

1

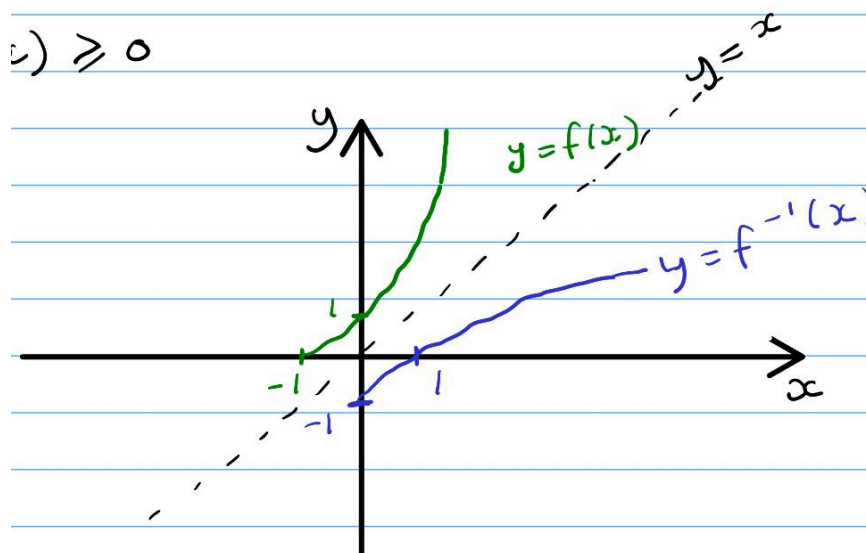
- (a)
- Substitute $x = 1$ into $g(x)$: $g(1) = \frac{2}{1+3} = \frac{1}{2}$ (M1)
 - Substitute result into $f(x)$: $f(\frac{1}{2}) = 3(\frac{1}{2}) + 4 = \frac{11}{2}$ (A1)
- (b)
- Find expression for $gf(x)$: $gf(x) = \frac{2}{(3x+4)+3} = \frac{2}{3x+7}$ (M1)
 - Set equal to 6: $\frac{2}{3x+7} = 6$ (M1)
 - Rearrange: $2 = 18x + 42 \Rightarrow 18x = -40$ (M1)
 - Final answer: $x = -\frac{20}{9}$ (A1)

(Total for question 1 is 6 marks)

2

- (a)
- Recognise $f(x) = (x+1)^2$ (M1)
 - Range: $f(x) \geq 0$ (A1)

(b)



- Correct shape for $f(x)$ starting at $(-1, 0)$ (B1)
 - Correct shape for $f^{-1}(x)$ starting at $(0, -1)$ (B1)
 - Reflection in $y = x$ shown (B1)
- (c)
- Set $y = (x+1)^2$ and swap variables: $x = (y+1)^2$ (M1)
 - Rearrange for y : $\sqrt{x} = y+1 \Rightarrow y = \sqrt{x} - 1$ (M1)
 - $f^{-1}(x) = \sqrt{x} - 1$ (A1)
 - Domain: $x \geq 0$ (B1)

(Total for question 2 is 8 marks)

3

- (a)
- $f(1) = 2 + \ln(2(1) - 1) = 2 + \ln(1) = 2$ (M1)
 - $f(2) = 2 + \ln(2(2) - 1) = 2 + \ln 3$ (A1)
- (b)
- $y = 2 + \ln(2x - 1) \Rightarrow x = 2 + \ln(2y - 1)$ (M1)
 - $x - 2 = \ln(2y - 1) \Rightarrow e^{x-2} = 2y - 1$ (M1)
 - $y = \frac{e^{x-2} + 1}{2}$ (A1)

(Total for question 3 is 5 marks)

4

- (a)
- $f(x) > 0$ (B1)
- (b)
- $gf(x) = 2(e^x) + \ln(e^x)$ (M1)
 - $gf(x) = 2e^x + x$ (A1)
- (c)
- $gf(x) \in \mathbb{R}$ (B1)

(Total for question 4 is 4 marks)

5

- (a)
- $f(x) \in \mathbb{R}, f(x) \neq 0$ (B2)
- (b)
- $y = \frac{1}{x+2} \Rightarrow x = \frac{1}{y+2}$ (M1)
 - $y + 2 = \frac{1}{x} \Rightarrow y = \frac{1}{x} - 2$ (A1)
 - Domain: $x \in \mathbb{R}, x \neq 0$ (B1)
- (c)
- $fg(x) = \frac{1}{(x^2-5)+2} = \frac{1}{x^2-3}$ (M1)
 - $\frac{1}{x^2-3} = \frac{1}{2} \Rightarrow x^2 - 3 = 2$ (M1)
 - $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$ (A1)

(Total for question 5 is 8 marks)

