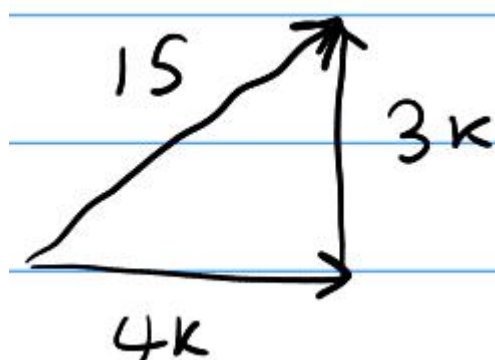


1



- Find the initial velocity vector u using the direction and speed: $(3k)^2 + (4k)^2 = 15^2 \implies 25k^2 = 225 \implies k = 3$. So $u = 12\mathbf{i} + 9\mathbf{j}$ (M1)
- Use $v = u + at$: $-12\mathbf{i} + 6\mathbf{j} = 12\mathbf{i} + 9\mathbf{j} + 6a$ (M1)
- Rearrange for $6a$: $-24\mathbf{i} - 3\mathbf{j} = 6a$ (M1)
- Solve for a : $a = -4\mathbf{i} - \frac{1}{2}\mathbf{j} \text{ ms}^{-2}$ (A2)

(Total for question 1 is 5 marks)

2

- Use $v = u + at$: $-6\mathbf{i} + \mathbf{j} = u + 3(2\mathbf{i} - 5\mathbf{j})$ (M1)
- Expand and rearrange: $-6\mathbf{i} + \mathbf{j} = u + 6\mathbf{i} - 15\mathbf{j} \implies u = -12\mathbf{i} + 16\mathbf{j}$ (M1)
- Calculate magnitude of u : speed = $\sqrt{(-12)^2 + 16^2}$ (M1)
- Final answer: 20 ms^{-1} (A2)

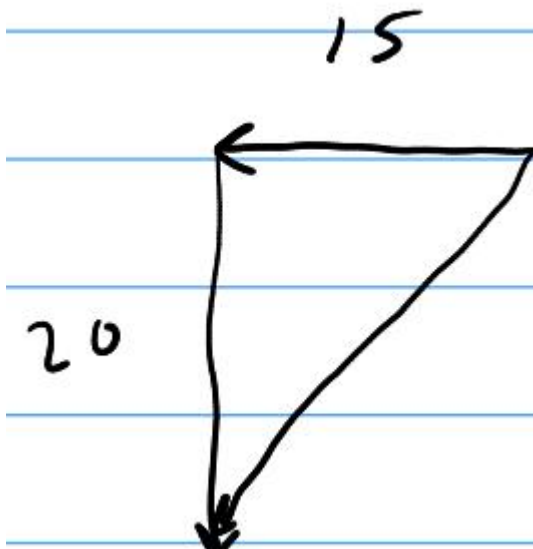
(Total for question 2 is 5 marks)

3

- (a)
- Find displacement $AB = \begin{pmatrix} -5 \\ 9 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} -9 \\ 12 \end{pmatrix}$ (M1)
 - Divide by time: velocity $= \frac{-9\mathbf{i}+12\mathbf{j}}{3}$ (M1)
 - Final answer: $-3\mathbf{i} + 4\mathbf{j}$ (A1)

- (b)
- Calculate displacement from A at $t = 5$: displacement $= 5 \times \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 20 \end{pmatrix}$ (M1)

•



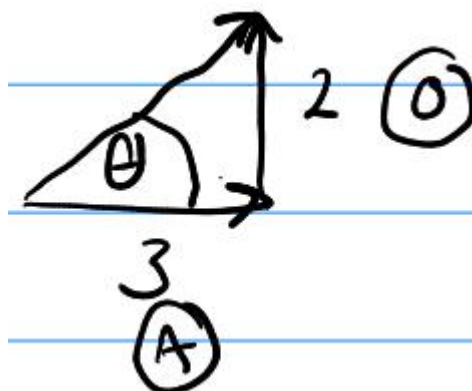
- Calculate distance using Pythagoras: $\sqrt{15^2 + 20^2}$ (M1)
- Final answer: 25m (A2)

(Total for question 3 is 7 marks)

4

(a)

•



- Use trigonometry: $\tan \theta = \frac{2}{3}$ (M1)

- Final answer: 33.7° (A1)

(b)

