

1 $f(x) = x^3 + 2x^2 + x + 1$

(a) Show that $f(x)$ has a root between -1.5 and -2

(2)

(b) Show that the equation $f(x) = 0$ can be written in the form $x = -\left(\frac{x+1}{x^2} + 2\right)$

(3)

(c) Use the iteration formula $x_{n+1} = -\left(\frac{x_n+1}{x_n^2} + 2\right)$ with $x_0 = -2$ to find, to 3 decimal places, the values of x_1 , x_2 and x_3 .

(3)

- (d) The root of $f(x) = 0$ is α . By choosing a suitable interval, prove that $\alpha = -1.755$ to 3 decimal places

(3)

(Total for question 1 is 11 marks)

2 $f(x) = e^{x+2} + x - 10$

(a) Show that the equation $f(x) = 0$ can be written in the form $x = \ln(10 - x) - 2$

(3)

(b) Use the iteration formula $x_{n+1} = \ln(10 - x_n) - 2$ with $x_0 = 0.5$ to find, to 3 decimal places, the values of x_1 , x_2 and x_3 .

(3)

(c) The root of $f(x) = 0$ is α . By choosing a suitable interval, prove that $\alpha = 0.275$ to 3 decimal places

(3)

(Total for question 2 is 9 marks)

3 The curve C has the equation $y = (12 - x) \ln x \quad x > 0$

(a) Find $\frac{dy}{dx}$

(3)

(b) C has a stationary point at P . Show that the x coordinate of P lies between 4.5 and 5

(2)

(c) Show that the x coordinate of P is a solution of $x = \frac{12}{1 + \ln x}$

(3)

- (d) Use the iteration formula $x_{n+1} = \frac{12}{1+\ln x_n}$ with $x_0 = 4.75$ to find, to 3 decimal places, the values of x_1, x_2 and x_3 .

(3)

(Total for question 3 is 11 marks)

4 The curve C with the equation $y = e^x$ meets the line L with equation $y = 5x - 1$ at the point P .

(a) Show that the x coordinate of P satisfies $x = \ln(5x - 1)$

(2)

(b) Use the iteration formula $x_{n+1} = \ln(5x_n - 1)$ with $x_0 = 2.5$ to find, to 3 decimal places, the values of x_1 , x_2 and x_3 .

(3)

(Total for question 4 is 5 marks)

5 $f(x) = x^5 + 3x^2 + x - 10$

(a) Find $f'(x)$

(2)

(b) The root of $f(x) = 0$ lies in the interval $[1, 1.5]$. Use the Newton-Raphson method, starting with $x_0 = 1.25$, to find the root of $f(x) = 0$ to 3 decimal places.

(5)

(Total for question 5 is 7 marks)

6 $f(x) = \ln(2x + 1) + x^2 - 5 \quad x > -0.5$

(a) Show that $f(x) = 0$ has a root in the interval $[1.8, 1.9]$

(2)

(b) Find $f'(x)$

(3)

(c) Using $x_0 = 1.8$ as a first approximation, apply the Newton-Raphson procedure to find a second approximation, giving your answer to 3 decimal places.

(3)

(Total for question 6 is 8 marks)

7 $f(x) = x^3 + 3x^2 - 2\sqrt{x} \quad x > 0$

(a) Show that $f(x) = 0$ has a root in the interval $[0.6, 0.7]$

(2)

(b) Find $f'(x)$

(3)

(c) Starting with $x_0 = 0.65$, apply the Newton-Raphson procedure once to find an approximate solution to the equation $f(x) = 0$ giving your answer to 3 decimal places.

(3)

8 $f(x) = \sin(2x) + \ln(x) \quad x > 0$

- (a) Show that $y = f(x)$ has a stationary point in the interval $[1, 1.1]$

(5)

- (b) Starting with $x_0 = 1.05$, apply the Newton-Raphson procedure twice to find an approximation for the x coordinate of the stationary point in the interval $[1, 1.1]$.

