

Solutions

1

- Let $f(x) = 4x$ and form the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) - 4x}{h}.$$

(M1)

- Expand the numerator to obtain

$$\frac{4x + 4h - 4x}{h}.$$

(M1)

- Simplify the quotient to 4. (A1)
- Take the limit as $h \rightarrow 0$ and conclude that $f'(x) = 4$. (A1)

(Total for question 1 is 4 marks)

2

- Let $f(x) = x^3$ and form the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}.$$

(M1)

- Expand $(x+h)^3$ correctly as $x^3 + 3hx^2 + 3h^2x + h^3$. (M1)
- Cancel x^3 to obtain

$$\frac{3hx^2 + 3h^2x + h^3}{h}.$$

(M1)

- Divide through by h to obtain $3x^2 + 3hx + h^2$. (A1)
- Take the limit as $h \rightarrow 0$ and conclude that $f'(x) = 3x^2$. (A1)

(Total for question 2 is 5 marks)

3

- Let $f(x) = 2x^3$ and form the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^3 - 2x^3}{h}.$$

(M1)

- Expand $2(x+h)^3$ correctly as $2x^3 + 6hx^2 + 6h^2x + 2h^3$. (M1)
- Cancel $2x^3$ to obtain

$$\frac{6hx^2 + 6h^2x + 2h^3}{h}.$$

(M1)

- Divide through by h to obtain $6x^2 + 6hx + 2h^2$. (A1)
- Take the limit as $h \rightarrow 0$ and conclude that $f'(x) = 6x^2$. (A1)

(Total for question 3 is 5 marks)

4

- Let $f(x) = 5x^2$ and form the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{5(x+h)^2 - 5x^2}{h}.$$

(M1)

- Expand and cancel terms to obtain

$$\frac{10hx + 5h^2}{h}.$$

(M1)

- Divide through by h to obtain $10x + 5h$. (A1)
- Take the limit as $h \rightarrow 0$ and conclude that $f'(x) = 10x$. (A1)

(Total for question 4 is 4 marks)

5

- Let $f(x) = kx^3$ and form the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{k(x+h)^3 - kx^3}{h}.$$

(M1)

- Expand $k(x+h)^3$ correctly as $kx^3 + 3hkkx^2 + 3h^2kx + h^3k$. **(M1)**
- Cancel kx^3 to obtain

$$\frac{3hkkx^2 + 3h^2kx + h^3k}{h}.$$

(M1)

- Divide through by h to obtain $3kx^2 + 3hkkx + h^2k$. **(A1)**
- Take the limit as $h \rightarrow 0$ and conclude that $f'(x) = 3kx^2$. **(A1)**

(Total for question 5 is 5 marks)