- Use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$: $\mathbf{r} = \begin{pmatrix} 2 \\ 11 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$ (M2)

- Calculate distance from origin: $\sqrt{8^2 + 15^2}$ (M1)

- Final answer: 17 m (A1)

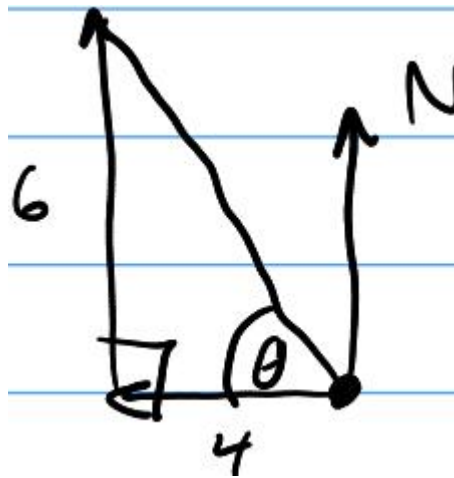
(Total for question 4 is 6 marks)

5

- (a)
- Use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ with $t = 2.5$: $\begin{pmatrix} -8 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + 2.5\mathbf{v}$ (M1)
 - Solve for $\mathbf{v}$: $\mathbf{v} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ km/h (A1)

(b)

•



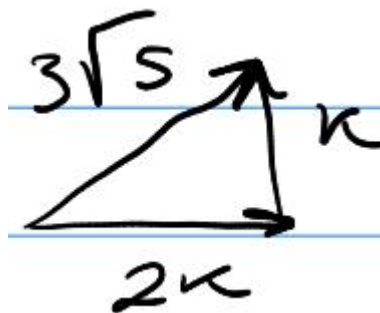
- Find angle with horizontal: $\tan \theta = \frac{6}{4} \Rightarrow \theta = 56^\circ$ (M1)
 - Calculate bearing: $270 + 56 = 326^\circ$ (A1)
- (c)
- Use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$: $\mathbf{r} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} t$ (M1)
 - Final answer: $(2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}$ (A1)

(Total for question 5 is 6 marks)

6

(a)

•



- Find acceleration vector: $k^2 + (2k)^2 = (3\sqrt{5})^2 \implies 5k^2 = 45 \implies k = 3$. So $a = 6\mathbf{i} + 3\mathbf{j}$ (M2)
- Use $v = u + at$: $v = \begin{pmatrix} 1 \\ -5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix} t$ (M2)
- Simplify to show: $v = (6t + 1)\mathbf{i} + (3t - 5)\mathbf{j}$ (A2)

(b)

- Expression for speed² = $(6t + 1)^2 + (3t - 5)^2$ (M1)
- Expand: $36t^2 + 12t + 1 + 9t^2 - 30t + 25 = 45t^2 - 18t + 26$ (M1)
- Differentiate or complete the square to find minimum: $90t - 18 = 0$ (M2)
- Final answer: $t = \frac{1}{5}$ (A1)

(Total for question 6 is 11 marks)

7

(a)

- For A: $\mathbf{r}_A = 5\mathbf{i} + (-6\mathbf{i} + 5\mathbf{j})t = (5 - 6t)\mathbf{i} + 5t\mathbf{j}$ (B2)
- For B: $\mathbf{r}_B = 10\mathbf{j} + (-4\mathbf{i} + \mathbf{j})t$ (M2)
- Final answer for B: $-4t\mathbf{i} + (10 + t)\mathbf{j}$ (A1)

(b)

- Relative position $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$ (M1)
- $\begin{pmatrix} -4t \\ 10 + t \end{pmatrix} - \begin{pmatrix} 5 - 6t \\ 5t \end{pmatrix} = \begin{pmatrix} 2t - 5 \\ 10 - 4t \end{pmatrix}$ (A1)

(c)

- For collision, relative position must be zero: $2t - 5 = 0$ and $10 - 4t = 0$ (M1)
- Solve both: $t = 2.5$ and $t = 2.5$ (M1)
- Since t is the same for both components, they collide at 1430 (2.5 hours after midday) (A1)

(Total for question 7 is 10 marks)