(4)

9 The equation $x^3 - 3x^2 + 5 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{2x_n^3 - 3x_n^2 - 5}{3x_n^2 - 6x_n}$$

(3)

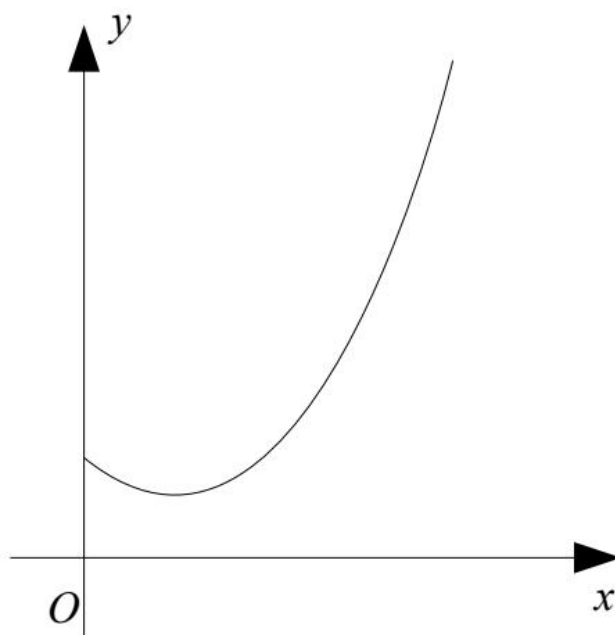
(b) Using the formula given in part (a) with $x_1 = -1$, find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

(Total for question 9 is 6 marks)



The diagram shows a sketch of the curve C with equation $y = 2x^{\frac{3}{2}} - 5\sqrt{x} + e^x + 4$

- (a) Show that $\frac{dy}{dx} = 3\sqrt{x} + e^x - \frac{5}{2\sqrt{x}}$

(3)

- (b) The point P is the minimum turning point on C . Show that the x coordinate of P is a solution of

$$x = \frac{5 - 2\sqrt{x}e^x}{6}$$

(2)

- (c) Use the iteration formula $x_{n+1} = \frac{5 - 2\sqrt{x_n}e^{x_n}}{6}$ with $x_1 = 0.5$ to find (i) the value of x_2 to 5 decimal places, (ii) the x coordinate of P to 5 decimal places.

(3)

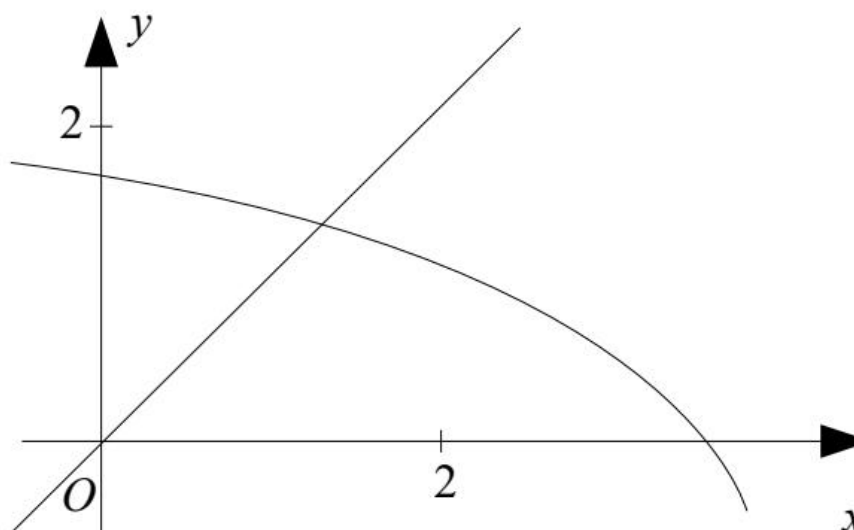
(Total for question 10 is 8 marks)

11 The curve with equation $y = \ln(5 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $1 < \alpha < 2$

(2)