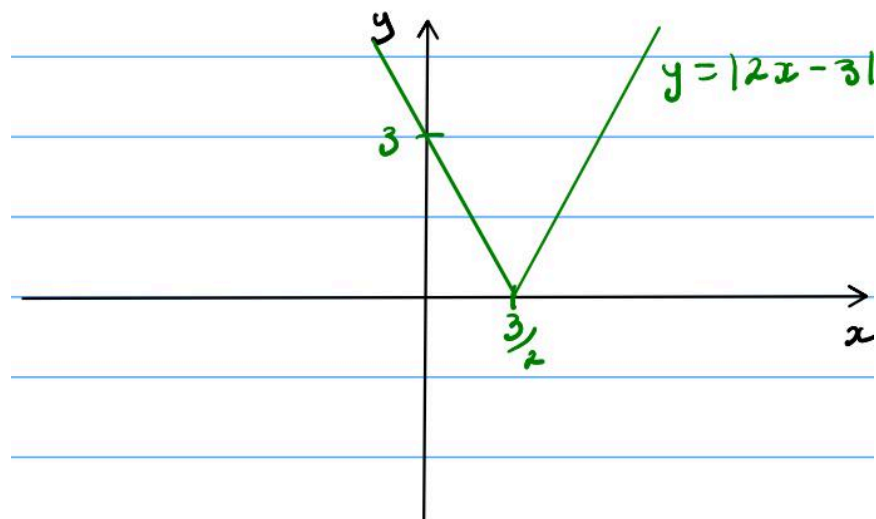
6

- (a)
- Complete the square: $(x + 2)^2 - 4 + 1 = (x + 2)^2 - 3$ (M1)
 - Identify minimum point at $(-2, -3)$ (M1)
 - Range: $f(x) \geq -3$ (A1)
- (b)
- It is a many-to-one function. (B1)

(Total for question 6 is 4 marks)

7

(a)



- V-shape with vertex on x-axis (B1)
- Intercepts at $(1.5, 0)$ and $(0, 3)$ (B1)

(b)

- $2x - 3 = 9 \Rightarrow 2x = 12 \Rightarrow x = 6$ (M1)
- $-(2x - 3) = 9 \Rightarrow 2x - 3 = -9 \Rightarrow 2x = -6 \Rightarrow x = -3$ (M1)
- Range: $-3 < x < 6$ (A1)

(c)

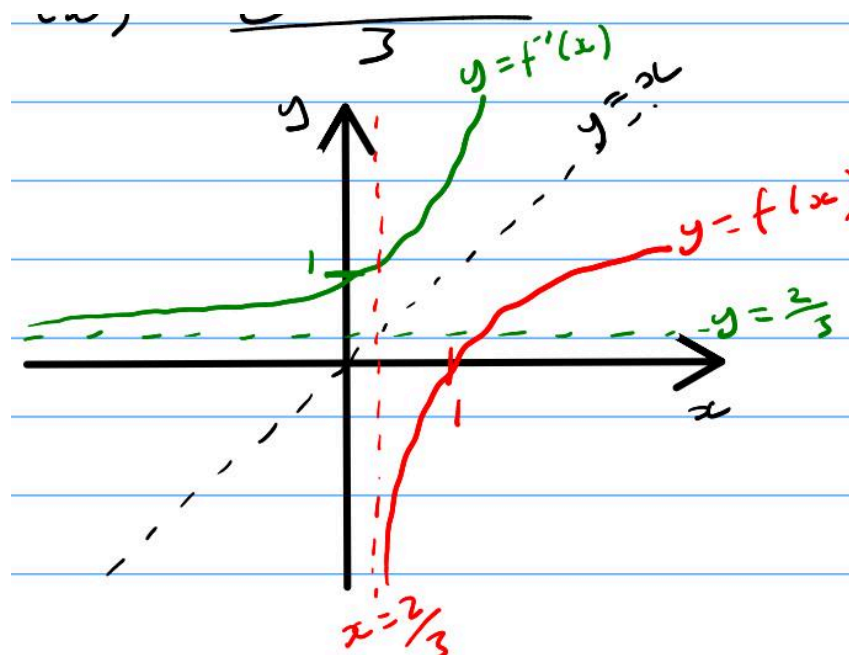
- $2x - 3 = x + 1 \Rightarrow x = 4$ (M1)
- $-(2x - 3) = x + 1 \Rightarrow -2x + 3 = x + 1 \Rightarrow 2 = 3x \Rightarrow x = \frac{2}{3}$ (M1)
- Range: $\frac{2}{3} < x < 4$ (A1)

(Total for question 7 is 6 marks)

- (a)
- $g(3) = \frac{3}{3-2} = 3$ (M1)
 - $f(3) = \ln(3(3) - 2) = \ln 7$ (A1)

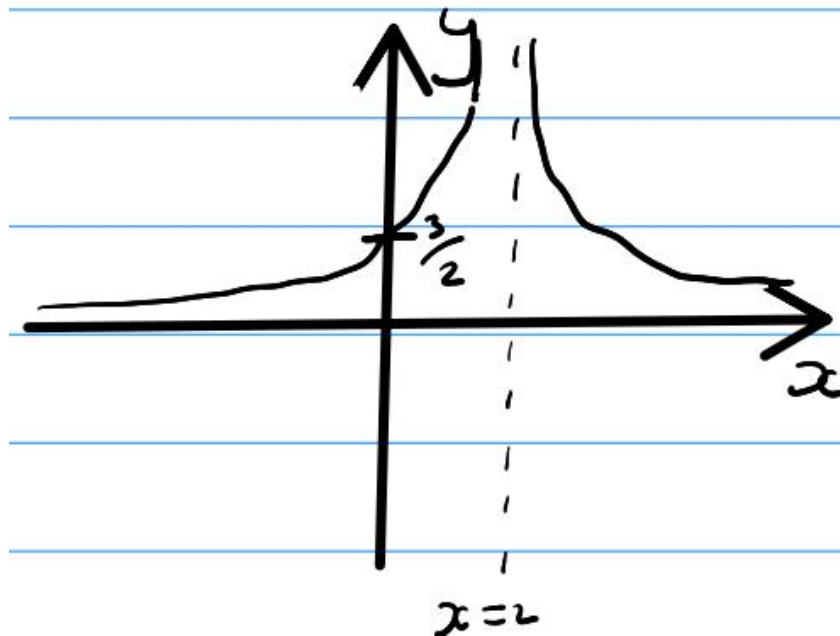
- (b)
- $y = \ln(3x - 2) \Rightarrow x = \ln(3y - 2)$ (M1)
 - $e^x = 3y - 2 \Rightarrow y = \frac{e^x + 2}{3}$ (M1)
 - $f^{-1}(x) = \frac{e^x + 2}{3}$ (A1)
 - Domain: $x \in \mathbb{R}$ (B1)

