

**1** The circle  $C$  has the equation  $x^2 + y^2 - 2x + 6y = 26$

(i) Find the coordinates of the centre of  $C$ .

**(2)**

(ii) Find the radius of  $C$ .

**(2)**

**(Total for question 1 is 4 marks)**

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- 2** The circle  $C$  has centre  $(2, 5)$  and passes through point  $(4, 9)$ .  
Find an equation for  $C$ .

**(Total for question 2 is 3 marks)**

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**3** The circle  $C$  has centre  $(-2, 3)$  and passes through point  $(1, 8)$ .

(a) Find an equation for  $C$ .

**(4)**

(b) Show that the point  $(3, 6)$  lies on  $C$ .

**(1)**

- (c) Find the equation to the tangent to  $C$  at  $(3, 6)$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**(5)**

**(Total for question 3 is 10 marks)**

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**4** The circle  $C$  has centre  $(2, 5)$  and radius 7.

(a) Find an equation for  $C$ .

**(2)**

(b) The line  $y = 3x - 1$  intersects  $C$  at the points  $A$  and  $B$ . Find the exact coordinates of  $A$  and  $B$ .

**(6)**

**(Total for question 4 is 8 marks)**

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- 5** The circle  $C$  has the equation  $x^2 + y^2 + 8x - 4y + k = 0$  where  $k$  is a constant. Given that the point  $(1, 5)$  lies on  $C$ .

(a) Find the value of  $k$ .

**(2)**

(b) Find the coordinates of the centre and the radius of  $C$ .

**(3)**

- (c) A straight line that passes through the point  $A(3, 7)$  is a tangent to the circle  $C$  at the point  $B$ . Find the exact length of the line  $AB$ .

**(5)**

**(Total for question 5 is 10 marks)**

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**6** The points  $D$ ,  $E$  and  $F$  have coordinates  $(-3, 2)$ ,  $(4, -1)$  and  $(1, -8)$  respectively.

(a) Show that angle  $DEF$  is a right angle.

**(4)**

(b) Given that  $D$ ,  $E$  and  $F$  all lie on the circle  $C$ , find the coordinates of the centre of  $C$ .

**(3)**

(c) Find the equation of the circle  $C$ .

**(3)**

**(Total for question 6 is 10 marks)**

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7 The circle  $C$  has the equation  $x^2 + y^2 - 6x + 2y = 6$

(a) Find the coordinates of the centre and the radius of  $C$ .

(3)

(b)  $C$  crosses the  $y$  axis at the points  $A$  and  $B$ . Find the coordinates of the points  $A$  and  $B$ .

(3)

(Total for question 7 is 6 marks)

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- 8** The points  $A$  and  $B$  have coordinates  $(-3, 5)$  and  $(13, -4)$  respectively. Given that  $AB$  is a diameter of the circle  $C$ .  
Find an equation for  $C$ .

**(Total for question 8 is 4 marks)**

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**9** The circle  $C$  has centre  $(1, 5)$  and passes through the point  $A(-4, 3)$ .

(a) Find an equation for  $C$ .

**(3)**

(b) Find an equation for the tangent to  $C$  at  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**(4)**

**(Total for question 9 is 7 marks)**

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- 10** The circle  $C$  has centre  $(5, k)$ , where  $k$  is a constant. The line  $y = 2x + 1$  is a tangent to the circle  $C$ , touching  $C$  at the point  $A(3, 7)$ .  
Find an equation for  $C$ .

**(Total for question 10 is 4 marks)**

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**11** The circle  $C$  has the equation  $x^2 + y^2 - 14x + 2y + 40 = 0$

(a) Find (i) the coordinates of the centre of  $C$  and (ii) the exact radius of  $C$ .

**(3)**

(b) The line with equation  $y = kx$  where  $k$  is a constant, meets  $C$  at two distinct points. Find the range of possible values of  $k$ .

**(6)**

**(Total for question 11 is 9 marks)**

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**12** The circle  $C$  has the equation  $x^2 + y^2 - 8x + 6y + 5 = 0$

(a) Find (i) the coordinates of the centre of  $C$  and (ii) the exact radius of  $C$ .

**(3)**

(b) The line with equation  $y = k$  where  $k$  is a constant, is a tangent to  $C$ . Find the possible values of  $k$ .

