

1 The functions f and g are defined by

$$f : x \rightarrow 3x + 4, \quad x \in \mathbb{R}$$

$$g : x \rightarrow \frac{2}{x+3}, \quad x \in \mathbb{R}, x \neq -3$$

(a) Evaluate $fg(1)$

(2)

(b) Solve the equation $gf(x) = 6$

(4)

(Total for question 1 is 6 marks)

2 The function f is defined by

$$f(x) = x^2 + 2x + 1, \quad x \in \mathbb{R}, x \geq -1$$

(a) State the range of f

(1)

(b) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same diagram

(3)

(c) Find an expression for $f^{-1}(x)$ and state its domain

(4)

(Total for question 2 is 8 marks)

3 The function f is defined by

$$f(x) = 2 + \ln(2x - 1), \quad x \in \mathbb{R}, x > 0.5$$

(a) Find the exact value of $ff(1)$

(2)

(b) Find an expression for $f^{-1}(x)$

(3)

(Total for question 3 is 5 marks)

4 The functions f and g are defined by

$$f : x \rightarrow e^x, \quad x \in \mathbb{R}$$

$$g : x \rightarrow 2x + \ln x, \quad x \in \mathbb{R}, x > 0$$

(a) Write down the range of f

(1)

(b) Find an expression for the composite function gf

(2)

(c) Write down the range of gf

(1)

(Total for question 4 is 4 marks)

5 The function f is defined by

$$f(x) = \frac{1}{x+2}, \quad x \in \mathbb{R}, x \neq -2$$

(a) Write down the range of $f(x)$

(2)

(b) Find an expression for $f^{-1}(x)$ and state its domain

(3)

(c) $g(x) = x^2 - 5, \quad x \in \mathbb{R}$

Solve $fg(x) = \frac{1}{2}$

(3)

(Total for question 5 is 8 marks)

6 The function f is defined by

$$f(x) = x^2 + 4x + 1, \quad x \in \mathbb{R}$$

(a) Find the range of $f(x)$

(3)

(b) Explain why the function $f(x)$ does not have an inverse

(1)

(Total for question 6 is 4 marks)

7

- (a) Sketch the graph with equation

$$y = |2x - 3|$$

stating the coordinates where the graph meets the coordinate axis.

(2)

- (b) Find the values of x which satisfy $|2x - 3| < 9$

(2)

- (c) Find the values of x which satisfy $|2x - 3| < x + 1$

(2)

(Total for question 7 is 6 marks)

8 The functions f and g are defined by

$$f : x \rightarrow \ln(3x - 2), \quad x \in \mathbb{R}, x > \frac{2}{3}$$

$$g : x \rightarrow \frac{3}{x - 2}, \quad x \in \mathbb{R}, x \neq 2$$

(a) Find the exact value of $fg(3)$

(2)

(b) Find an expression for $f^{-1}(x)$ and state its domain

(4)

(c) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same diagram

(3)

(d) Sketch the graph of $y = |g(x)|$

(3)

(e) Find the exact values of $|\frac{3}{x-2}| = 4$

(3)

(Total for question 8 is 15 marks)

9

$$f(x) = \frac{3x+1}{x-2} \quad x \geq 4$$

(a) Find $ff(4)$

(2)

(b) State the range of f .

(1)

(c) Find $f^{-1}(x)$ and state its domain

(3)

(Total for question 9 is 6 marks)

- 10** (i) Sketch the graph of $y = |x| + 2$ (ii) Explain why $|x| + 2 \geq |x + 2|$ for all real values of x .

(Total for question 10 is 3 marks)

11 $f(x) = 2x^2 + 8x + 11 \quad x \in \mathbb{R}$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

- (c) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 8x - 5 \quad x \in \mathbb{R}$$

(2)

- (d) Find the range of the function

$$h(x) = \frac{18}{2x^2 + 8x + 11} \quad x \in \mathbb{R}$$

(2)

(Total for question 11 is 10 marks)

12 The function f is defined by

$$f(x) = \frac{3x - 5}{x - 2}, \quad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(4)$

(2)