(c)



- $f(x)$ with asymptote $x = 2/3$ and intercept $(1, 0)$ (B1)
- $f^{-1}(x)$ with asymptote $y = 2/3$ and intercept $(0, 1)$ (B1)
- Reflection in $y = x$ (B1)

(d)



- Asymptote at $x = 2$ (B1)
- Both branches above x-axis (B1)
- Intercept at $(0, 1.5)$ (B1)

(e)

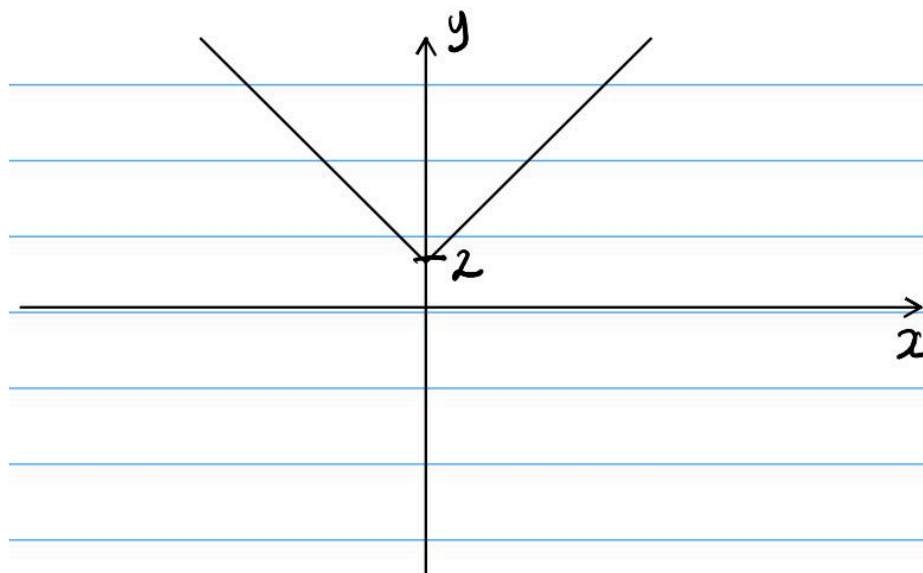
- $\frac{3}{x-2} = 4 \Rightarrow 3 = 4x - 8 \Rightarrow 11 = 4x \Rightarrow x = \frac{11}{4}$ (M1)
- $\frac{3}{x-2} = -4 \Rightarrow 3 = -4x + 8 \Rightarrow -5 = -4x \Rightarrow x = \frac{5}{4}$ (M1)
- Final answers: $x = \frac{11}{4}, x = \frac{5}{4}$ (A1)

(Total for question 8 is 15 marks)

9

- (a)
- $f(4) = \frac{3(4)+1}{4-2} = \frac{13}{2}$ (M1)
 - $f(\frac{13}{2}) = \frac{3(\frac{13}{2})+1}{(\frac{13}{2})-2} = \frac{41/2}{9/2} = \frac{41}{9}$ (A1)
- (b)
- At $x = 4, f(x) = 6.5$. As $x \rightarrow \infty, f(x) \rightarrow 3$. (M1)
 - Range: $3 < f(x) \leq \frac{13}{2}$ (A1)
- (c)
- $y = \frac{3x+1}{x-2} \Rightarrow x = \frac{3y+1}{y-2}$ (M1)
 - $xy - 2x = 3y + 1 \Rightarrow y(x - 3) = 1 + 2x \Rightarrow y = \frac{1+2x}{x-3}$ (M1)
 - $f^{-1}(x) = \frac{1+2x}{x-3}$ with domain $3 < x \leq \frac{13}{2}$ (A1)

(Total for question 9 is 6 marks)



