

- 1
- (i)
- Completes the square for x and y : $(x - 1)^2 - 1 + (y + 3)^2 - 9 = 26$ (M1)
 - Identifies centre as $(1, -3)$ (A1)
- (ii)
- Rearranges to $(x - 1)^2 + (y + 3)^2 = 36$ and identifies $r^2 = 36$ (M1)
 - Radius = 6 (A1)

(Total for question 1 is 4 marks)

- 2
- Uses circle equation form $(x - 2)^2 + (y - 5)^2 = r^2$ (M1)
 - Substitutes $(4, 9)$ to find r^2 : $(4 - 2)^2 + (9 - 5)^2 = 20$ (M1)
 - Final equation $(x - 2)^2 + (y - 5)^2 = 20$ (A1)

(Total for question 2 is 3 marks)

- 3
- (a)
- $(x + 2)^2 + (y - 3)^2 = r^2$ (M1)
 - $(1 + 2)^2 + (8 - 3)^2 = r^2 \Rightarrow 3^2 + 5^2 = 34$ (M1)
 - $r^2 = 34$ (A1)
 - $(x + 2)^2 + (y - 3)^2 = 34$ (A1)
- (b)
- Substitutes $(3, 6)$ into the equation: $(3 + 2)^2 + (6 - 3)^2 = 5^2 + 3^2 = 34$ (B1)
- (c)
- Gradient of radius = $\frac{6-3}{3-(-2)} = \frac{3}{5}$ (M1)
 - Gradient of tangent = $-\frac{5}{3}$ (M1)
 - $y - 6 = -\frac{5}{3}(x - 3)$ (M1)
 - $3y - 18 = -5x + 15$ (M1)
 - $5x + 3y - 33 = 0$ (A1)

(Total for question 3 is 10 marks)

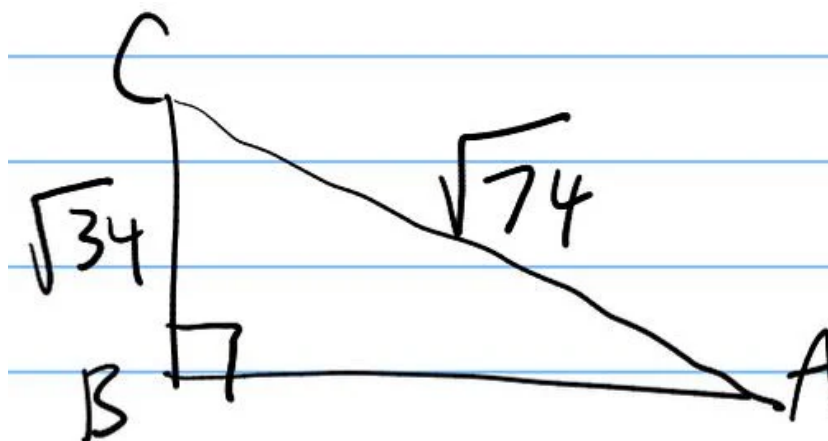
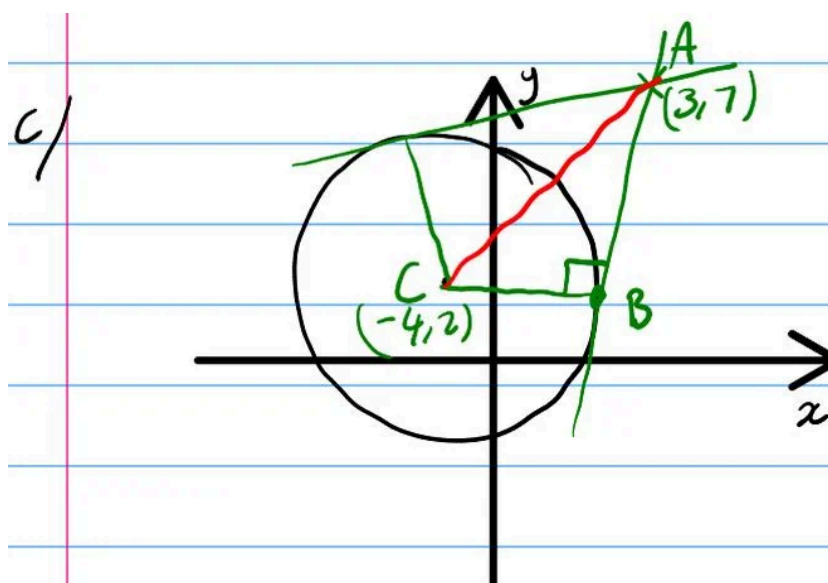
4