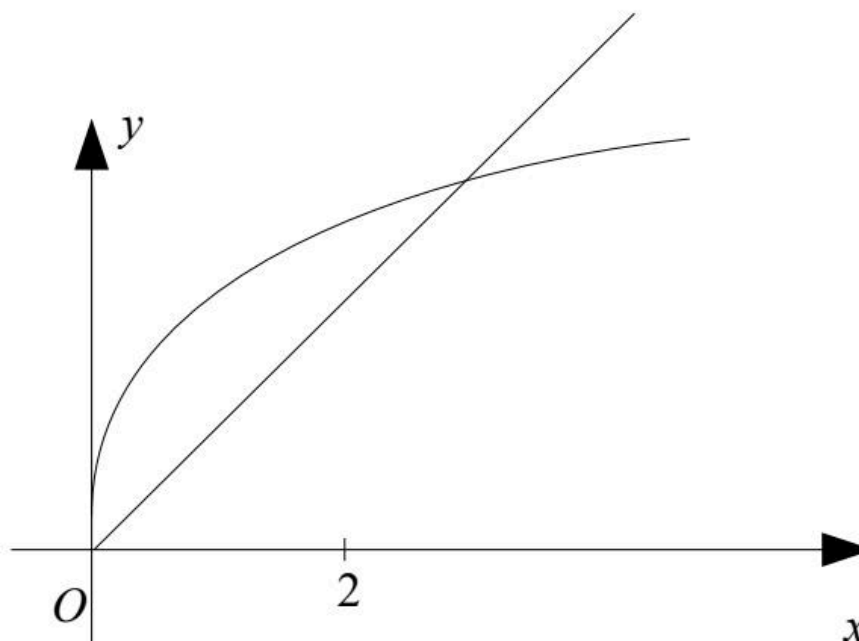
(b)



The diagram shows the graph of $y = \ln(5 - x)$ and the graph of $y = x$. Use the diagram, starting with $x_1 = 2$, to determine whether the iteration formula $x_{n+1} = \ln(5 - x_n)$ can be used to find an approximation for α , justifying your answer.

(2)

(Total for question 11 is 4 marks)

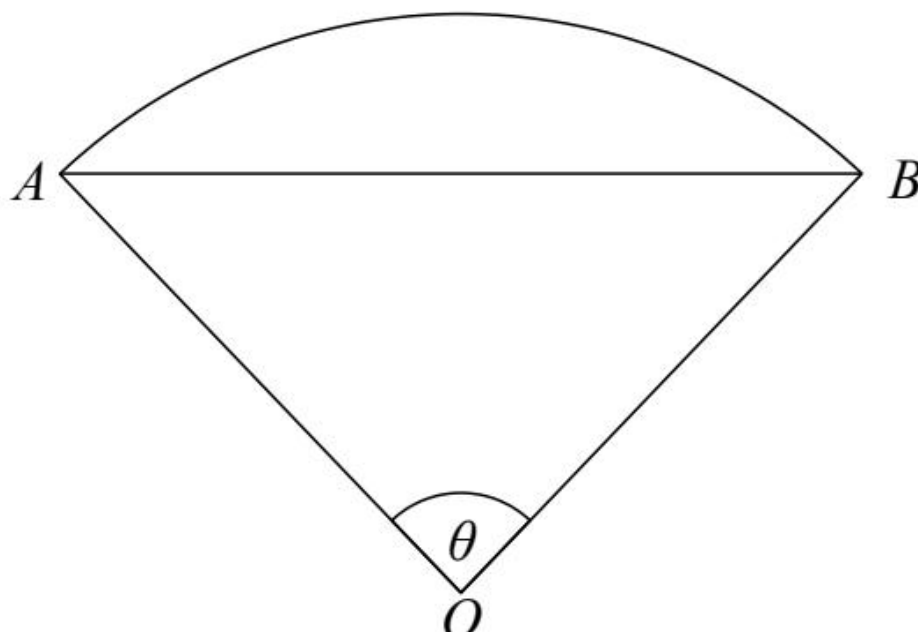


The diagram shows the graph of $y = \ln(4x + 3)$ and the graph of $y = x$.

