

- 1** A particle P moves with constant acceleration. At time $t = 0$, P is moving in the direction $(4\mathbf{i} + 3\mathbf{j})$ with speed 15 ms^{-1} . 6 seconds later P is moving with velocity $(-12\mathbf{i} + 6\mathbf{j}) \text{ ms}^{-1}$. Find the acceleration of P .

(Total for question 1 is 5 marks)

- 2** A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j}) \text{ ms}^{-2}$. At time $t = 0$, P has speed $u \text{ ms}^{-1}$. At time $t = 3 \text{ s}$, P has velocity $(-6\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$. Find the value of u .

(Total for question 2 is 5 marks)

- 3** A particle P moves in a straight line with constant velocity. Initially P is at the point A with position vector $(4\mathbf{i} - 3\mathbf{j})$ m. At time $t = 3$, P is at the point B with position vector $(-5\mathbf{i} + 9\mathbf{j})$ m.

(a) Find the velocity of P .

(3)

(b) When $t = 5$ P is at the point C . Find the distance AC .

(4)

(Total for question 3 is 7 marks)

- 4** A particle P moves with a constant velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ with respect to a fixed origin O . It passes through the point A whose position vector is $(2\mathbf{i} + 11\mathbf{j}) \text{ m}$ at $t = 0$.

(a) Find the angle in degrees that the velocity vector of P makes with the vector $\mathbf{i}$.

(2)

(b) Calculate the distance of P from O when $t = 2$.

(4)

(Total for question 4 is 6 marks)

- 5** A boat B moves with a constant velocity. At noon, B is at the point with position vector $(2\mathbf{i} - 5\mathbf{j})$ km with respect to a fixed origin O . At 1430 the boat is at the point with position vector $(-8\mathbf{i} + 10\mathbf{j})$ km.

(a) Find the velocity of B .

(2)

(b) Find the bearing of the velocity vector

(2)

(c) Find an expression, in terms of t , for the position of B t hours after noon.

(2)

(Total for question 5 is 6 marks)

- 6** A particle has an initial velocity of $(\mathbf{i} - 5\mathbf{j}) \text{ ms}^{-1}$ and is accelerating uniformly in the direction $(2\mathbf{i} + \mathbf{j})$ where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Given that the magnitude of the acceleration is $3\sqrt{5} \text{ ms}^{-2}$,
- (a) show that, after t seconds, the velocity vector of the particle is $[(6t + 1)\mathbf{i} + (3t - 5)\mathbf{j}] \text{ ms}^{-1}$.

(6)

- (b) Using your answer to part (a), or otherwise, find the value of t for which the speed of the particle is at its minimum.

(5)

(Total for question 6 is 11 marks)

- 7 The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are due east and due north respectively. At midday a boat A is 5 km east of a fixed origin O and is moving with constant velocity $(-6\mathbf{i} + 5\mathbf{j}) \text{ km h}^{-1}$. At the same time, another boat B is 10 km north of O and is moving with uniform velocity $(-4\mathbf{i} + \mathbf{j}) \text{ km h}^{-1}$.

- (a) Show that, at time t hours after midday, the position vector of A is $[(5 - 6t)\mathbf{i} + 5t\mathbf{j}] \text{ km}$ and find a similar expression for the position vector of B at this time.

(5)

- (b) Hence show that, at time t , the position vector of B relative to A is $[(2t - 5)\mathbf{i} + (10 - 4t)\mathbf{j}] \text{ km}$

(2)

- (c) By using your answer to part (b), or otherwise, show that the boats would collide if they continued at the same velocities and find the time at which the collision would occur.

(3)

(Total for question 7 is 10 marks)

- 8** At 6 a.m. a boat A has position vector $(12\mathbf{i} - 11\mathbf{j})$ km relative to a fixed origin O and moves with constant velocity $(9\mathbf{i} - 6\mathbf{j})$ km h⁻¹. Another boat B has position vector $(40\mathbf{i} - 39\mathbf{j})$ km relative to a fixed origin O and moves with constant velocity $(-12\mathbf{i} + 15\mathbf{j})$ km h⁻¹.

(a) Find expressions for the position vectors of A and B , in terms of t hours after 6 a.m.

(5)

(b) Show that if both boats maintain their course and speed, they will collide and find the time and position vector at which this occurs.

(4)

- (c) At 7 a.m. boat A realises that a collision is imminent and changes course so that it now has velocity $(-18\mathbf{i} + 21\mathbf{j}) \text{ km h}^{-1}$. Find the distance between the two ships at the time when they would have collided.

(4)

(Total for question 8 is 13 marks)

- 9 At 10 a.m. plane A has position vector $(2\mathbf{i} - 5\mathbf{j})$ km and moves with constant velocity $(-4\mathbf{i} + 6\mathbf{j})$ km h⁻¹. Another plane B has position vector $(-3\mathbf{i} - 9\mathbf{j})$ km and moves with constant velocity $(\mathbf{i} + 8\mathbf{j})$ km h⁻¹.

(a) Find the relative displacement of plane A from plane B after t hours.

(6)

(b) Find the time when A is due west of B

(2)

- (c) Find the time, after 10 a.m. when the planes are exactly 37 km apart.

