the statement is true. (A1)
- (b)
- Choose values where one is negative, e.g., $a = -2, b = 1$. (M1)
 - Evaluate: $\frac{9(-2)}{1} + \frac{4(1)}{-2} = -18 - 2 = -20$. (A1)
 - Show $-20 < 12$.

(Total for question 17 is 6 marks)

18

- (a)
- Let $n = 2k + 1$. (M1)
 - Substitute: $(2k + 1)^3 - (2k + 1)$. (M1)
 - Expand and simplify: $(8k^3 + 12k^2 + 6k + 1) - (2k + 1) = 8k^3 + 12k^2 + 4k$. (A1)
 - Factor out 4: $4(2k^3 + 3k^2 + k)$, which is a multiple of 4. (A1)
- (b)
- Let $n = 2$. (M1)
 - Evaluate: $2^3 - 2 = 6$. (A1)
 - Show 6 is not a multiple of 4.

(Total for question 18 is 6 marks)

19

- Identify that prime numbers > 5 must end in 1, 3, 7, or 9. (M1)
- Test $1^4 = 1$ (ends in 1). (M1)
- Test $3^4 = 81$ (ends in 1). (M1)
- Test $7^4 = 2401$ (ends in 1) and $9^4 = 6561$ (ends in 1). (A1)
- Conclude all cases result in a final digit of 1. (A1)

(Total for question 19 is 5 marks)

20

- Test $n = 4$: $4^3 + 4 + 2 = 64 + 4 + 2 = 70$. (B1)
- 70 is not a multiple of 4.

(Total for question 20 is 1 mark)

21

- Use $f(-3) = 0$: $p(-3)^3 - 7(-3)^2 - 43(-3) + q = 0 \Rightarrow -27p + 66 + q = 0 \Rightarrow 27p - q = 66$. (M1)
- Use $f(5) = 0$: $p(5)^3 - 7(5)^2 - 43(5) + q = 0 \Rightarrow 125p - 390 + q = 0 \Rightarrow 125p + q = 390$. (M1)
- Solve simultaneously: $152p = 456 \Rightarrow p = 3$. (A1)
- Find q : $q = 390 - 125(3) = 15$. (A1)

(Total for question 21 is 4 marks)

22

- (a)
- Evaluate $f(2) = 2^3 - 4(2)^2 + 9(2) - 10$. (M1)
 - Show $f(2) = 8 - 16 + 18 - 10 = 0$ and conclude $(x - 2)$ is a factor. (A1)
- (b)
- Divide $f(x)$ by $(x - 2)$ to get $x^2 - 2x + 5$. (M1) (A1)
 - Check discriminant of $x^2 - 2x + 5$: $b^2 - 4ac = (-2)^2 - 4(1)(5) = -16$. (M1)
 - Since $-16 < 0$, there are no real roots for the quadratic, so $x = 2$ is the only real root. (A1)

(Total for question 22 is 6 marks)

23

- (a)
- Factor out n : $n(n^2 - 1)$. (M1)
 - Factorise difference of squares: $n(n - 1)(n + 1)$. (A1)
- (b)
- State that $(n - 1)n(n + 1)$ is the product of three consecutive integers. (M1)
 - Explain that at least one must be a multiple of 2 and exactly one must be a multiple of 3. (M1)
 - Conclude that the product must be a multiple of $2 \times 3 = 6$. (A1)

(Total for question 23 is 5 marks)

24

- (a)
- Rearrange to $x^2 + 2x + 4 > 0$. (M1)
 - Complete the square: $(x + 1)^2 + 3 > 0$. (M1)
 - State that $(x + 1)^2 \geq 0$, so the minimum value is 3, which is > 0 . (A1)
- (b)
- Test positive: $n = 2$, $(2 + 2)^2 = 16$, $16 > 4$ (True). (M1)
 - Test negative: $n = -3$, $(-3 + 2)^2 = 1$, 1 is not > 9 (False). (A1)
 - Conclude: Sometimes true.