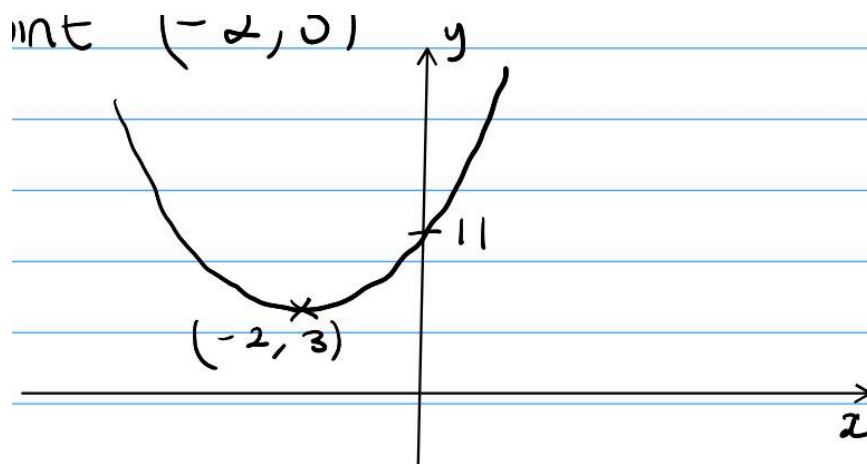
- (i) V-shape vertex at $(0, 2)$ (B1)
- (ii) When $x \geq 0$, $|x| + 2 = |x + 2|$. When $x < 0$, e.g., $x = -2$, $|-2| + 2 = 4$ but $|-2 + 2| = 0$, so $|x| + 2 > |x + 2|$. (B2)

(Total for question 10 is 3 marks)

11

- (a)
- $2(x^2 + 4x) + 11$ (M1)
 - $2[(x + 2)^2 - 4] + 11$ (M1)
 - $2(x + 2)^2 + 3$ (A1)

(b)



- Turning point at $(-2, 3)$ (B1)
 - y-intercept at $(0, 11)$ (B1)
 - Correct parabola shape (B1)
- (c)
- $f(x - 2) = 2((x - 2) + 2)^2 + 3 = 2x^2 + 3$ (incorrect logic in source solution, source says translation 2 right) (M1)
 - Translation by vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (A1)
- (d)
- Max value when denominator is min: $\frac{18}{3} = 6$ (M1)
 - As $x \rightarrow \infty, h(x) \rightarrow 0$ (M1)
 - Range: $0 < h(x) \leq 6$ (A1)

(Total for question 11 is 10 marks)

12

- (a)
- Set $f(x) = 4$: $4 = \frac{3x-5}{x-2}$ (M1)
 - $4x - 8 = 3x - 5 \Rightarrow x = 3$ (A1)
- (b)
- $f\left(\frac{3x-5}{x-2}\right) = \frac{3\left(\frac{3x-5}{x-2}\right)-5}{\left(\frac{3x-5}{x-2}\right)-2}$ (M1)
 - Multiply numerator and denominator by $(x - 2)$: $\frac{3(3x-5)-5(x-2)}{3x-5-2(x-2)}$ (M1)
 - $\frac{9x-15-5x+10}{3x-5-2x+4} = \frac{4x-5}{x-1}$ (A1)

(Total for question 12 is 5 marks)

- 13
- $y = e^{x+1} \Rightarrow x = e^{y+1}$ (M1)
 - $\ln x = y + 1 \Rightarrow y = \ln x - 1$ (A1)
 - Domain: $x > 0$ (B1)

(Total for question 13 is 3 marks)

- 14
- (a)
- $g(0) = 5 - 3(0) = 5$ (M1)
 - $g(5) = (5 - 1)^2 + 2 = 18$ (A1)
- (b)
- $5 - 3x = 18 \Rightarrow -13 = 3x \Rightarrow x = -13/3$ (M1)
 - $(x - 1)^2 + 2 = 18 \Rightarrow (x - 1)^2 = 16 \Rightarrow x - 1 = 4 \Rightarrow x = 5$ (M1)
 - $x < -13/3$ or $x > 5$ (A2)
- (c)
- h is a one-to-one function but g is a many-to-one function. (B1)
- (d)
- $h(2) = (2 - 1)^2 + 2 = 3$ (M1)
 - $x = 3$ (A2)

(Total for question 14 is 10 marks)

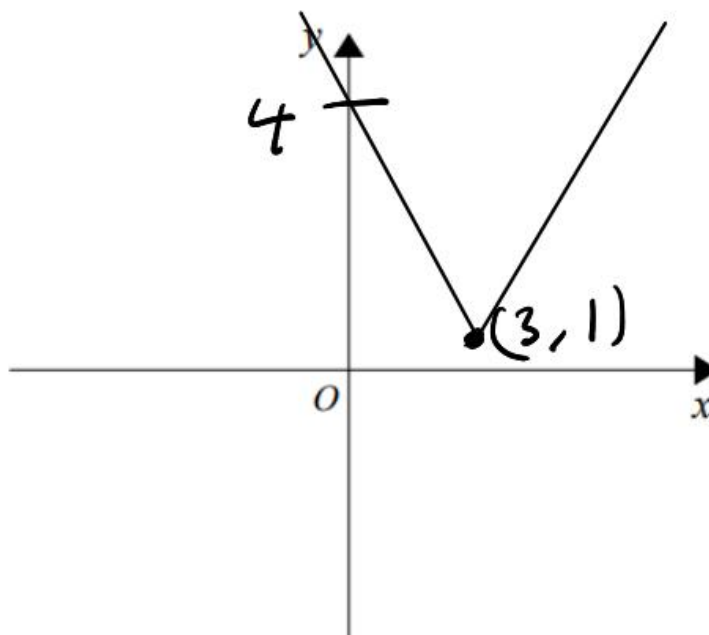
- 15
- (a)
- $y = \frac{2x+1}{x-1} \Rightarrow x = \frac{2y+1}{y-1}$ (M1)
 - $xy - x = 2y + 1 \Rightarrow y(x - 2) = 1 + x$ (M1)
 - $f^{-1}(x) = \frac{x+1}{x-2}$ (A1)
- (b)
- $f\left(\frac{2x+1}{x-1}\right) = \frac{2\left(\frac{2x+1}{x-1}\right)+1}{\left(\frac{2x+1}{x-1}\right)-1} = \frac{2(2x+1)+(x-1)}{2x+1-(x-1)} = \frac{5x+1}{x+2}$ (B1)