8

- (a)
- $R_A = (12 + 9t)\mathbf{i} + (-11 - 6t)\mathbf{j}$ (B2)
 - $R_B = (40 - 12t)\mathbf{i} + (15t - 39)\mathbf{j}$ (B3)
- (b)
- Equate components: $12 + 9t = 40 - 12t \implies 21t = 28 \implies t = 4/3$ (M1)
 - Check other component: $-11 - 6(4/3) = -19$ and $15(4/3) - 39 = -19$ (M1)
 - Time is 7:20 am (A1)
 - Position vector: $R = 24\mathbf{i} - 19\mathbf{j}$ (A1)
- (c)
- Position of A at 7am ($t = 1$): $R_A = 21\mathbf{i} - 17\mathbf{j}$ (B1)
 - New position of A at $t = 4/3$ (1/3 hour later): $R_A = \begin{pmatrix} 21 \\ -17 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} -18 \\ 21 \end{pmatrix} = \begin{pmatrix} 15 \\ -10 \end{pmatrix}$ (M1)
 - Position of B at $t = 4/3$: $R_B = 24\mathbf{i} - 19\mathbf{j}$ (B1)
 - Distance: $\sqrt{(24 - 15)^2 + (-19 - (-10))^2} = \sqrt{9^2 + (-9)^2} = 12.7 \text{ km}$ (A1)

(Total for question 8 is 13 marks)

9

- (a)
- $R_A = (2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}$ (B1)
 - $R_B = (t - 3)\mathbf{i} + (8t - 9)\mathbf{j}$ (B1)
 - $R_A - R_B = (2 - 4t - (t - 3))\mathbf{i} + (6t - 5 - (8t - 9))\mathbf{j}$ (M2)
 - Final answer: $(5 - 5t)\mathbf{i} + (4 - 2t)\mathbf{j}$ (A2)
- (b)
- Due west means $\mathbf{j}$ component of relative displacement is zero: $4 - 2t = 0$ (M1)
 - $t = 2$, so time is 12 pm (A1)
- (c)
- Distance squared: $(5 - 5t)^2 + (4 - 2t)^2 = 37^2$ (M1)
 - Expand: $25 - 50t + 25t^2 + 16 - 16t + 4t^2 = 1369$ (M2)
 - Simplify: $29t^2 - 66t - 1328 = 0$ (M1)
 - Solve quadratic: $t = 8$ (ignore negative root) (M1)
 - Time is 6 pm (A1)

(Total for question 9 is 14 marks)

10

- (a)
- Differentiate $\mathbf{r}$ with respect to t : $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ (M1)
 - Final answer: $\mathbf{v} = (9t^2 - 1)\mathbf{i} + (4t + 2)\mathbf{j} \text{ ms}^{-1}$ (A1)
- (b)
- Differentiate $\mathbf{v}$ with respect to t : $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ (M1)
 - Final answer: $\mathbf{a} = 18t\mathbf{i} + 4\mathbf{j} \text{ ms}^{-2}$ (A1)

(Total for question 10 is 4 marks)

11

- (a)
- Substitute $t = 2$ into $\mathbf{r}$: $(2^2 + 5(2) - 2)\mathbf{i} + (2^3 + 2(2^2))\mathbf{j}$ (M1)
 - Final answer: $12\mathbf{i} + 16\mathbf{j}$ (A1)
- (b)
- Differentiate $\mathbf{r}$: $\mathbf{v} = (2t + 5)\mathbf{i} + (3t^2 + 4t)\mathbf{j}$ (M1)
 - Substitute $t = 3$: $(2(3) + 5)\mathbf{i} + (3(3^2) + 4(3))\mathbf{j}$ (M1)
 - Final answer: $11\mathbf{i} + 39\mathbf{j} \text{ ms}^{-1}$ (A1)

(Total for question 11 is 5 marks)

12

- (a)
- Differentiate to find velocity: $\mathbf{v} = 6t\mathbf{i} + (4t + 12)\mathbf{j}$ (M1)
 - For bearing 045° , $\mathbf{i}$ component = $\mathbf{j}$ component: $6t = 4t + 12$ (M2)
 - Solve for t : $2t = 12 \implies t = 6$ (A1)
- (b)
- Differentiate velocity to find acceleration: $\mathbf{a} = 6\mathbf{i} + 4\mathbf{j}$ (M2)
 - Calculate magnitude: $\sqrt{6^2 + 4^2} = \sqrt{52}$ (M1)
 - Final answer: $2\sqrt{13} \text{ m s}^{-2}$ (A1)

(Total for question 12 is 8 marks)