- (a)
- $(x - 2)^2 + (y - 5)^2 = 7^2$ (M1)
 - $(x - 2)^2 + (y - 5)^2 = 49$ (A1)
- (b)
- Substitutes $y = 3x - 1$ into circle equation: $(x - 2)^2 + (3x - 1 - 5)^2 = 49$ (M1)
 - Expands and simplifies to $10x^2 - 40x - 9 = 0$ (M1)
 - Uses quadratic formula to find $x = \frac{20 \pm 7\sqrt{10}}{10}$ (M1)
 - Substitutes x back into $y = 3x - 1$ (M1)
 - $y = \frac{50 \pm 21\sqrt{10}}{10}$ (A1)
 - Coordinates: $(\frac{20+7\sqrt{10}}{10}, \frac{50+21\sqrt{10}}{10})$ and $(\frac{20-7\sqrt{10}}{10}, \frac{50-21\sqrt{10}}{10})$ (A1)

(Total for question 4 is 8 marks)

5

- (a)
- Substitutes $(1, 5)$ into equation: $1^2 + 5^2 + 8(1) - 4(5) + k = 0$ (M1)
 - $1 + 25 + 8 - 20 + k = 0 \Rightarrow 14 + k = 0 \Rightarrow k = -14$ (A1)
- (b)
- Completes the square: $(x + 4)^2 - 16 + (y - 2)^2 - 4 - 14 = 0$ (M1)
 - $(x + 4)^2 + (y - 2)^2 = 34$ (A1)
 - Centre $(-4, 2)$, Radius $\sqrt{34}$ (A1)
- (c)
- Identifies right-angled triangle ABC where C is the centre $(-4, 2)$ (M1)
 - Calculates $AC^2 = (3 - (-4))^2 + (7 - 2)^2 = 7^2 + 5^2 = 74$ (M1)
 - Uses Pythagoras: $AB^2 = AC^2 - BC^2$ where BC is radius (M1)
 - $AB^2 = 74 - 34 = 40$ (M1)
 - $AB = \sqrt{40}$ (A1)



(Total for question 5 is 10 marks)

6

- (a)
- Gradient $DE = \frac{-1-2}{4-(-3)} = -\frac{3}{7}$ (M1)
 - Gradient $EF = \frac{-8-(-1)}{1-4} = \frac{-7}{-3} = \frac{7}{3}$ (M1)
 - Product of gradients: $-\frac{3}{7} \times \frac{7}{3} = -1$ (M1)
 - Conclusion that lines are perpendicular, so angle is 90° (A1)
- (b)
- Recognises DF is the diameter (angle in semi-circle) (M1)
 - Centre is midpoint of DF : $(\frac{-3+1}{2}, \frac{2-8}{2})$ (M1)
 - Centre = $(-1, -3)$ (A1)
- (c)
- $(x+1)^2 + (y+3)^2 = r^2$ (M1)
 - $r^2 = (-3 - (-1))^2 + (2 - (-3))^2 = (-2)^2 + 5^2 = 29$ (M1)
 - $(x+1)^2 + (y+3)^2 = 29$ (A1)

(Total for question 6 is 10 marks)

7

- (a)
- $(x-3)^2 - 9 + (y+1)^2 - 1 = 6$ (M1)
 - $(x-3)^2 + (y+1)^2 = 16$ (A1)
 - Centre $(3, -1)$, Radius 4 (A1)
- (b)
- Sets $x = 0$: $y^2 + 2y = 6$ (M1)
 - $y^2 + 2y - 6 = 0 \Rightarrow y = \frac{-2 \pm \sqrt{4 - 4(1)(-6)}}{2} = -1 \pm \sqrt{7}$ (M1)
 - Coordinates: $(0, -1 + \sqrt{7})$ and $(0, -1 - \sqrt{7})$ (A1)