Use the diagram, starting with $x_1 = 2$, to determine whether the iteration formula $x_{n+1} = \ln(4x_n + 3)$ can be used to find a root, justifying your answer.

(Total for question 12 is 2 marks)

- 13 The diagram shows a sector of a circle AOB . Angle AOB is θ radians.



Given that the area of triangle AOB is $\frac{3}{4}$ the area of sector AOB .

- (a) Show that $\sin \theta - \frac{3}{4}\theta = 0$

(3)

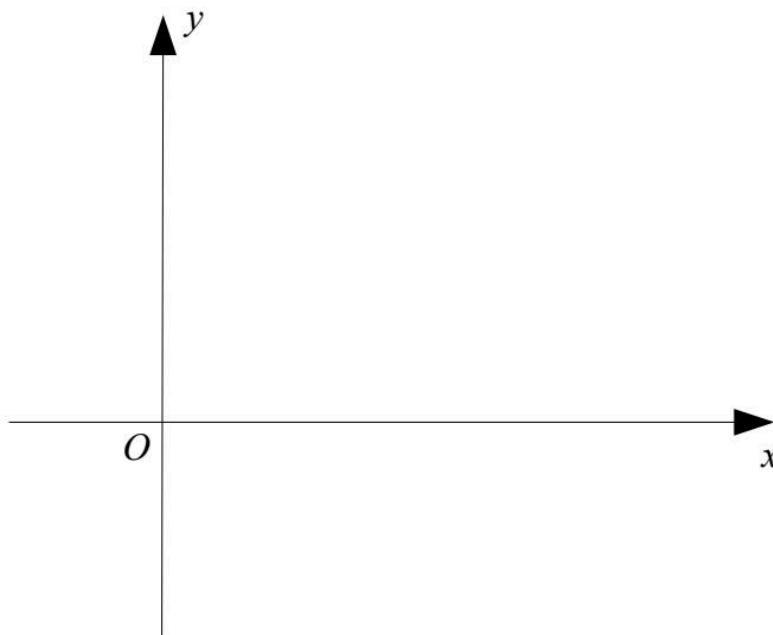
- (b) Use the Newton-Raphson method with $\theta_0 = 2$, to find θ_4 as an approximation for θ . Give your answer to be 4 decimal places.

(3)

(Total for question 13 is 6 marks)

14

- (a) Sketch the graphs of $y = \ln x$ and $y = \frac{1}{x}$ for $x > 0$ on the same axes.



- (b) Explain why the equation $\ln x = \frac{1}{x}$ has exactly one solution.

(2)

(1)

- (c) By considering a suitable change of sign, show that the solution to the equation lies between 1.7 and 1.8

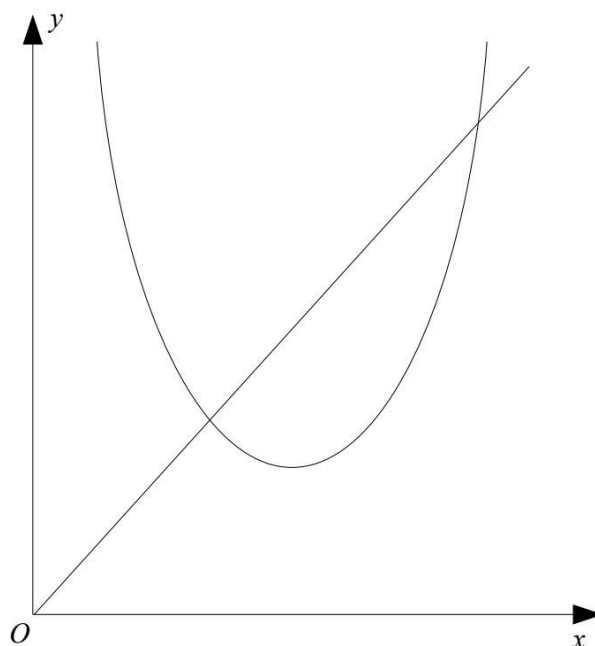
(2)

- (d) Use the iterative formula $x_{n+1} = e^{\frac{1}{x_n}}$ with $x_1 = 1.8$, to find x_2 , x_3 and x_4 , giving your answers to four decimal places.