(Total for question 24 is 5 marks)

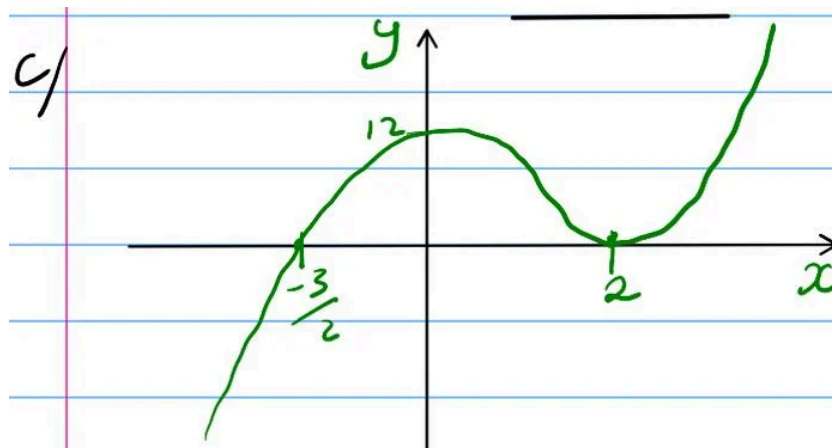
25

- (a)
- Evaluate $f(-3) = 4(-3)^3 - 8(-3)^2 - 35(-3) + 75$. (M1)
 - Show $f(-3) = 0$ and conclude $(x + 3)$ is a factor. (A1)
- (b)
- Divide $f(x)$ by $(x + 3)$ to get $4x^2 - 20x + 25$. (M1) (A1)
 - Factorise quadratic: $(2x - 5)^2$. (M1)
 - State $a = 2$, $b = -5$. (A1)

(Total for question 25 is 6 marks)

26

- (a)
- Evaluate $f(2) = 2(2)^3 - 5(2)^2 - 4(2) + 12 = 16 - 20 - 8 + 12 = 0$. (M1)
 - Conclude $(x - 2)$ is a factor. (A1)
- (b)
- Divide $f(x)$ by $(x - 2)$ to get $2x^2 - x - 6$. (M1) (A1)
 - Factorise quadratic: $(2x + 3)(x - 2)$. (M1)
 - State roots: $x = 2$ (repeated) and $x = -1.5$. These are two distinct roots. (A1)
- (c)
- Sketch showing y-intercept at $(0, 12)$. (B1)
 - Sketch showing x-intercepts at $x = -1.5$ and a turning point touching the x-axis at $x = 2$. (B1)
 - Correct cubic shape (positive x^3). (B1)



- (d)
- Recognise this is $f(x) - 2 = 0$. (M1)
 - Explain that shifting the graph down by 2 units will cause the local minimum (previously at $y = 0$) to go below the x-axis, resulting in 3 intersections. (A1)
- (e)
- For the curve to pass through the origin, one of the roots must be shifted to $x = 0$. (M1)
 - If $x = -1.5$ shifts to 0, $k = 1.5$. (A1)
 - If $x = 2$ shifts to 0, $k = -2$. (A1)

(Total for question 26 is 14 marks)

27

- Case 1: n is even ($n = 2k$). $(2k)^3 + 2 = 8k^3 + 2$. This is 2 more than a multiple of 8, so not divisible. (M1) (A1)
- Case 2: n is odd ($n = 2k + 1$). $(2k + 1)^3 + 2 = 8k^3 + 12k^2 + 6k + 1 + 2 = 8k^3 + 12k^2 + 6k + 3$. (M1)
- This simplifies to $2(4k^3 + 6k^2 + 3k) + 3$, which is an odd number. Odd numbers are not divisible by 8. (A1)

(Total for question 27 is 4 marks)

28

- Consider numbers in forms $3n, 3n - 1, 3n + 1$. (M1)
- $(3n)^3 = 27n^3 = 9(3n^3)$ (multiple of 9). (A1)
- $(3n - 1)^3 = 27n^3 - 27n^2 + 9n - 1 = 9(3n^3 - 3n^2 + n) - 1$ (one less). (A1)
- $(3n + 1)^3 = 27n^3 + 27n^2 + 9n + 1 = 9(3n^3 + 3n^2 + n) + 1$ (one more). (A1)