(Total for question 7 is 6 marks)

8

- Centre = midpoint of AB : $(\frac{-3+13}{2}, \frac{5-4}{2}) = (5, 0.5)$ (M1)
- $r^2 = (13 - 5)^2 + (-4 - 0.5)^2 = 8^2 + (-4.5)^2 = 64 + 20.25 = 84.25$ (M1)
- $r^2 = \frac{337}{4}$ (A1)
- $(x-5)^2 + (y-0.5)^2 = 84.25$ (A1)

(Total for question 8 is 4 marks)

9

- (a)
- $(x - 1)^2 + (y - 5)^2 = r^2$ (M1)
 - $r^2 = (-4 - 1)^2 + (3 - 5)^2 = 25 + 4 = 29$ (M1)
 - $(x - 1)^2 + (y - 5)^2 = 29$ (A1)
- (b)
- Gradient of radius = $\frac{3-5}{-4-1} = \frac{-2}{-5} = \frac{2}{5}$ (M1)
 - Gradient of tangent = $-\frac{5}{2}$ (M1)
 - $y - 3 = -\frac{5}{2}(x + 4)$ (M1)
 - $2y - 6 = -5x - 20 \Rightarrow 5x + 2y + 14 = 0$ (A1)

(Total for question 9 is 7 marks)

10

- Gradient of tangent = 2, so gradient of radius = $-\frac{1}{2}$ (M1)
- $\frac{7-k}{3-5} = -\frac{1}{2} \Rightarrow \frac{7-k}{-2} = -\frac{1}{2} \Rightarrow 7 - k = 1 \Rightarrow k = 6$ (M1)
- $r^2 = (3 - 5)^2 + (7 - 6)^2 = (-2)^2 + 1^2 = 5$ (M1)
- Equation: $(x - 5)^2 + (y - 6)^2 = 5$ (A1)

(Total for question 10 is 4 marks)

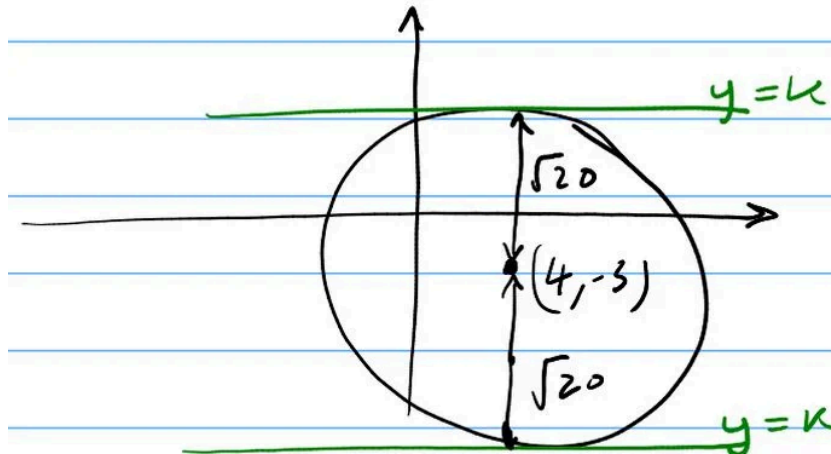
11

- (a)
- $(x - 7)^2 - 49 + (y + 1)^2 - 1 + 40 = 0$ (M1)
 - $(x - 7)^2 + (y + 1)^2 = 10$ (A1)
 - Centre $(7, -1)$, Radius $\sqrt{10}$ (A1)
- (b)
- Substitutes $y = kx$ into circle equation: $x^2 + (kx)^2 - 14x + 2(kx) + 40 = 0$ (M1)
 - $(1 + k^2)x^2 + (2k - 14)x + 40 = 0$ (M1)
 - For two distinct points, discriminant $b^2 - 4ac > 0$ (M1)
 - $(2k - 14)^2 - 4(1 + k^2)(40) > 0$ (M1)
 - $4k^2 - 56k + 196 - 160 - 160k^2 > 0 \Rightarrow -156k^2 - 56k + 36 > 0$ (M1)
 - Solves quadratic inequality: $-\frac{9}{13} < k < \frac{1}{3}$ (A1)

(Total for question 11 is 9 marks)