(2)

(Total for question 14 is 7 marks)

- 15 The diagram shows the part of the graphs of $y = \operatorname{cosec} x$ and $y = x$.



- (a) Write down the coordinates of the minimum point and the equations of the two vertical asymptotes for $y = \operatorname{cosec} x$

(2)

- (b) Show that $x = \operatorname{cosec} x$ has a root between $x = 2.5$ and $x = 3$

(1)

- (c) Use the diagram, starting with $x_1 = 2.5$, to determine whether the iteration formula can be used to find an approximation for this root, justifying your answer.

(3)

(Total for question 15 is 6 marks)

16 A curve has equation $y = \ln(3 - 2x) + x^2 - 2x$. Given that the curve intersects the x -axis at exactly one point.

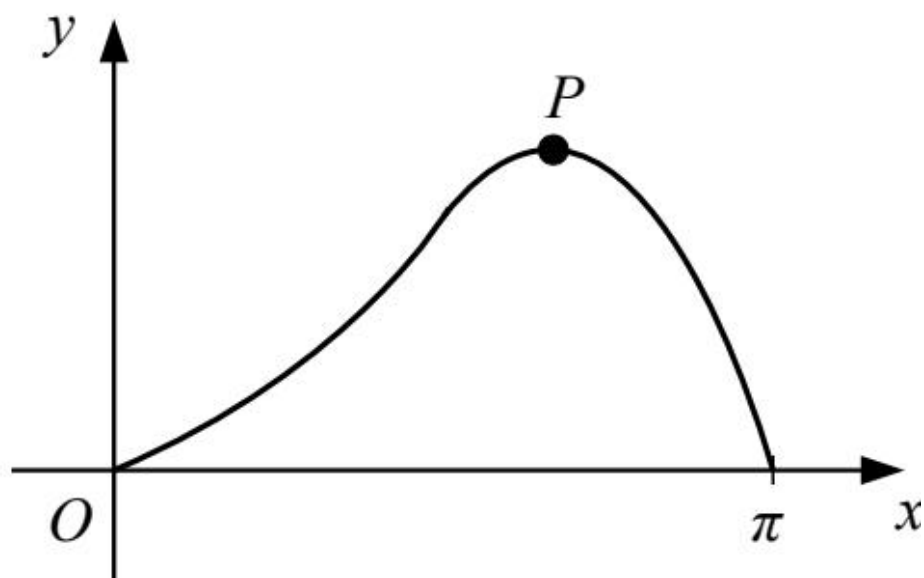
(a) Show that the x -coordinate of this point lies between 0 and 1

(2)

(b) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x -coordinate of the point where the curve intersects the x -axis, giving your answer correct to 5 decimal places.

(6)

(Total for question 16 is 8 marks)



The diagram shows the part of the curve $y = 2x \sin x$. The maximum point on the curve is denoted by P .

- (a) Show that the x -coordinate of P satisfies the equation $\tan x + x = 0$

(3)

- (b) Use the Newton-Raphson method, with a suitable initial value, to find the x -coordinate of P , giving your answer correct to 4 decimal places. Show the result of each iteration.