13

- (a)
- Integrate acceleration: $\mathbf{v} = (t^3 - 4t)\mathbf{i} + (\frac{2}{3}t^3 + 2t^2)\mathbf{j} + \mathbf{C}$ (M1)
 - Use $t = 3, \mathbf{v} = 10\mathbf{i} + 20\mathbf{j}$ to find $\mathbf{C}$: $10\mathbf{i} + 20\mathbf{j} = (27 - 12)\mathbf{i} + (18 + 18)\mathbf{j} + \mathbf{C}$ (M1)
 - $\mathbf{C} = -5\mathbf{i} - 16\mathbf{j}$ (A1)
 - Final answer: $\mathbf{v} = (t^3 - 4t - 5)\mathbf{i} + (\frac{2}{3}t^3 + 2t^2 - 16)\mathbf{j}$ (A1)
- (b)
- Integrate velocity: $\mathbf{s} = (\frac{1}{4}t^4 - 2t^2 - 5t)\mathbf{i} + (\frac{1}{6}t^4 + \frac{2}{3}t^3 - 16t)\mathbf{j} + \mathbf{C}$ (M1)
 - Use $t = 0, \mathbf{s} = 2\mathbf{i} - 5\mathbf{j}$ to find $\mathbf{C}$: $\mathbf{C} = 2\mathbf{i} - 5\mathbf{j}$ (M2)
 - Final answer: $\mathbf{s} = (\frac{1}{4}t^4 - 2t^2 - 5t + 2)\mathbf{i} + (\frac{1}{6}t^4 + \frac{2}{3}t^3 - 16t - 5)\mathbf{j}$ (A1)

(Total for question 13 is 8 marks)

14

- (a)
- Integrate acceleration: $v = (\frac{3}{2}t^2 - 3t)\mathbf{i} + (t^2 - 4t)\mathbf{j} + \mathbf{c}$ (M1)
 - Use $t = 0, v = -4\mathbf{i} + 4\mathbf{j}$ to find $\mathbf{c}$: $\mathbf{c} = -4\mathbf{i} + 4\mathbf{j}$ (M1)
 - Final answer: $v = (\frac{3}{2}t^2 - 3t - 4)\mathbf{i} + (t^2 - 4t + 4)\mathbf{j}$ (A1)
- (b)
- Parallel to $2\mathbf{i} + \mathbf{j}$ means $\mathbf{i}$ component is double the $\mathbf{j}$ component: $\frac{3}{2}t^2 - 3t - 4 = 2(t^2 - 4t + 4)$ (M1)
 - Expand and simplify: $3t^2 - 6t - 8 = 4t^2 - 16t + 16 \implies t^2 - 10t + 24 = 0$ (M1)
 - Solve for t : $(t - 4)(t - 6) = 0 \implies t = 4, t = 6$ (M1)
 - Substitute $t = 4$: $v = 8\mathbf{i} + 4\mathbf{j}$ (A1)
 - Substitute $t = 6$: $v = 32\mathbf{i} + 16\mathbf{j}$ (A1)

(Total for question 14 is 8 marks)

15

- Integrate velocity to find position: $\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j} + \mathbf{c}$ (M2)
- Find displacement vector $\mathbf{AB} = \mathbf{r}(4) - \mathbf{r}(1)$ (M1)
- $\mathbf{AB} = \begin{pmatrix} 16 \\ -16 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 14 \\ -15 \end{pmatrix}$ (M1)
- Calculate distance: $\sqrt{14^2 + (-15)^2} = \sqrt{196 + 225}$ (M1)
- Final answer: $\sqrt{421}$ (A1)

(Total for question 15 is 6 marks)

16

- (a)
- Use $v = u + at$: $v = (2\mathbf{i} - 2\mathbf{j}) + 2(2\mathbf{i} - 3\mathbf{j}) = 6\mathbf{i} - 8\mathbf{j}$ (M1)
 - Calculate magnitude: $\sqrt{6^2 + (-8)^2}$ (M1)
 - Final answer: 10 ms^{-1} (A1)
- (b)
- Use $s = ut + \frac{1}{2}at^2$: $s = 2(2\mathbf{i} - 2\mathbf{j}) + \frac{1}{2}(2)^2(2\mathbf{i} - 3\mathbf{j})$ (M2)
 - $s = (4\mathbf{i} - 4\mathbf{j}) + (4\mathbf{i} - 6\mathbf{j}) = 8\mathbf{i} - 10\mathbf{j}$ (A1)