12

- (a)
- $(x - 4)^2 - 16 + (y + 3)^2 - 9 + 5 = 0$ (M1)
 - $(x - 4)^2 + (y + 3)^2 = 20$ (A1)
 - Centre $(4, -3)$, Radius $\sqrt{20}$ (A1)
- (b)
- Tangent $y = k$ is horizontal, so $k = y_{\text{centre}} \pm \text{radius}$ (M1)
 - $k = -3 + \sqrt{20}$ or $k = -3 - \sqrt{20}$ (A1)



(Total for question 12 is 5 marks)

13

- Completes square to find centre: $(x - 3)^2 + (y - 6)^2 = 45$, so centre is $(3, 6)$ (M1)
- Gradient of radius $= \frac{6-3}{3-8} = -\frac{3}{5}$ (M1)
- Gradient of tangent $= \frac{5}{3}$ (M1)
- $y - 3 = \frac{5}{3}(x - 8) \Rightarrow 3y - 9 = 5x - 40$ (M1)
- $5x - 3y - 31 = 0$ (A1)

(Total for question 13 is 5 marks)

14

- Completes square: $(x - 4)^2 + (y - 5)^2 = 41 - k$ (M1)
- Centre is $(4, 5)$. For circle to exist, $41 - k > 0 \Rightarrow k < 41$ (M1)
- For circle to be in 1st quadrant, radius must be less than the distance to the nearest axis ($x = 4$ is closer than $y = 5$) (M1)
- $r < 4 \Rightarrow r^2 < 16 \Rightarrow 41 - k < 16 \Rightarrow k > 25$ (M1)
- Range: $25 < k < 41$ (A1)

(Total for question 14 is 5 marks)

- 15
- Substitutes $y = mx - 2$ into circle equation: $x^2 + 6x + (mx - 2)^2 - 8(mx - 2) + 5 = 0$ (M1)
 - Expands and simplifies to $(1 + m^2)x^2 + (6 - 12m)x + 25 = 0$ (M1)
 - For tangent, discriminant $b^2 - 4ac = 0$ (M1)
 - $(6 - 12m)^2 - 4(1 + m^2)(25) = 0$ (M1)
 - $144m^2 - 144m + 36 - 100 - 100m^2 = 0 \Rightarrow 44m^2 - 144m - 64 = 0$ (M1)
 - Solves for m : $m = \frac{18 \pm 10\sqrt{5}}{11}$ (A2)

(Total for question 15 is 7 marks)

- 16
- Substitutes $y = mx + 2$ into circle equation: $(x - 5)^2 + (mx + 3)^2 = 15$ (M1)
 - Expands: $x^2 - 10x + 25 + m^2x^2 + 6mx + 9 = 15$ (M1)
 - Simplifies: $(1 + m^2)x^2 + (6m - 10)x + 19 = 0$ (M1)
 - Discriminant $b^2 - 4ac = 0$: $(6m - 10)^2 - 4(1 + m^2)(19) = 0$ (M1)
 - $36m^2 - 120m + 100 - 76 - 76m^2 = 0 \Rightarrow -40m^2 - 120m + 24 = 0$ (M1)
 - Solves for m : $m = \frac{-15 \pm \sqrt{285}}{10}$ (A2)

(Total for question 16 is 7 marks)

- 17
- (a)
- $(x - 2)^2 - 4 + (y - 3)^2 - 9 = 48$ (M1)
 - Centre $(2, 3)$ (A1)
- (b)
- Gradient of radius $= \frac{9-3}{7-2} = \frac{6}{5}$ (M1)
 - Gradient of tangent $= -\frac{5}{6}$ (M1)
 - $y - 9 = -\frac{5}{6}(x - 7) \Rightarrow 6y - 54 = -5x + 35$ (M1)
 - $5x + 6y - 89 = 0$ (A1)

(Total for question 17 is 6 marks)

