

Solutions

- 1
- Use the inverse cosine to obtain $2x + 15 = \cos^{-1}(0.3) = 72.5^\circ$. (M1)
 - Find the second solution $2x + 15 = 360^\circ - 72.5^\circ = 287.5^\circ$. (M1)
 - Rearrange using $x = \frac{(2x+15)-15}{2}$. (M1)
 - Obtain $x = 28.8^\circ$. (A1)
 - Obtain $x = 136.2^\circ$. (A1)

(Total for question 1 is 5 marks)

- 2
- Use the inverse sine to obtain $3\theta - 15 = \sin^{-1}(0.7) = 44.4^\circ$. (M1)
 - Use sine symmetry and periodicity to obtain the relevant angles 44.4° , 135.6° , 404.4° and 495.6° . (M1)
 - Rearrange using $\theta = \frac{(3\theta-15)+15}{3}$. (M1)
 - Obtain $\theta = 19.81^\circ$ and 50.19° . (A1)
 - Obtain $\theta = 139.81^\circ$ and 170.19° . (A1)

(Total for question 2 is 5 marks)

- 3
- Use the inverse tangent to obtain $\theta + 30 = \tan^{-1}(-2.5) = -68.2^\circ$. (M1)
 - Use the 180° period of tangent to obtain the other relevant angle 111.8° . (M1)
 - Subtract 30° from the relevant angles. (M1)
 - Obtain $\theta = -98.2^\circ$ and $\theta = 81.8^\circ$. (A1)

(Total for question 3 is 4 marks)

- 4
- Rearrange to obtain $\cos(x - 40) = 0.4$. (M1)
 - Use inverse cosine and cosine symmetry to obtain $x - 40 = 66.4^\circ$ or 293.6° . (M1)
 - Add 40° to the relevant angles. (M1)
 - Obtain $x = 106.42^\circ$ and $x = 333.58^\circ$. (A1)

(Total for question 4 is 4 marks)

- 5
- Take square roots to obtain $\tan x = \pm\sqrt{3}$. (M1)
 - Solve $\tan x = \sqrt{3}$ using an inverse tangent. (M1)
 - Obtain $x = 60^\circ$ and $x = 240^\circ$ from the positive case. (A1)
 - Solve $\tan x = -\sqrt{3}$ using the tangent period. (M1)
 - Obtain the complete solution set $x = 60^\circ, 120^\circ, 240^\circ$ and 300° . (A1)

(Total for question 5 is 5 marks)

- 6
- (a)
- Substitute $\sin^2 x = 1 - \cos^2 x$ to obtain $2(1 - \cos^2 x) = 7 \cos x + 5$. (M1)
 - Expand to obtain $2 - 2 \cos^2 x = 7 \cos x + 5$. (A1)
 - Rearrange to obtain $2 \cos^2 x + 7 \cos x + 3 = 0$. (A1)
- (b)
- Factorise to obtain $(2 \cos x + 1)(\cos x + 3) = 0$. (M1)
 - Obtain $\cos x = -\frac{1}{2}$ or $\cos x = -3$. (A1)
 - Reject $\cos x = -3$ because it has no real solutions. (B1)
 - Solve $\cos x = -\frac{1}{2}$ in the given interval. (M1)
 - Obtain $x = 120^\circ$ and $x = 240^\circ$. (A1)

(Total for question 6 is 8 marks)

- 7
- (a)
- Substitute $\cos^2 x = 1 - \sin^2 x$ to obtain $6(1 - \sin^2 x) = 4 - \sin x$. (M1)
 - Expand to obtain $6 - 6 \sin^2 x = 4 - \sin x$. (A1)
 - Rearrange to obtain $6 \sin^2 x - \sin x - 2 = 0$. (A1)
- (b)
- Factorise to obtain $(3 \sin x - 2)(2 \sin x + 1) = 0$. (M1)
 - Obtain $\sin x = \frac{2}{3}$ or $\sin x = -\frac{1}{2}$. (A1)
 - Solve $\sin x = \frac{2}{3}$ in the given interval. (M1)
 - Obtain $x = 41.8^\circ$ and $x = 138.2^\circ$. (A1)
 - Solve $\sin x = -\frac{1}{2}$ in the given interval. (M1)
 - Obtain $x = 210^\circ$ and $x = 330^\circ$. (A1)