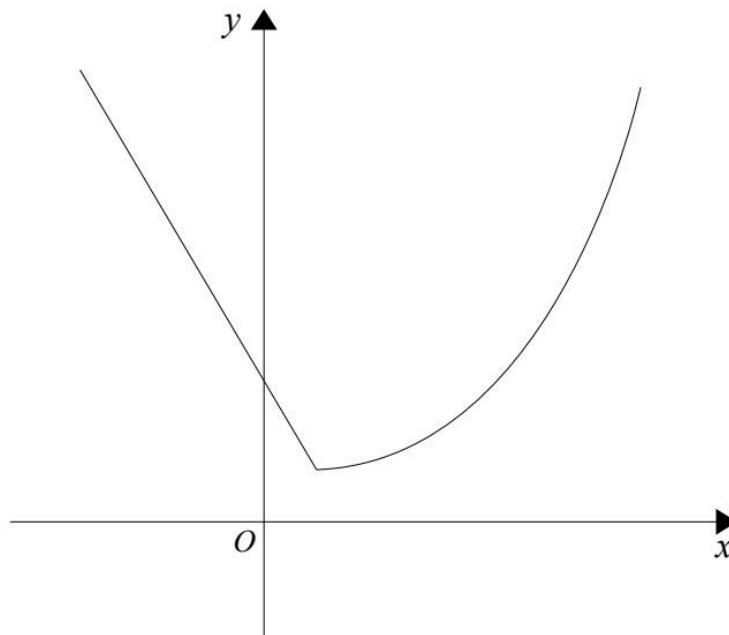
(b) Show that $ff(x) = \frac{ax+b}{x-1}$ where a and b are integers to be found.

(3)

(Total for question 12 is 5 marks)

- 13** The function f is defined by $f(x) = e^{x+1}$, $x \in \mathbb{R}$
Find $f^{-1}(x)$ and state its domain.

(Total for question 13 is 3 marks)



The diagram shows a sketch of $y = g(x)$, where

$$g(x) = \begin{cases} 5 - 3x & x < 1 \\ (x - 1)^2 + 2 & x \geq 1 \end{cases}$$

- (a) Find the value of $gg(0)$

(2)

(b) Find all the values of x for which $g(x) > 18$

(4)

(c) The function h is defined as $h(x) = (x - 1)^2 + 2$ $x \geq 1$. Explain why h has an inverse but g does not.

(1)

(d) Solve the equation $h^{-1}(x) = 2$

(3)

15

$$f(x) = \frac{2x+1}{x-1} \quad x \neq 1$$

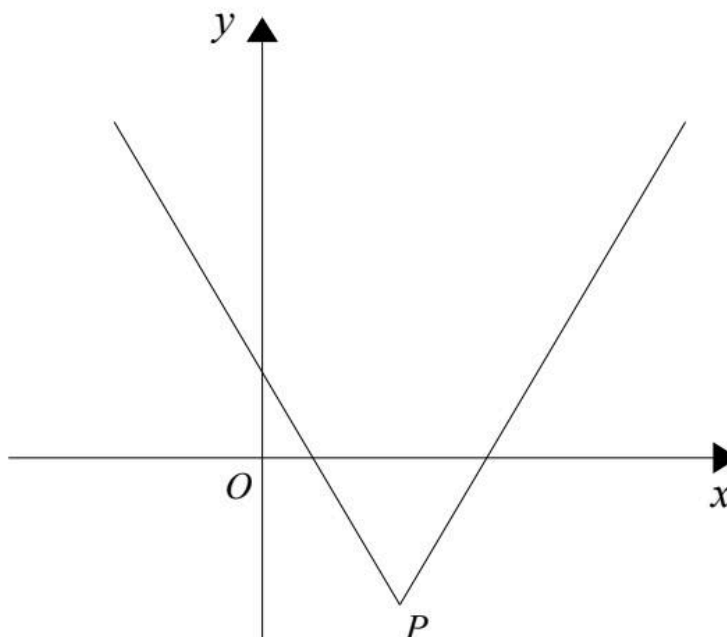
(a) Find $f^{-1}(x)$

(3)

(b) Find an expression for $ff(x)$

(1)

(Total for question 15 is 4 marks)



The diagram shows a sketch of $y = 2|x - 3| - 5$

The vertex of the graph is at point P .

(a) Find the coordinates of point P .