18

- (a)
- Circle equation: $(x - 10)^2 + (y - 6)^2 = (2\sqrt{17})^2 = 68$ (M1)
 - Substitutes $y = mx$: $(x - 10)^2 + (mx - 6)^2 = 68$ (M1)
 - Expands: $x^2 - 20x + 100 + m^2x^2 - 12mx + 36 = 68$ (M1)
 - Simplifies to $(m^2 + 1)x^2 - (12m + 20)x + 68 = 0$ (A1)
- (b)
- Discriminant $b^2 - 4ac = 0$: $(12m + 20)^2 - 4(m^2 + 1)(68) = 0$ (M1)
 - $144m^2 + 480m + 400 - 272m^2 - 272 = 0 \Rightarrow -128m^2 + 480m + 128 = 0$ (M1)
 - $4m^2 - 15m - 4 = 0 \Rightarrow (4m + 1)(m - 4) = 0$ (M1)
 - $m = 4$ or $m = -\frac{1}{4}$ (A1)
- (c)
- For $m = 4$: $(4^2 + 1)x^2 - (12(4) + 20)x + 68 = 0 \Rightarrow 17x^2 - 68x + 68 = 0 \Rightarrow x = 2$ (M1)
 - $y = 4(2) = 8$, so point is $(2, 8)$ (A1)
 - For $m = -\frac{1}{4}$: $((\frac{1}{4})^2 + 1)x^2 - (12(-\frac{1}{4}) + 20)x + 68 = 0 \Rightarrow \frac{17}{16}x^2 - 17x + 68 = 0 \Rightarrow x = 8$ (M1)
 - $y = -\frac{1}{4}(8) = -2$, so point is $(8, -2)$ (A1)

(Total for question 18 is 12 marks)

19

- Gradient of radius at $(3, 2)$ is $-\frac{3}{2}$. Equation: $y - 2 = -\frac{3}{2}(x - 3) \Rightarrow 3x + 2y = 13$ (M1)
- Gradient of radius at $(3, -2)$ is $\frac{3}{2}$. Equation: $y + 2 = \frac{3}{2}(x - 3) \Rightarrow 3x - 2y = 13$ (M1)
- Solve simultaneously: $x = \frac{13}{3}, y = 0$. Centre is $(\frac{13}{3}, 0)$ (M1)
- $r^2 = (3 - \frac{13}{3})^2 + 2^2 = (-\frac{4}{3})^2 + 4 = \frac{16}{9} + \frac{36}{9} = \frac{52}{9}$ (M1)
- Equation: $(x - \frac{13}{3})^2 + y^2 = \frac{52}{9}$ (A2)

(Total for question 19 is 6 marks)

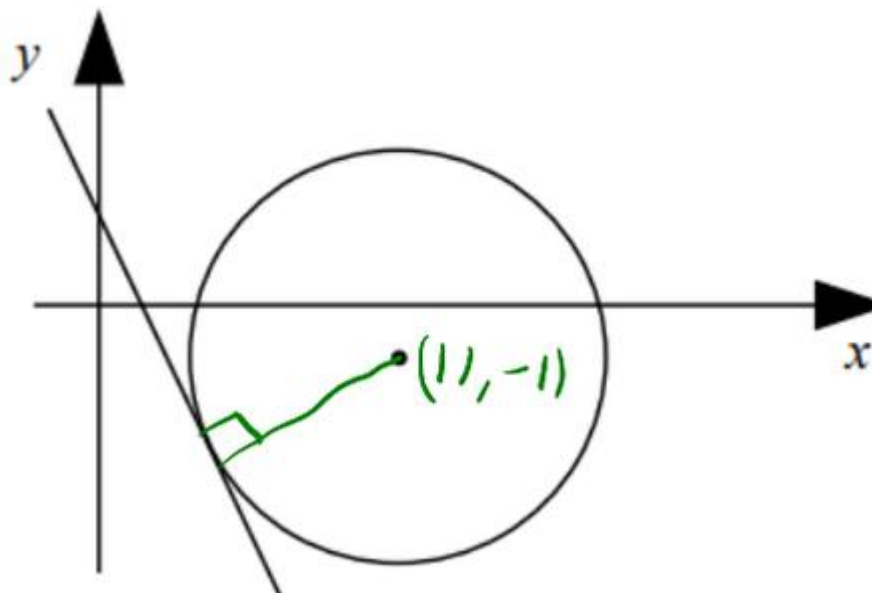
20

- Completes square: $(x - 5)^2 - 25 + (y + \frac{k}{2})^2 - \frac{k^2}{4} + 20 = 0$ (M1)
- $(x - 5)^2 + (y + \frac{k}{2})^2 = \frac{k^2}{4} + 5$ (M1)
- $r^2 = 9$, so $\frac{k^2}{4} + 5 = 9$ (M1)
- $\frac{k^2}{4} = 4 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$ (A1)