(Total for question 15 is 4 marks)

- 16
- (a)
- Vertex occurs when $x - 3 = 0 \Rightarrow x = 3$ (M1)
 - $y = 2(0) - 5 = -5$. $P = (3, -5)$ (A1)
- (b)
- $10 - x = 2(x - 3) - 5 \Rightarrow 10 - x = 2x - 11 \Rightarrow 21 = 3x \Rightarrow x = 7$ (M1)
 - $10 - x = -2(x - 3) - 5 \Rightarrow 10 - x = -2x + 1 \Rightarrow x = -9$ (M1)
- (c)
- Gradient of line through $(0, 0)$ and $(3, -5)$ is $m = -5/3$ (M1)
 - Gradient of left branch is -2 (M1)
 - Range: $\{a : -2 < a < -5/3\}$ (A1)

(Total for question 16 is 7 marks)

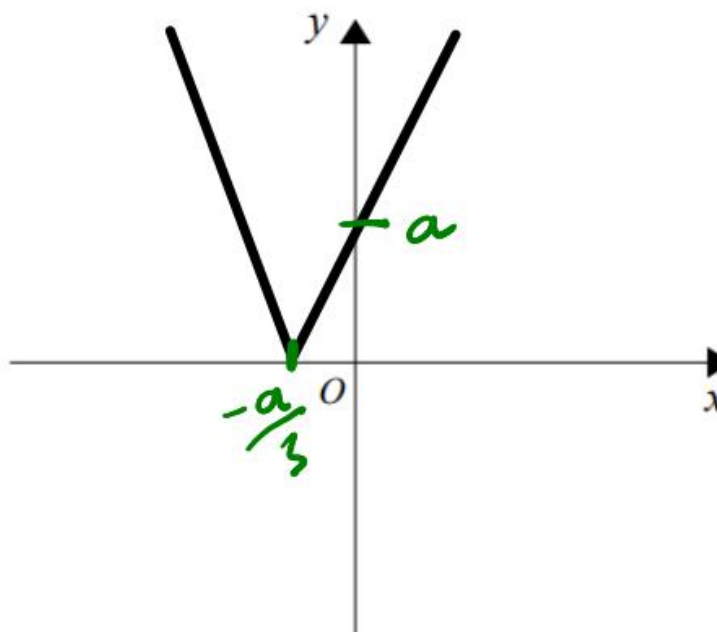
17



- V-shape with vertex at $(3, 1)$ (B1)
- y-intercept at $(0, 4)$ (B1)

(Total for question 17 is 2 marks)

18



- V-shape with vertex at $(-a/3, 0)$ (B2)
- y-intercept at $(0, a)$ (B1)

(Total for question 18 is 3 marks)

19

- (a) • $f(x) \geq 5$ (B1)
- (b) • $y = 2x^2 + 5 \Rightarrow y - 5 = 2x^2 \Rightarrow x = \sqrt{\frac{y-5}{2}}$ (M2)
- $f^{-1}(x) = \sqrt{\frac{x-5}{2}}$ (A1)
- (c) • $f^{-1}(x) \geq 0$ (B1)

(Total for question 19 is 5 marks)

20

- $-1 \leq x \leq 1$ (B2)

(Total for question 20 is 2 marks)

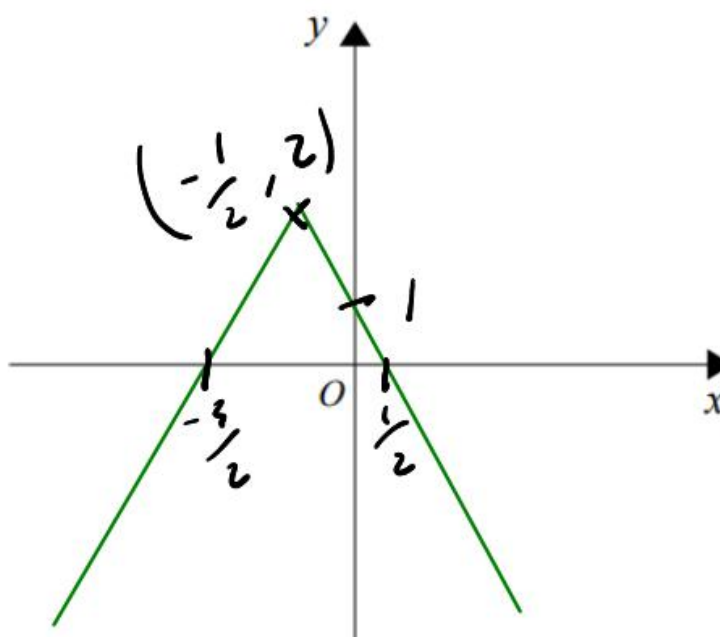
21

- $x + 2 > 0 \Rightarrow x > -2$ (B2)

(Total for question 21 is 2 marks)

22

(a)