(Total for question 7 is 9 marks)

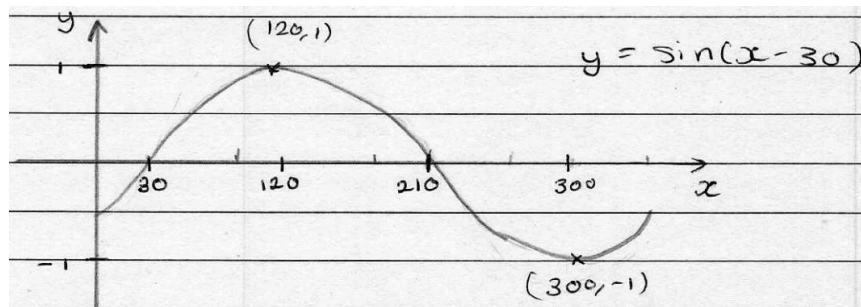
8

- Substitute $\sin^2 x = 1 - \cos^2 x$. (M1)
- Obtain $2 \cos^2 x - 3(1 - \cos^2 x) = 14 \cos x$. (A1)
- Expand and rearrange to obtain $5 \cos^2 x - 14 \cos x - 3 = 0$. (M1)
- Factorise to obtain $(5 \cos x + 1)(\cos x - 3) = 0$. (A1)
- Obtain $\cos x = -\frac{1}{5}$ or $\cos x = 3$. (A1)
- Reject $\cos x = 3$ as impossible for real x . (B1)
- Solve $\cos x = -\frac{1}{5}$ in the given interval. (M1)
- Obtain $x = 101.5^\circ$ and $x = 258.5^\circ$. (A1)

(Total for question 8 is 8 marks)

9

- (a)
- Sketch a sine curve with amplitude 1 and period 360° . (B1)
 - Position the curve with zeros at $x = 30^\circ$ and $x = 210^\circ$, a maximum at $(120^\circ, 1)$ and a minimum at $(300^\circ, -1)$. (B1)



- (b)
- Use inverse sine to obtain $x - 30 = \sin^{-1}(0.3) = 17.5^\circ$. (M1)
 - Use sine symmetry to obtain the second relevant angle $x - 30 = 162.5^\circ$. (M1)
 - Add 30° to both relevant angles. (M1)
 - Obtain $x = 47.5^\circ$ and $x = 192.5^\circ$. (A1)

(Total for question 9 is 6 marks)

10

- Substitute $\tan x = \frac{\sin x}{\cos x}$. (M1)
- Multiply through by $\cos x$ to obtain $3 \sin x = 4 \sin x \cos x$. (M1)
- Rearrange and factorise to obtain $\sin x(4 \cos x - 3) = 0$. (A1)
- Obtain $\sin x = 0$ or $\cos x = \frac{3}{4}$. (A1)
- Solve $\sin x = 0$ in the given interval. (M1)
- Solve $\cos x = \frac{3}{4}$ in the given interval. (M1)
- Obtain the complete solution set $x = 0^\circ, 41.4^\circ, 180^\circ$ and 318.6° . (A1)

(Total for question 10 is 7 marks)

11

(a)

- Use $\tan 2x = \frac{\sin 2x}{\cos 2x}$ to obtain $\frac{3 \sin^2 2x}{\cos 2x} = \cos 2x + 2$. (M1)
- Multiply by $\cos 2x$ to obtain $3 \sin^2 2x = \cos^2 2x + 2 \cos 2x$. (M1)
- Substitute $\sin^2 2x = 1 - \cos^2 2x$. (M1)
- Rearrange correctly to obtain $4 \cos^2 2x + 2 \cos 2x - 3 = 0$. (A1)

(b)

- Apply the quadratic formula to $4 \cos^2 2x + 2 \cos 2x - 3 = 0$. (M1)
- Obtain $\cos 2x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-3)}}{8}$, giving approximately 0.651 and -1.15 . (A1)
- Reject $\cos 2x \approx -1.15$ because it is outside the range of cosine. (B1)
- Use inverse cosine to obtain $2x \approx 49.4^\circ$. (M1)
- Find the second solution $2x \approx 310.6^\circ$ in $0 \leq 2x < 360^\circ$. (A1)
- Divide by 2 and give $x = 24.68^\circ, 155.32^\circ$. (A1)

(Total for question 11 is 10 marks)