(Total for question 28 is 4 marks)

29

- (a)
- Evaluate $g(-5) = 2(-5)^3 + (-5)^2 - 38(-5) + 35 = -250 + 25 + 190 + 35 = 0$. (M1)
 - Conclude $(x + 5)$ is a factor. (A1)
- (b)
- Divide $g(x)$ by $(x + 5)$ to get $2x^2 - 9x + 7$. (M1) (A1)
 - Factorise quadratic: $(2x - 7)(x - 1)$. (M1)
 - State final form: $(x + 5)(2x - 7)(x - 1)$. (A1)

(Total for question 29 is 6 marks)

30

- Let $N = 3n \pm 1$. (M1)
- Case 1: $N = 3n + 1 \Rightarrow N^2 = 9n^2 + 6n + 1 = 3(3n^2 + 2n) + 1$. (M1) (A1)
- Case 2: $N = 3n - 1 \Rightarrow N^2 = 9n^2 - 6n + 1 = 3(3n^2 - 2n) + 1$. (M1)
- Conclude both are in form $3p + 1$. (A1)

(Total for question 30 is 5 marks)

31

- (a)
- Evaluate $f(1) = 2(1)^3 + 9(1)^2 - 6(1) - 5 = 2 + 9 - 6 - 5 = 0$. (B1)
- (b)
- Divide $f(x)$ by $(x - 1)$ to get $2x^2 + 11x + 5$. (M1) (A1)
 - Factorise quadratic: $(2x + 1)(x + 5)$. (M1)
 - State solutions: $x = 1, x = -0.5, x = -5$. (A1)

(Total for question 31 is 5 marks)

32

- (a)
- Counterexample: let $m = 9$. (M1)
 - $9^2 + 4 = 85$. $85 = 5 \times 17$, so not prime. (A1)
- (b)
- Case 1: n is even ($n = 2m$). $(2m)^2 + 1 = 4m^2 + 1$. This is 1 more than a multiple of 4. (M1) (A1)
 - Case 2: n is odd ($n = 2m + 1$). $(2m + 1)^2 + 1 = 4m^2 + 4m + 1 + 1 = 4(m^2 + m) + 2$. This is 2 more than a multiple of 4. (M1) (A1)

(Total for question 32 is 6 marks)

33

- (a)
- Use $f(-1.5) = 0$: $6(-1.5)^3 + k(-1.5)^2 + 25(-1.5) - 12 = 0$. (M1)
 - Simplify: $2.25k - 69.75 = 0 \Rightarrow k = 31$. (A1)
- (b)
- Divide $6x^3 + 31x^2 + 25x - 12$ by $(2x + 3)$ to get $3x^2 + 11x - 4$. (M1) (A1)
 - Factorise quadratic: $(3x - 1)(x + 4)$. (M1)
 - Final answer: $(2x + 3)(3x - 1)(x + 4)$. (A1)

(Total for question 33 is 5 marks)

34

- (a)
- Evaluate $f(-3) = 3(-3)^3 - 19(-3) + 24 = -81 + 57 + 24 = 0$. (M1)
 - Conclude $(x + 3)$ is a factor. (A1)
- (b)
- Divide $f(x)$ by $(x + 3)$ to get $3x^2 - 9x + 8$. (M1) (A1)
 - Check discriminant of $3x^2 - 9x + 8$: $(-9)^2 - 4(3)(8) = 81 - 96 = -15$. (M1)
 - Since discriminant < 0 , there are no other real roots. Solution is $x = -3$. (A1)

(Total for question 34 is 6 marks)

35

- (a)
- Let $n = 2$ (the only even prime). (M1)
 - $2^2 = 4$, which is even. (A1)
- (b)
- Let $x = -2$. (M1)
 - LHS: $(-2 + 2)^2 = 0$. (A1)
 - RHS: $(-2)^2 + 2 = 6$. (A1)
 - 0 is not > 6 .

(Total for question 35 is 4 marks)