- Inverted V-shape with vertex at $(-0.5, 2)$ (B1)
 - x-intercepts at -1.5 and 0.5 (B1)
 - y-intercept at $(0, 1)$ (B1)
- (b) • From graph, find x-intercepts: $2 - |2x + 1| = 0 \Rightarrow |2x + 1| = 2$ (M1)
- $2x + 1 = 2 \Rightarrow x = 0.5$; $2x + 1 = -2 \Rightarrow x = -1.5$ (M1)
- Range: $-1.5 < x < 0.5$ (A1)

(Total for question 22 is 5 marks)

23

- (a)
- Complete square: $2(x - 3/4)^2 - 9/8$ (M1)
 - Min value is $-9/8$ at $x = 3/4$. Max value at $x = 4$: $2(16) - 12 = 20$ (M1)
 - Range: $-9/8 \leq f(x) \leq 20$ (A1)
- (b)
- No, because it is a many-to-one function on the given domain. (B2)

(Total for question 23 is 5 marks)

24

- (i)
- $f(x) = (x - 3)^2 - 9 + c$ (M1)
 - Min value is $c - 9$. Set $c - 9 = 2$ (M1)
 - $c = 11$ (A1)
- (ii)
- $f(2) = 2^2 - 6(2) + c = c - 8$ (M1)
 - $f(c - 8) = (c - 8)^2 - 6(c - 8) + c = 12$ (M1)
 - $c^2 - 16c + 64 - 6c + 48 + c = 12 \Rightarrow c^2 - 21c + 100 = 0$ (M1)
 - $c = \frac{21 \pm \sqrt{441 - 400}}{2} = \frac{21 \pm \sqrt{41}}{2}$ (A1)

(Total for question 24 is 7 marks)

25

- $x + 4 = 2x + 3 \Rightarrow x = 1$ (M1)
- $x + 4 = -(2x + 3) \Rightarrow 3x = -7 \Rightarrow x = -7/3$ (M1)
- Final answers: $x = 1, x = -7/3$ (A1)

(Total for question 25 is 3 marks)

26

- (a)
- $fg(x) = (x^2 - 1)^3$ (B1)
 - $gf(x) = (x^3)^2 - 1 = x^6 - 1$ (B1)
- (b)
- $x^6 - 1 - (x^2 - 1)^3 = 18$ (M1)
 - Expand $(x^2 - 1)^3 = x^6 - 3x^4 + 3x^2 - 1$ (M1)
 - $x^6 - 1 - (x^6 - 3x^4 + 3x^2 - 1) = 18 \Rightarrow 3x^4 - 3x^2 = 18$ (M1)
 - $x^4 - x^2 - 6 = 0 \Rightarrow (x^2 - 3)(x^2 + 2) = 0$ (M1)
 - $x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ (A2)

(Total for question 26 is 8 marks)

27

- (a) • Vertex is at $x = 2$, so $k = 2$ (B1)
- (b) • $f(5) = (5 - 2)^2 - 9 = 0$ (M1)
• $f(0) = (0 - 2)^2 - 9 = -5$ (A1)
- (c) • $(x - 2)^2 - 9 = x \Rightarrow x^2 - 4x + 4 - 9 = x$ (M1)
• $x^2 - 5x - 5 = 0$ (M1)
• $x = \frac{5 + \sqrt{25 - 4(-5)}}{2} = \frac{5 + 3\sqrt{5}}{2}$ (rejecting negative root as $x \geq 2$) (A1)
- (d) • The graphs of $f(x)$ and $f^{-1}(x)$ reflect in the line $y = x$. Any point where $f(x)$ crosses $y = x$, $f^{-1}(x)$ will also cross $y = x$. (B1)

(Total for question 27 is 7 marks)

28

- (a) • $x - 1 = 2x + 4 \Rightarrow x = -5$ (M1)
• $x - 1 = -(2x + 4) \Rightarrow 3x = -3 \Rightarrow x = -1$ (M1)
• Test regions or use graph: $x \leq -5$ or $x \geq -1$ (A2)
- (b) • 1. Stretch with scale factor $1/2$ in the x direction (B1)
• 2. Translation by vector $\begin{pmatrix} -2.5 \\ 0 \end{pmatrix}$ (B2)

(Total for question 28 is 7 marks)

29

- (a) • $f(x) = 2(x - 3/4)^2 - 9/8$ (M2)
• $f(x) \geq -9/8$ (A1)
- (b) • It is a many-to-one function. (B1)
- (c) • $f(2) = 2(4) - 6 = 2$ (M1)
• $gf(2) = g(2) = 4(2) + 6 = 14$ (M1)
• $g^{-1}(a) = 14 \Rightarrow a = g(14) = 4(14) + 6 = 62$ (A1)
- (d) • $2x^2 - 3x = 4x + 6 \Rightarrow 2x^2 - 7x - 6 = 0$ (M1)
• $x = \frac{7 \pm \sqrt{49 - 4(2)(-6)}}{4} = \frac{7 \pm \sqrt{97}}{4}$ (M1)
• $\{x : x < \frac{7 - \sqrt{97}}{4}\} \cup \{x : x > \frac{7 + \sqrt{97}}{4}\}$ (A1)

(Total for question 29 is 10 marks)