**(2)**

**(Total for question 12 is 5 marks)**

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- 13** The circle  $C$  has the equation  $x^2 + y^2 - 6x - 12y = 0$ . The line  $l$  is a tangent to  $C$  at the point  $(8, 3)$ .  
Find the equation of  $l$  in the form  $ax + by + c = 0$ .

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**(Total for question 13 is 5 marks)**

- 14** The circle  $C$  has the equation  $x^2 + y^2 - 8x - 10y + k = 0$ , where  $k$  is a constant.  
Given the  $C$  lies entirely in the first quadrant, find the range of possible values of  $k$ .

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**(Total for question 14 is 5 marks)**

- 15** The line  $y = mx - 2$  is a tangent to the circle  $x^2 + 6x + y^2 - 8y + 5 = 0$ .  
Find the two possible values of  $m$ , giving your answers in exact form.

(Total for question 15 is 7 marks)

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- 16** The line  $y = mx + 2$  is a tangent to the circle  $(x - 5)^2 + (y + 1)^2 = 15$ .  
Find the two possible values of  $m$ , giving your answers in exact form.

(Total for question 16 is 7 marks)

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**17** The circle  $C$  has the equation  $x^2 + y^2 - 4x - 6y = 48$

(a) Find the coordinates of the centre of  $C$ .

**(2)**

(b) Find equation of the tangent to the circle at the point  $(7, 9)$ .

**(4)**

**(Total for question 17 is 6 marks)**

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**18** A circle  $C$  has centre  $(10, 6)$  and radius  $2\sqrt{17}$ . A line  $L$  has equation  $y = mx$ .

- (a) Show that the  $x$ -coordinate of any points of intersection of  $C$  and  $L$  satisfies the equation  $(m^2 + 1)x^2 - (12m + 20)x + 68 = 0$ .

**(4)**

- (b) Find values of  $m$  for which the equation in (a) has equal roots.

**(4)**

- (c) Two lines drawn from the origin which are tangents to  $C$ . Find the coordinates of the points of contact between the tangents and  $C$ .

(4)

(Total for question 18 is 12 marks)

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- 19** The lines with equations  $y = \frac{2}{3}x$  and  $y = -\frac{2}{3}x$  are tangents to a circle at  $(3, 2)$  and  $(3, -2)$  respectively.  
Find an equation for the circle.

(Total for question 19 is 6 marks)

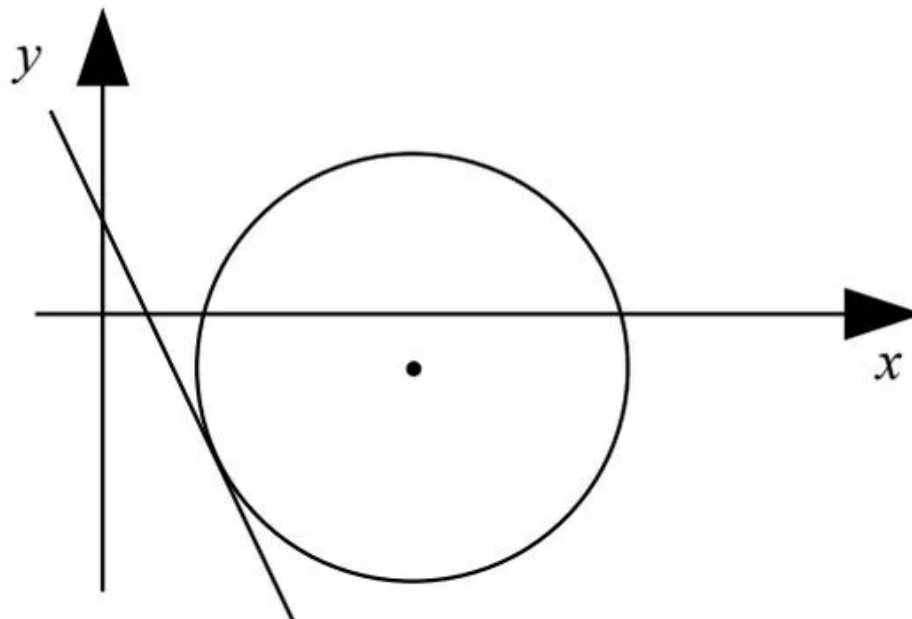
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- 20** The circle with equation  $x^2 + y^2 - 10x + ky + 20 = 0$  has radius 3.  
Find the two possible values of the constant  $k$ .