(Total for question 20 is 4 marks)

21

- Gradient of tangent is -3 , so gradient of radius is $\frac{1}{3}$ (M1)
- Equation of radius line: $y + 1 = \frac{1}{3}(x - 11) \Rightarrow x - 3y = 14$ (M1)
- Intersection of $x - 3y = 14$ and $3x + y = 2$: $x = 2, y = -4$ (M1)
- $r^2 = (11 - 2)^2 + (-1 - (-4))^2 = 9^2 + 3^2 = 90$ (M1)
- Equation: $(x - 11)^2 + (y + 1)^2 = 90$ (A1)



(Total for question 21 is 5 marks)

22

- $(x - 4)^2 - 16 + (y + 5)^2 - 25 = 15$ (M1)
- Centre $(4, -5)$ (A1)

(Total for question 22 is 2 marks)

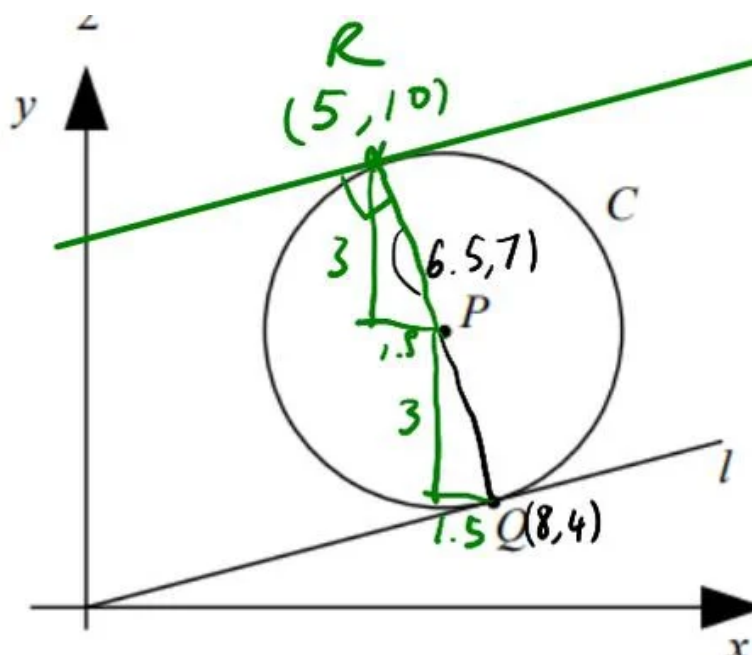
23

- Gradient $AB = \frac{6-a}{13-4} = \frac{1}{3} \Rightarrow \frac{6-a}{9} = \frac{1}{3} \Rightarrow 6 - a = 3 \Rightarrow a = 3$ (M1)
- Centre (c, d) is midpoint of AB : $(\frac{4+13}{2}, \frac{3+6}{2}) = (8.5, 4.5)$ (M1)
- $c = 8.5, d = 4.5$ (A1)
- $e = r^2 = (13 - 8.5)^2 + (6 - 4.5)^2 = 4.5^2 + 1.5^2 = 20.25 + 2.25 = 22.5$ (M1)
- $a = 3, c = 8.5, d = 4.5, e = 22.5$ (A2)

(Total for question 23 is 6 marks)

24

- (a)
- $(x - 6.5)^2 + (y - 7)^2 = r^2$ (M1)
 - $r^2 = (8 - 6.5)^2 + (4 - 7)^2 = 1.5^2 + (-3)^2 = 2.25 + 9 = 11.25$ (M1)
 - $r^2 = \frac{45}{4}$ (A1)
 - $(x - 6.5)^2 + (y - 7)^2 = 11.25$ (A1)