30

- (a) • $(0, 2)$ and $(2/3, 0)$ (B2)
- (b) • (i) Gradient must be less than 3. Line through $(2/3, 0)$ and $(0, 1)$ has gradient $a = -3/2$. (M2)
- Range: $-3/2 < a < 3$ (A2)
- (ii) $3x - 2 = ax + 1 \Rightarrow x(3 - a) = 3 \Rightarrow x = \frac{3}{3-a}$ (M1)
- $-(3x - 2) = ax + 1 \Rightarrow 1 = x(a + 3) \Rightarrow x = \frac{1}{a+3}$ (A2)

(Total for question 30 is 9 marks)

31

- (a) • $y = \frac{e^x}{2-e^x} \Rightarrow x = \frac{e^y}{2-e^y}$ (M1)
- $2x - xe^y = e^y \Rightarrow 2x = e^y(1 + x)$ (M1)
- $e^y = \frac{2x}{1+x} \Rightarrow y = \ln\left(\frac{2x}{1+x}\right)$ (A1)
- (b) • $f^{-1}(x) \in \mathbb{R}, f^{-1}(x) \neq \ln 2$ (B1)

(Total for question 31 is 4 marks)

32

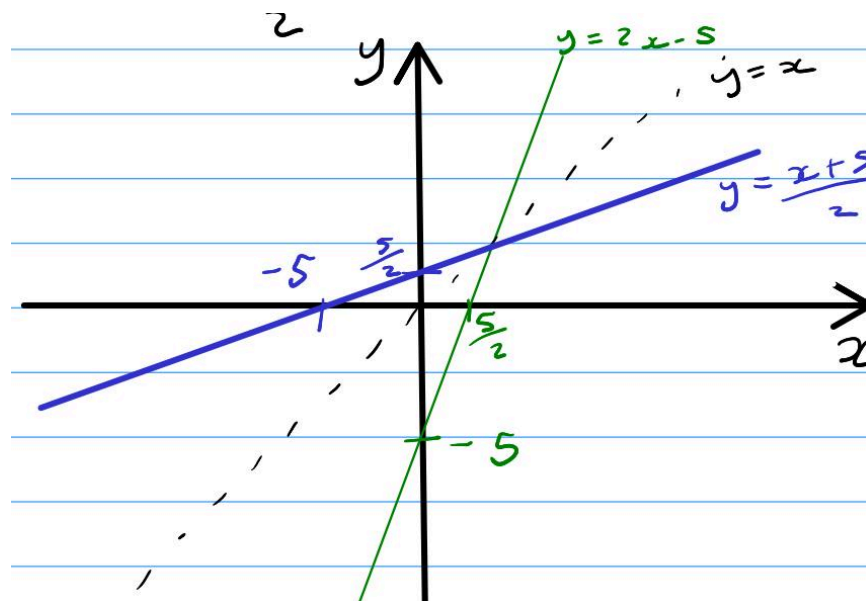
- (a) • $gf(x) = 2(x^2 - 4x - 5) + 1 = 2x^2 - 8x - 9$ (M1)
- Domain: $x > 2$ (B2)
- (b) • At $x = 2, gf(2) = 2(4) - 16 - 9 = -17$. (M1)
- Range: $gf(x) > -17$ (A1)
- (c) • $f(x) = (x - 2)^2 - 9 \Rightarrow x = (y - 2)^2 - 9 \Rightarrow y = 2 + \sqrt{x + 9}$ (M2)
- $gf^{-1}(x) = 2(2 + \sqrt{x + 9}) + 1 = 5 + 2\sqrt{x + 9}$ (M2)
- Domain: $x > -9$ (A1)

(Total for question 32 is 9 marks)

33

- (a)
- $y = 2x - 5 \Rightarrow x = \frac{y+5}{2}$ (M1)
 - $f^{-1}(x) = \frac{x+5}{2}$ (A1)

(b)



- Both lines drawn correctly (B1)
 - Intersection at (5, 5) shown (B1)
- (c)
- $2x - 5 = x \Rightarrow x = 5$ (M1)
 - $x > 5$ (A1)

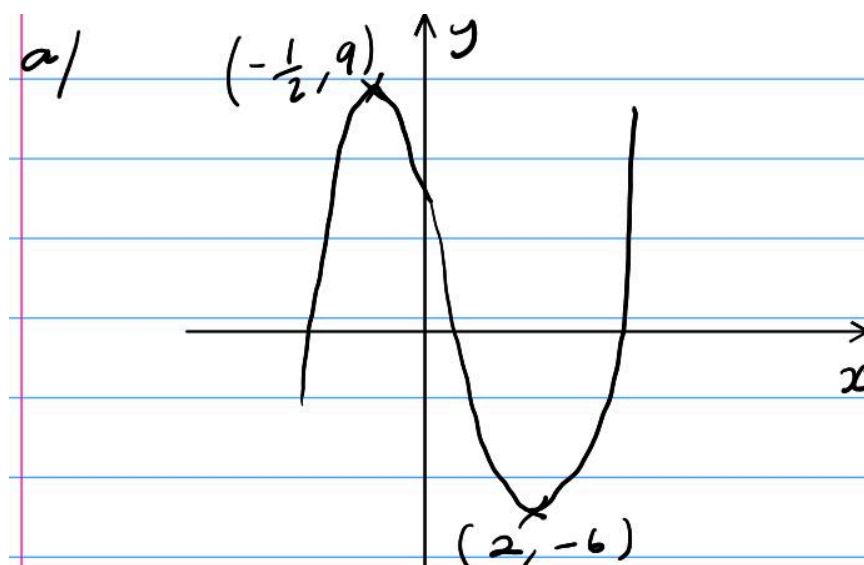
(Total for question 33 is 6 marks)