(4)

(Total for question 17 is 7 marks)

18 The function f is defined by $f(x) = e^x\sqrt{x} - 1$

- (a) $f(x) = 0$ has a single solution at the point $x = \alpha$. By considering a suitable change of sign, show that α lies between 0 and 1

(2)

- (b) Show that $f'(x) = \frac{e^x(2x+1)}{2\sqrt{x}}$

(3)

- (c) Use the Newton-Raphson method with $x_1 = 1$ to find x_3 , an approximation for α . Give your answer to five decimal places

(2)

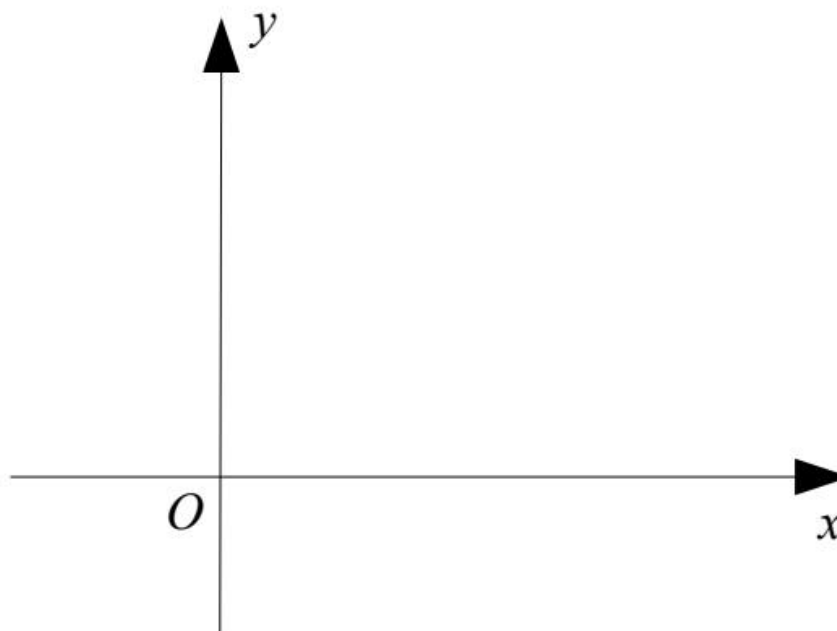
(d) Explain why the Newton-Raphson method fails to find α with $x_1 = 0$

(2)

(Total for question 18 is 9 marks)

19

- (a) Sketch the graphs of $y = \ln x$ and $y = \frac{1}{x}$ for $x > 0$ on the same axes.



(2)

- (b) Explain why the equation $\ln x = \frac{1}{x}$ has exactly one solution.

(1)

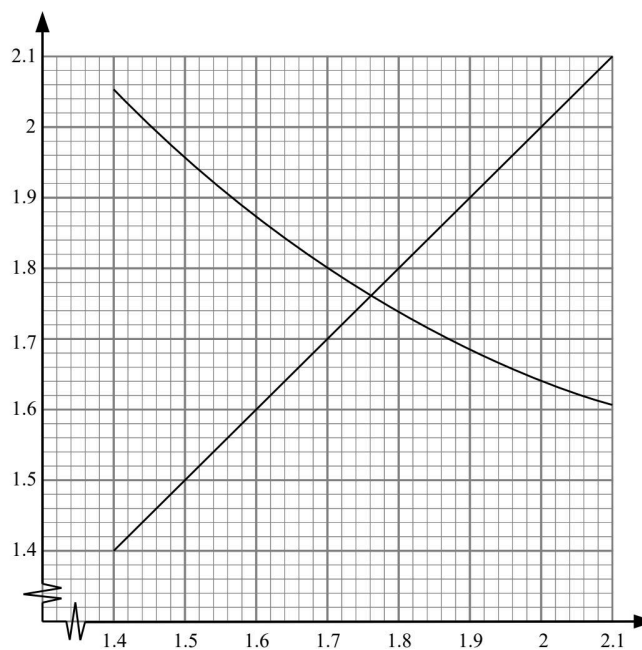
- (c) By considering a suitable change of sign, show that the solution to the equation lies between 1.7 and 1.8

(2)

- (d) Use the iterative formula $x_{n+1} = e^{\frac{1}{x_n}}$ with $x_1 = 1.8$, to find x_2 , x_3 and x_4 , giving your answers to four decimal places.

(2)

- (e) On the graph below, draw a cobweb or staircase diagram to show how the convergence takes place.



(2)

(Total for question 19 is 9 marks)