(6)

(Total for question 9 is 14 marks)

10 After t seconds, a particle P has position vector $\mathbf{r} = [(3t^3 - t + 3)\mathbf{i} + (2t^2 + 2t - 1)\mathbf{j}]$ m

(a) Find an expression for the velocity of P in terms of t

(2)

(b) Find an expression for the acceleration of P in terms of t

(2)

(Total for question 10 is 4 marks)

11 After t seconds, a particle P has position vector $\mathbf{r} = [(t^2 + 5t - 2)\mathbf{i} + (t^3 + 2t^2)\mathbf{j}]$ m

(a) Find the displacement of P when $t = 2$

(2)

(b) Find the velocity of P when $t = 3$

(3)

(Total for question 11 is 5 marks)

12 After t seconds, a particle has position vector $\mathbf{r} = [(3t^2 - 4)\mathbf{i} + (2t^2 + 12t)\mathbf{j}]$ m

(a) Find the time at which the particle is travelling on a bearing of 045°

(4)

(b) Find the magnitude of the acceleration of the particle.

(4)

(Total for question 12 is 8 marks)

- 13** A particle P has acceleration $[(3t^2 - 4)\mathbf{i} + (2t^2 + 4t)\mathbf{j}] \text{ ms}^{-2}$ where t is the time in seconds. When $t = 3$ the velocity of the particle is $(10\mathbf{i} + 20\mathbf{j}) \text{ ms}^{-1}$

(a) Find an expression for the velocity of P in terms of t

(4)

(b) Initially the particle is at the point with position vector $(2\mathbf{i} - 5\mathbf{j})$. Find an expression for the displacement of P in terms of t

(4)

(Total for question 13 is 8 marks)

- 14** A particle P has acceleration $[(3t - 3)\mathbf{i} + (2t - 4)\mathbf{j}] \text{ ms}^{-2}$ where t is the time in seconds. Initially P has velocity $(-4\mathbf{i} + 4\mathbf{j}) \text{ ms}^{-1}$

(a) Find an expression for the velocity of P in terms of t

(3)

(b) Find the velocities at the two times P is moving parallel to the vector $(2\mathbf{i} + \mathbf{j})$

(5)

(Total for question 14 is 8 marks)

- 15** After t seconds, a particle P has velocity $\mathbf{v}$ ms^{-1} where $\mathbf{v} = 3t^{\frac{1}{2}}\mathbf{i} - 2t\mathbf{j}$. When $t = 1$, P is at the point A and when $t = 4$, P is at the point B .
Find the exact distance AB .

(Total for question 15 is 6 marks)

16 A particle P moves with constant acceleration of $2\mathbf{i} - 3\mathbf{j} \text{ ms}^{-2}$. At time $t = 0$, the particle is at the origin, O , and moving with a velocity of $2\mathbf{i} - 2\mathbf{j} \text{ ms}^{-1}$

(a) Find the speed of P at time $t = 2$ seconds

(3)

(b) Show that the position vector of P at time $t = 2$ seconds is $8\mathbf{i} - 10\mathbf{j} \text{ m}$

(3)

(Total for question 16 is 6 marks)

17 After t seconds, a particle P has velocity $\mathbf{v}$ ms^{-1} where $\mathbf{v} = 4t\mathbf{i} - 2t^{-\frac{1}{2}}\mathbf{j}$. When $t = 0$, the position vector of P is $10\mathbf{i} + 5\mathbf{j}$ m

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

(Total for question 17 is 6 marks)

- 18** A particle P moves with constant acceleration $2\mathbf{i} - 3\mathbf{j} \text{ ms}^{-2}$. When $t = 0$, the particle is at point A and moving with a velocity $-3\mathbf{i} + 5\mathbf{j} \text{ ms}^{-1}$. At time $t = T$ the particle is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T .

(4)

(b) At time $t = 4$ seconds, P is at the point B . Find the distance AB .

(4)

(Total for question 18 is 8 marks)

- 19** A particle P moves with constant acceleration $3\mathbf{i} - 4\mathbf{j} \text{ ms}^{-2}$. When $t = 0$, the particle is at the origin, O , and moving with a velocity $2\mathbf{i} - 3\mathbf{j} \text{ ms}^{-1}$

(a) Find the velocity of P at time $t = 5$

(2)

(b) At time $t = T$ seconds, where $T > 0$, P passes through the point A . The position vector of A is $k\mathbf{i} - 10\mathbf{j} \text{ m}$ relative to O , where k is a constant. Find the value of T .

(4)

(c) Hence, find the value of k .

(2)

(Total for question 19 is 8 marks)

20 At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a} \text{ ms}^{-2}$ is given by $\mathbf{a} = (1 - 2t)\mathbf{i} + (5 - t^2)\mathbf{j}$. At the instant when $t = 0$, the velocity of P is $20\mathbf{i} \text{ ms}^{-1}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(Total for question 20 is 6 marks)

- 21** At time t seconds, where $t \geq 0$, a particle P moves so that its position vector $\mathbf{r}$ m is given by $\mathbf{r} = (t^2 - 4t)\mathbf{i} + 3t\mathbf{j}$.

Find the value of t at the instant when the speed of P is 5ms^{-1}

(Total for question 21 is 5 marks)