(2)

(b) Solve the equation $10 - x = 2|x - 3| - 5$

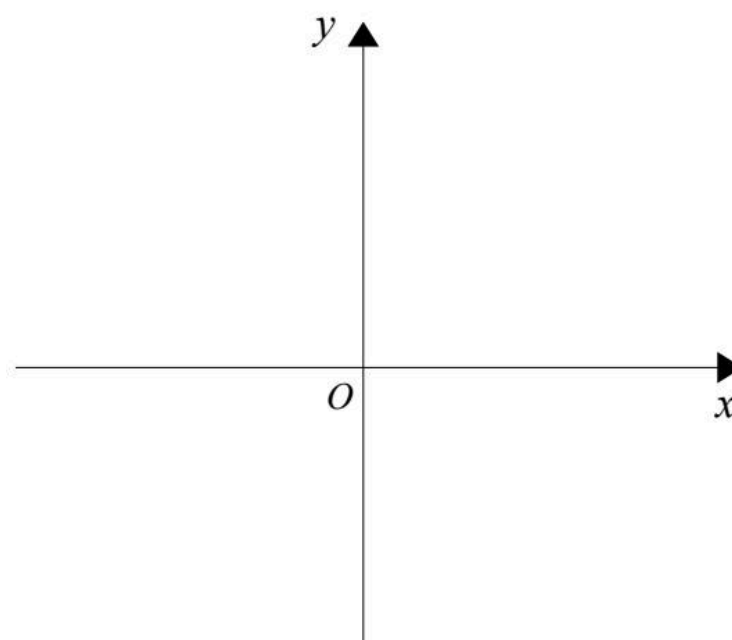
(2)

- (c) A line l has equation $y = ax$, where a is a constant. Given that l intersects $y = 2|x - 3| - 5$ at least once, find the range of possible values of a , writing your answer in set notation.

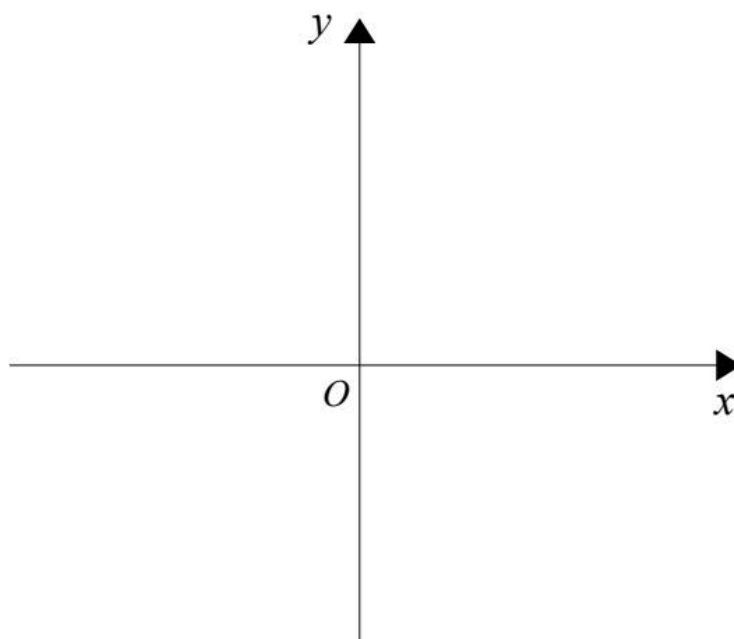
(3)

(Total for question 16 is 7 marks)

- 17 Sketch the graph of $y = 1 + |x - 3|$. Show clearly where the graph intersects the axes.



(Total for question 17 is 2 marks)



Sketch the graph of $y = |3x + a|$, where a is a positive constant. Show clearly where the graph intersects the axes.

(Total for question 18 is 3 marks)

19 The function f is defined by $f(x) = 2x^2 + 5, x \geq 0$

(a) Find the range of f .

(1)

(b) Find $f^{-1}(x)$

(3)

(c) State the range of $f^{-1}(x)$

(1)

(Total for question 19 is 5 marks)

20 $f(x) = \arcsin x$

State the maximum possible domain of f .