(Total for question 20 is 4 marks)

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- 21** The diagram shows line with equation  $y + 3x = 2$  which is a tangent to a circle with centre  $(11, -1)$ .



Find an equation for the circle.

(Total for question 21 is 5 marks)

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- 22** Find the centre of the circle with equation  $x^2 + y^2 - 8x + 10y = 15$ .

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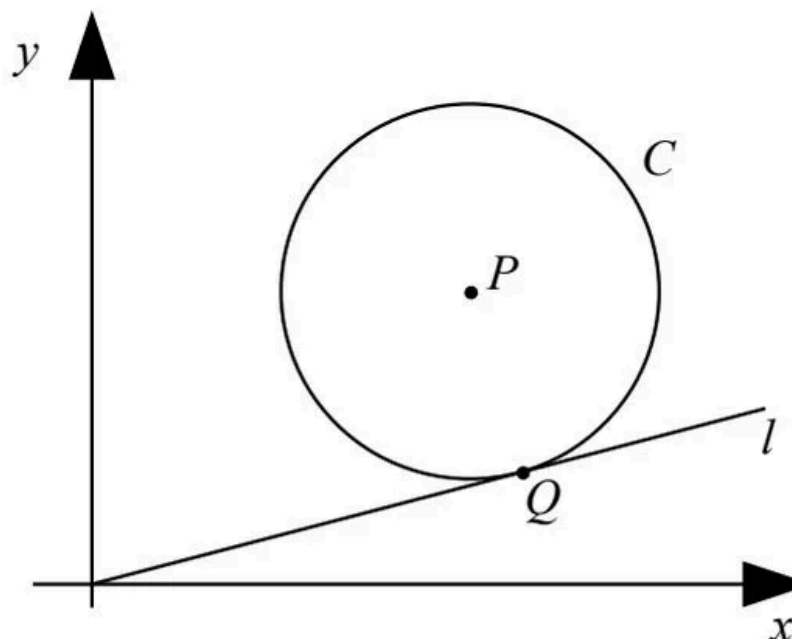
**(Total for question 22 is 2 marks)**

- 23** The points  $A(4, a)$  and  $B(13, 6)$  lie on a circle.  $AB$  is a diameter of the circle and has a gradient of  $\frac{1}{3}$ . The circle has equation  $(x - c)^2 + (y - d)^2 = e$  where  $c$ ,  $d$  and  $e$  are rational numbers. Find the values of  $a$ ,  $c$ ,  $d$  and  $e$ .

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**(Total for question 23 is 6 marks)**

- 24** The sketch shows circle  $C$  with centre  $P(6.5, 7)$ . The line  $l$  with equation  $y = \frac{1}{2}x$  is a tangent to  $C$  at point  $Q(8, 4)$ .



- (a) Find an equation for  $C$ .

(4)

- (b) The line with equation  $y = \frac{1}{2}x + k$ , where  $k$  is a non-zero constant, is also a tangent to  $C$ . Find the value of  $k$ .

(4)

(Total for question 24 is 8 marks)

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- 25 A circle has equation  $(x + 3)^2 + (y - 4)^2 = 25$ .  
Find the gradient of the tangent to the circle at the origin.

(Total for question 25 is 3 marks)

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**26** A circle  $C$  with radius 5 passes through  $(0, 0)$  and  $(0, 8)$ .

(a) Sketch the two possible positions of  $C$ .

**(1)**

(b) Find the equations of the two circles.

**(3)**

**(Total for question 26 is 4 marks)**

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- 27** A circle  $C$  with centre  $(2, 1)$  passes through the point  $(-1, 3)$ . A line  $L$  passes through the points  $(1, -4)$  and  $(10, 2)$ .  
Show that  $L$  is a tangent to  $C$ .

**(Total for question 27 is 7 marks)**

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- 28** A circle  $C$  with centre  $(4, -1)$  and radius 10. A line  $L$  passes through the points  $(-2, 1)$  and  $(14, 9)$ . Find the points of intersection of  $L$  and  $C$ .

(Total for question 28 is 8 marks)

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