- (b)
- The other tangent is parallel to l and on the opposite side of the centre. (M1)
 - Vector $QP = \begin{pmatrix} -1.5 \\ 3 \end{pmatrix}$. Vector $PR = \begin{pmatrix} -1.5 \\ 3 \end{pmatrix}$ to find other contact point R . (M1)
 - $R = (6.5 - 1.5, 7 + 3) = (5, 10)$ (M1)
 - Substitutes R into $y = \frac{1}{2}x + k$: $10 = \frac{1}{2}(5) + k \Rightarrow k = 7.5$ (A1)

(Total for question 24 is 8 marks)

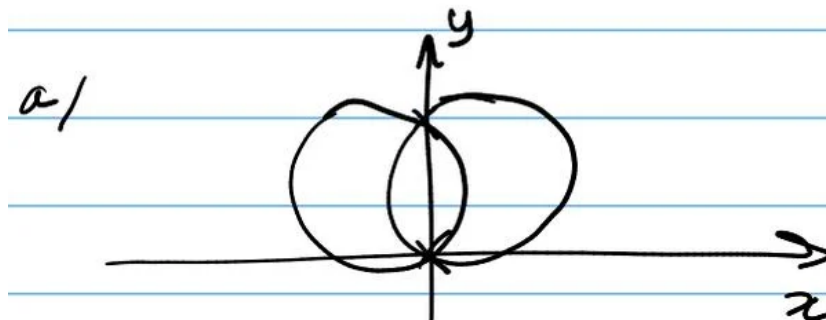
25

- Centre is $(-3, 4)$, origin is $(0, 0)$ (M1)
- Gradient of radius $= \frac{4-0}{-3-0} = -\frac{4}{3}$ (M1)
- Gradient of tangent $= \frac{3}{4}$ (A1)

(Total for question 25 is 3 marks)

26

- (a) • Sketch showing two circles passing through $(0, 0)$ and $(0, 8)$ with centres at $(\pm 3, 4)$ (B1)



- (b) • y -coordinate of centre must be 4 (midpoint of chord on y -axis) (M1)
 • $(x - a)^2 + (y - 4)^2 = 25$. Substitutes $(0, 0)$: $a^2 + 16 = 25 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$ (M1)
 • Equations: $(x - 3)^2 + (y - 4)^2 = 25$ and $(x + 3)^2 + (y - 4)^2 = 25$ (A1)

(Total for question 26 is 4 marks)

27

- Circle equation: $(x - 2)^2 + (y - 1)^2 = (-1 - 2)^2 + (3 - 1)^2 = 13$ (M1)
- Line gradient: $m = \frac{2 - (-4)}{10 - 1} = \frac{6}{9} = \frac{2}{3}$ (M1)
- Line equation: $y - 2 = \frac{2}{3}(x - 10) \Rightarrow y = \frac{2}{3}x - \frac{14}{3}$ (M1)
- Substitute line into circle: $(x - 2)^2 + (\frac{2}{3}x - \frac{14}{3} - 1)^2 = 13$ (M1)
- Expand and simplify to $13x^2 - 104x + 208 = 0 \Rightarrow x^2 - 8x + 16 = 0$ (M1)
- $(x - 4)^2 = 0$. Since there is only one solution ($x = 4$), the line is a tangent. (A2)

(Total for question 27 is 7 marks)

28

- Circle equation: $(x - 4)^2 + (y + 1)^2 = 100$ (M1)
- Line gradient: $m = \frac{9 - 1}{14 - (-2)} = \frac{8}{16} = 0.5$ (M1)
- Line equation: $y - 1 = 0.5(x + 2) \Rightarrow y = 0.5x + 2$ (M1)
- Substitute line into circle: $(x - 4)^2 + (0.5x + 2 + 1)^2 = 100$ (M1)
- Expand: $x^2 - 8x + 16 + 0.25x^2 + 3x + 9 = 100$ (M1)
- $1.25x^2 - 5x - 75 = 0 \Rightarrow x^2 - 4x - 60 = 0$ (M1)
- $(x - 10)(x + 6) = 0 \Rightarrow x = 10, x = -6$ (M1)
- y values: $y = 0.5(10) + 2 = 7$ and $y = 0.5(-6) + 2 = -1$ (A1)
- Points: $(10, 7)$ and $(-6, -1)$

(Total for question 28 is 8 marks)