(Total for question 20 is 2 marks)

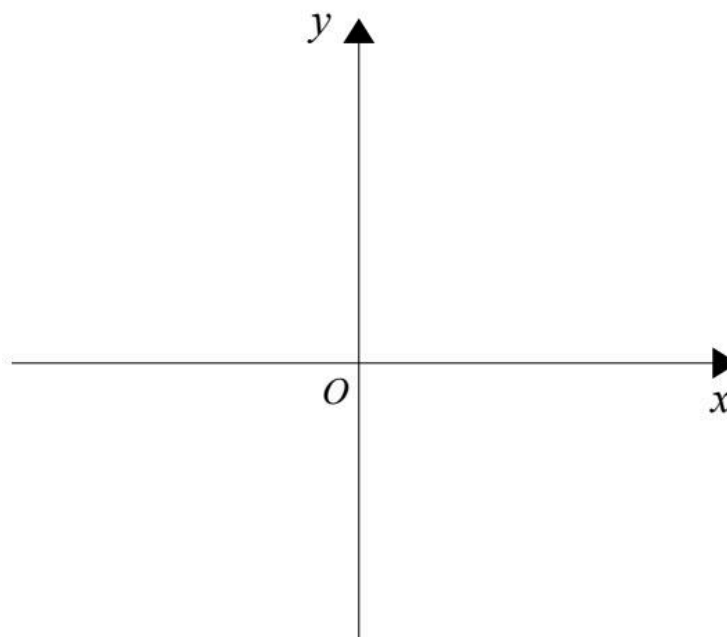
21 $f(x) = \frac{1}{\sqrt{x+2}}$

State the maximum possible domain of f .

(Total for question 21 is 2 marks)

22

(a) Sketch the graph of $y = 2 - |2x + 1|$



(b) Solve the inequality $2 - |2x + 1| > 0$

(3)

(2)

(Total for question 22 is 5 marks)

23 The function f is defined by $f(x) = 2x^2 - 3x$ $x \in \mathbb{R}, 0 \leq x \leq 4$

(a) Find the range of f .

(3)

(b) Determine whether f has an inverse. Fully justify your answer.

(2)

(Total for question 23 is 5 marks)

24 The function f is defined for all real values of x as $f(x) = x^2 - 6x + c$, where c is a constant.

(i) Given that the range of f is $f(x) \geq 2$, find the value of c .

(3)

(ii) Given instead that $ff(2) = 12$, find the possible values of c .

(4)

(Total for question 24 is 7 marks)

25 Solve the equation $|x + 4| = |2x + 3|$

(Total for question 25 is 3 marks)

26 The functions f and g are defined for all real values of x by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 - 1$$

(a) Write down expressions for $fg(x)$ and $gf(x)$

(2)

(b) Hence find the values of x for which $gf(x) - fg(x) = 18$

(6)

(Total for question 26 is 8 marks)

27 The function f is defined by $f(x) = (x - 2)^2 - 9$ for $x \geq k$, where k is a constant.

(a) Given that $f^{-1}(x)$ exists, state the least possible value of k .

(1)

(b) Evaluate $ff(5)$

(2)

(c) Solve the equation $f(x) = x$

(3)

(d) Explain why your answer to part (c) is also the solution to the equation $f(x) = f^{-1}(x)$

(1)

(Total for question 27 is 7 marks)

28

- (a) Solve the inequality $|x - 1| \leq |2x + 4|$

(4)

- (b) Give full details of a sequence of two transformations needed to transform the graph of $y = |x - 1|$ onto the graph of $y = |2x + 4|$

(3)

(Total for question 28 is 7 marks)

29 The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 - 3x \text{ and } g(x) = 4x + 6$$

(a) Find the range of f .

(3)

(b) Give a reason why f has no inverse.

(1)

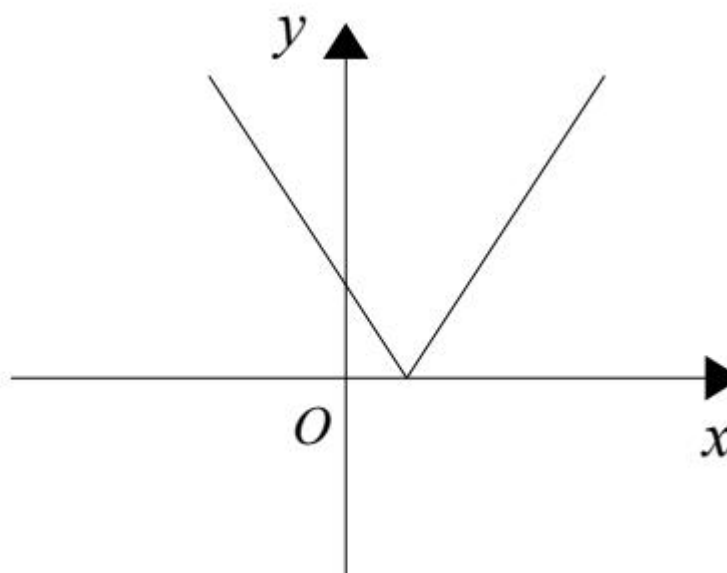
(c) Given that $gf(2) = g^{-1}(a)$, where a is a constant, determine the value of a .