(Total for question 16 is 6 marks)

17

- (a)
- Differentiate velocity: $a = 4\mathbf{i} + t^{-\frac{3}{2}}\mathbf{j}$ (M1)
 - Substitute $t = 4$: $a = 4\mathbf{i} + 4^{-\frac{3}{2}}\mathbf{j}$ (M1)
 - Final answer: $4\mathbf{i} + \frac{1}{8}\mathbf{j}$ (A1)
- (b)
- Integrate velocity: $\mathbf{r} = 2t^2\mathbf{i} - 4t^{\frac{1}{2}}\mathbf{j} + \mathbf{c}$ (M1)
 - Use $t = 0, \mathbf{r} = 10\mathbf{i} + 5\mathbf{j}$ to find $\mathbf{c} = 10\mathbf{i} + 5\mathbf{j}$ (M1)
 - Substitute $t = 4$: $\mathbf{r} = (2(16) + 10)\mathbf{i} + (-4(2) + 5)\mathbf{j} = 42\mathbf{i} - 3\mathbf{j}$ (A1)

(Total for question 17 is 6 marks)

18

- (a)
- Velocity vector $v = (-3 + 2t)\mathbf{i} + (5 - 3t)\mathbf{j}$ (M1)
 - Direction condition: $\frac{5-3t}{-4} = \frac{-3+2t}{3}$ (M2)
 - Solve for t : $-4(-3 + 2t) = 3(5 - 3t) \implies 12 - 8t = 15 - 9t \implies t = 3$ (A1)
- (b)
- Integrate velocity for displacement from A: $s = (t^2 - 3t)\mathbf{i} + (-\frac{3}{2}t^2 + 5t)\mathbf{j}$ (M2)
 - Substitute $t = 4$: $s = (16 - 12)\mathbf{i} + (-24 + 20)\mathbf{j} = 4\mathbf{i} - 4\mathbf{j}$ (M1)
 - Distance: $\sqrt{4^2 + (-4)^2} = 4\sqrt{2}$ m (A1)

(Total for question 18 is 8 marks)

19

- (a)
- $v = (3t + 2)\mathbf{i} + (-4t - 3)\mathbf{j}$ (M1)
 - Substitute $t = 5$: $v = 17\mathbf{i} - 23\mathbf{j}$ (A1)
- (b)
- Position vector $r = (\frac{3}{2}t^2 + 2t)\mathbf{i} + (-2t^2 - 3t)\mathbf{j}$ (M2)
 - Equate $\mathbf{j}$ component: $-2t^2 - 3t = -10 \implies 2t^2 + 3t - 20 = 0$ (M1)
 - Solve for t : $(2t - 5)(t + 4) = 0 \implies T = 2.5$ (A1)
- (c)
- Substitute $t = 2.5$ into $\mathbf{i}$ component: $k = \frac{3}{2}(2.5)^2 + 2(2.5)$ (M1)
 - Final answer: $k = 14.375$ (or $115/8$) (A1)

(Total for question 19 is 8 marks)

20

- (a)
- Integrate acceleration: $v = (t - t^2)\mathbf{i} + (5t - \frac{1}{3}t^3)\mathbf{j} + c$ (M1)
 - Use $t = 0, v = 20\mathbf{i}$ to find $c = 20\mathbf{i}$ (M1)
 - Substitute $t = 4$: $v = (4 - 16 + 20)\mathbf{i} + (20 - \frac{64}{3})\mathbf{j} = 8\mathbf{i} - \frac{4}{3}\mathbf{j}$ (A1)
- (b)
- Perpendicular to $\mathbf{i}$ means $\mathbf{i}$ component of velocity is zero: $t - t^2 + 20 = 0$ (M1)
 - Solve quadratic: $t^2 - t - 20 = 0 \implies (t - 5)(t + 4) = 0$ (M1)
 - Final answer: $t = 5$ (since $t \geq 0$) (A1)

(Total for question 20 is 6 marks)

21

- Differentiate position to find velocity: $v = (2t - 4)\mathbf{i} + 3\mathbf{j}$ (M2)
- Speed squared: $(2t - 4)^2 + 3^2 = 5^2$ (M1)
- $(2t - 4)^2 + 9 = 25 \implies (2t - 4)^2 = 16$ (M1)
- $2t - 4 = 4 \implies 2t = 8 \implies t = 4$ (A1)

(Total for question 21 is 5 marks)