34

- $|x - 2| - 3 = 0 \Rightarrow x = 5, x = -1$ (M2)
- $\{x : x > -1\} \cap \{x : x < 5\}$ (A2)

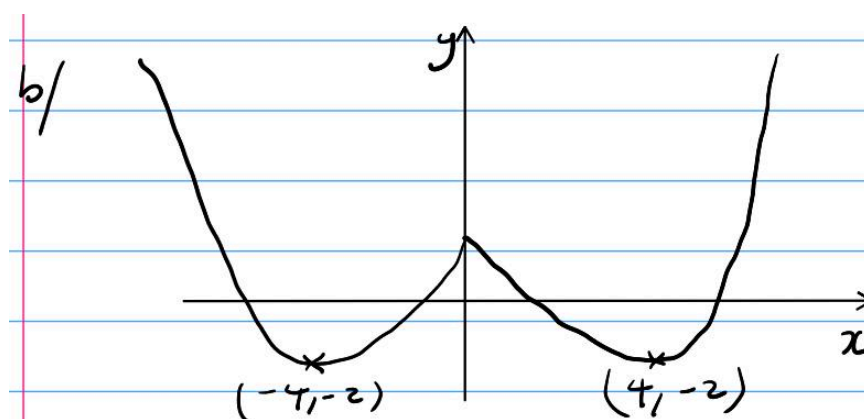
(Total for question 34 is 4 marks)

(a)



- Max point at $(-0.5, 9)$ (B1)
- Min point at $(2, -6)$ (B1)
- Correct shape (B1)

(b)



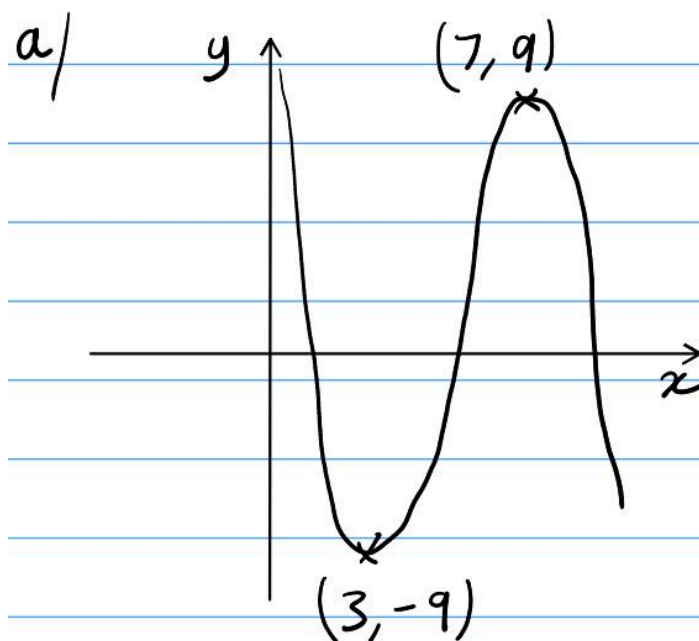
- Symmetric about y-axis (B1)
- Min points at $(4, -2)$ and $(-4, -2)$ (B2)

(c)

- Original min at $(4, -2)$. To move to $(0, 0)$, shift 4 left and 2 up. (M1)
- $a = 4, b = 2$ (A1)

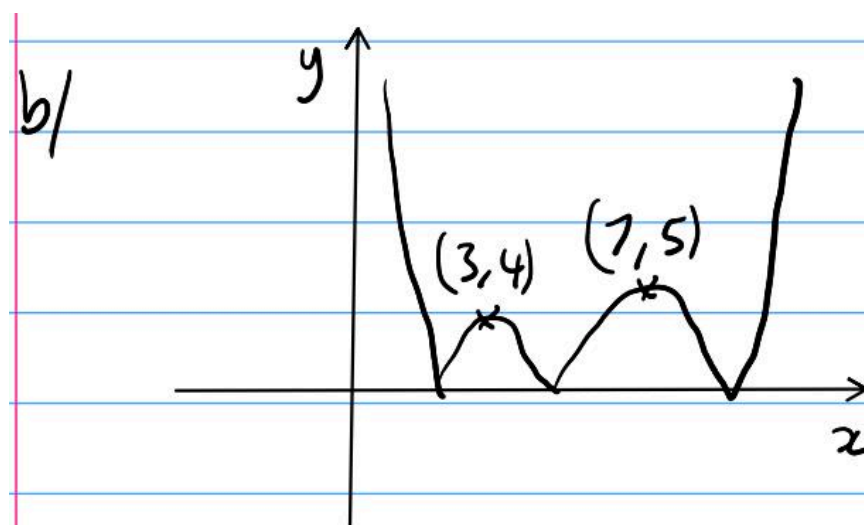
(Total for question 35 is 8 marks)

(a)



- Min point at $(3, -9)$ (B1)
- Max point at $(7, 9)$ (B1)
- Correct shape (B1)

(b)



- Negative parts reflected above x-axis (B1)
- Turning points at $(3, 4)$ and $(7, 5)$ (B2)

(Total for question 36 is 6 marks)