(3)

(d) Determine the set of values for which $f(x) > g(x)$. Give your answer in set notation.

(3)

(Total for question 29 is 10 marks)



The graph shows a sketch of $y = |3x - 2|$

- (a) State the coordinates of the points of intersection with the axes.

(2)

- (b) Given that the graphs of $y = |3x - 2|$ and $y = ax + 1$ have two distinct points of intersection, determine (i) the set of possible values of a , (ii) the x coordinates of the points of intersection of these graphs, giving your answers in terms of a .

(7)

(Total for question 30 is 9 marks)

31 The function $f(x) = \frac{e^x}{2-e^x}$ is defined on the domain $x \in \mathbb{R}, x \neq \ln 2$

(a) Find $f^{-1}(x)$

(3)

(b) Write down the range of $f^{-1}(x)$

(1)

(Total for question 31 is 4 marks)

32 You are given that

$$f(x) = x^2 - 4x - 5 \quad \text{for } x > 2 \quad \text{and}$$

$$g(x) = 2x + 1$$

- (a) Find $gf(x)$ and state its domain

(3)

- (b) State the range of $gf(x)$.

(1)

(c) Find $gf^{-1}(x)$.

(5)

(Total for question 32 is 9 marks)

33 The function f is defined for all real values of x by

$$f(x) = 2x - 5$$

(a) Find an expression for $f^{-1}(x)$

(2)

(b) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram.

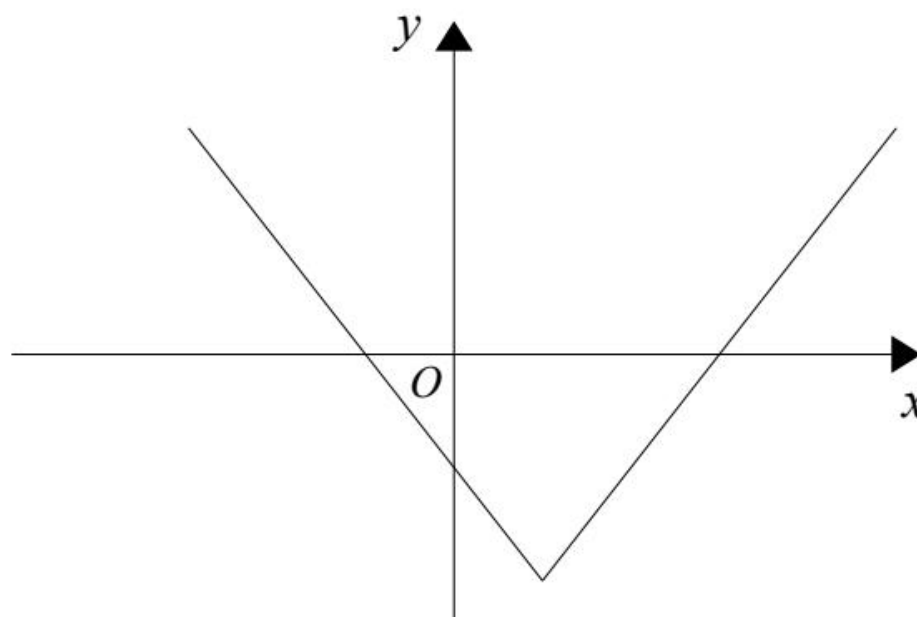
(2)

(c) Find the set of values for which $f(x) > f^{-1}(x)$.

(2)

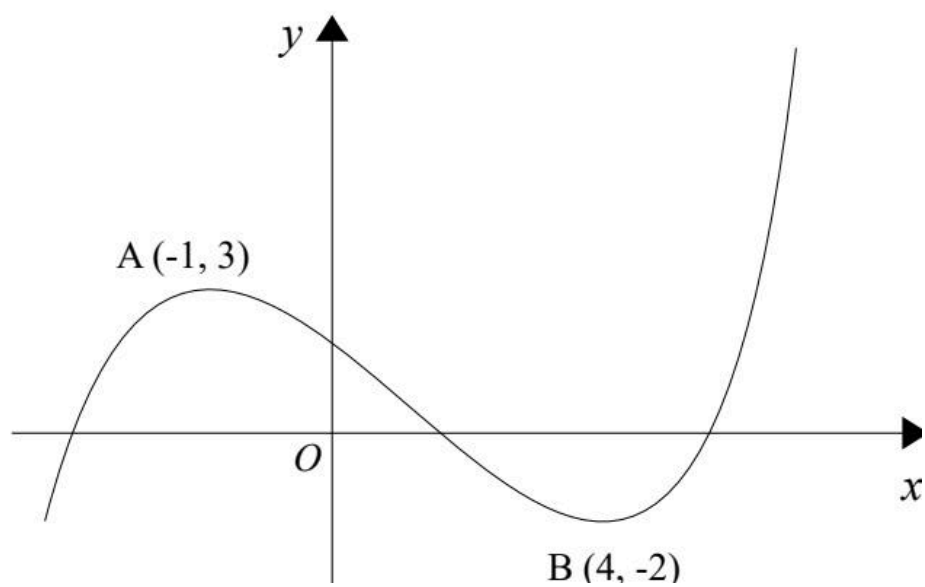
(Total for question 33 is 6 marks)

- 34** The graph shows a sketch of $y = |x - 2| - 3$



Find the set of values for which $|x - 2| < 3$

(Total for question 34 is 4 marks)



The curve has a maximum point A $(-1, 3)$ and a minimum point B $(4, -2)$. Showing the coordinates of any stationary points, on separate diagrams sketch the graphs of

(a) $y = 3f(2x)$

(3)

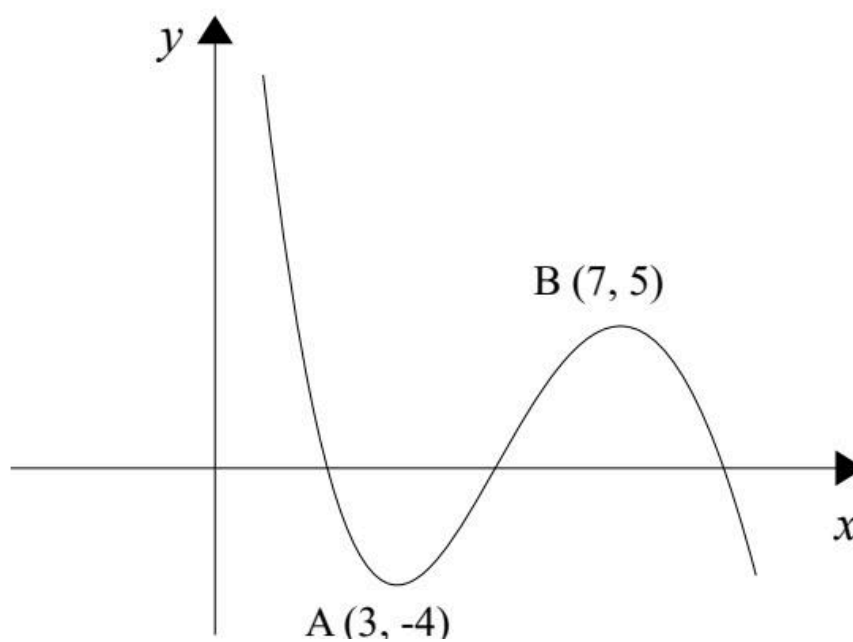
(b) $y = f(|x|)$

(3)

- (c) Write down the constants a and b such that the curve with equation $y = f(x + a) + b$ has a minimum point at the origin O .

(2)

(Total for question 35 is 8 marks)



The points A (3, -4) and B (7, 5) are the turning points of the graph. On separate diagrams sketch the graphs of

(a) $y = 2f(x) - 1$

(3)

(b) $y = |f(x)|$

(3)

(Total for question 36 is 6 marks)
