

1

- (a)
- $f(-1.5) = (-1.5)^3 + 2(-1.5)^2 + (-1.5) + 1 = \frac{5}{8}$ (M1)
 - $f(-2) = (-2)^3 + 2(-2)^2 + (-2) + 1 = -1$ (M1)
 - Change of sign and continuous function $\therefore$ root between -1.5 and -2 (A1)
- (b)
- $x^3 + 2x^2 + x + 1 = 0 \implies x^3 = -2x^2 - x - 1$ (M1)
 - $x^3 = -(x + 1 + 2x^2)$ (M1)
 - Divide by x^2 : $x = -\left(\frac{x+1}{x^2} + 2\right)$ (A1)
- (c)
- $x_1 = -\left(\frac{-2+1}{(-2)^2} + 2\right) = -1.75$ (B1)
 - $x_2 = -1.755$ (B1)
 - $x_3 = -1.755$ (B1)
- (d)
- $f(-1.7545) = 1.215 \times 10^{-3}$ (M1)
 - $f(-1.7555) = -2.005 \times 10^{-3}$ (M1)
 - Change of sign and continuous function $\therefore \alpha = -1.755$ (A1)

(Total for question 1 is 11 marks)

2

- (a)
- $e^{x+2} + x - 10 = 0 \implies e^{x+2} = 10 - x$ (M1)
 - $x + 2 = \ln(10 - x)$ (M1)
 - $x = \ln(10 - x) - 2$ (A1)
- (b)
- $x_1 = \ln(10 - 0.5) - 2 = 0.251$ (B1)
 - $x_2 = \ln(10 - 0.251) - 2 = 0.277$ (B1)
 - $x_3 = 0.274$ (B1)
- (c)
- $f(0.2745) = -2.44 \times 10^{-3}$ (M1)
 - $f(0.2755) = 8.28 \times 10^{-3}$ (M1)
 - Change of sign and continuous function $\therefore \alpha = 0.275$ to 3dp. (A1)

(Total for question 2 is 9 marks)

3

- (a)
- $u = 12 - x, v = \ln x \implies \frac{du}{dx} = -1, \frac{dv}{dx} = \frac{1}{x}$ (M1)
 - $\frac{dy}{dx} = -\ln x + \frac{1}{x}(12 - x)$ (A2)
- (b)
- When $x = 4.5, \frac{dy}{dx} = 0.163$ (M1)
 - When $x = 5, \frac{dy}{dx} = -0.209$ (M1)
 - Change of sign and continuous function $\therefore x$ coordinate of P lies between 4.5 and 5. (A1)
- (c)
- $-\ln x + \frac{12-x}{x} = 0 \implies -x \ln x + 12 - x = 0$ (M1)
 - $12 = x + x \ln x \implies 12 = x(1 + \ln x)$ (M1)
 - $x = \frac{12}{1+\ln x}$ (A1)
- (d)
- $x_1 = 4.691$ (B1)
 - $x_2 = 4.714$ (B1)
 - $x_3 = 4.705$ (B1)

(Total for question 3 is 11 marks)

4

- (a)
- $e^x = 5x - 1$ (M1)
 - $\ln(e^x) = \ln(5x - 1) \implies x = \ln(5x - 1)$ (A1)
- (b)
- $x_1 = \ln(5(2.5) - 1) = 2.442$ (B1)
 - $x_2 = \ln(5(2.442) - 1) = 2.417$ (B1)
 - $x_3 = 2.406$ (B1)

(Total for question 4 is 5 marks)

5

- (a)
- $f'(x) = 5x^4 + 6x + 1$ (B2)
- (b)
- $f(1.25) = -1.011, f'(1.25) = 20.707$ (M1)
 - $x_1 = 1.25 - \frac{-1.011}{20.707} = 1.299$ (M1)
 - $f(1.299) = 0.0555, f'(1.299) = 23.02$ (M1)
 - $x_2 = 1.299 - \frac{0.0555}{23.02} = 1.296$ (A1)
 - $x_3 = 1.296$ (A1)

(Total for question 5 is 7 marks)

6

- (a)
- $f(1.8) = \ln(2(1.8) + 1) + (1.8)^2 - 5 = -0.234$ (M1)
 - $f(1.9) = \ln(2(1.9) + 1) + (1.9)^2 - 5 = 0.179$ (M1)
 - Change of sign and continuous function $\therefore$ root in interval $[1.8, 1.9]$ (A1)
- (b)
- $f'(x) = \frac{2}{2x+1} + 2x$ (B3)
- (c)
- $f'(1.8) = \frac{2}{2(1.8)+1} + 2(1.8) = 4.035$ (M1)
 - $x_1 = 1.8 - \frac{-0.234}{4.035} = 1.858$ (A2)

(Total for question 6 is 8 marks)

7

- (a)
- $f(0.6) = (0.6)^3 + 3(0.6)^2 - 2\sqrt{0.6} = -0.253$ (M1)
 - $f(0.7) = 0.7^3 + 3(0.7)^2 - 2\sqrt{0.7} = 0.140$ (M1)
 - Change of sign and continuous function $\therefore$ root in interval $[0.6, 0.7]$ (A1)
- (b)
- $f'(x) = 3x^2 + 6x - x^{-\frac{1}{2}}$ (B3)
- (c)
- $f(0.65) = -0.0703, f'(0.65) = 3.927$ (M1)
 - $x_1 = 0.65 - \frac{-0.0703}{3.927} = 0.668$ (A2)

(Total for question 7 is 8 marks)

8

- (a)
- $f'(x) = 2 \cos 2x + \frac{1}{x}$ (M1)
 - $f'(1) = 2 \cos 2 + 1 = 0.168$ (M1)
 - $f'(1.1) = 2 \cos(2.2) + \frac{1}{1.1} = -0.268$ (M1)
 - Change of sign and continuous function $\therefore$ stationary point in interval $[1, 1.1]$ (A2)
- (b)
- $f''(x) = -4 \sin 2x - x^{-2}$ (M1)
 - $f'(1.05) = -0.0573, f''(1.05) = -4.360 \implies x_1 = 1.037$ (M1)
 - $f'(1.037) = 0.000495, f''(1.037) = -4.435 \implies x_2 = 1.037$ (A2)

(Total for question 8 is 9 marks)

9

- (a)
- $f(x) = x^3 - 3x^2 + 5, f'(x) = 3x^2 - 6x$ (M1)
 - $x_{n+1} = x_n - \frac{x_n^3 - 3x_n^2 + 5}{3x_n^2 - 6x_n}$ (M1)
 - $x_{n+1} = \frac{x_n(3x_n^2 - 6x_n) - (x_n^3 - 3x_n^2 + 5)}{3x_n^2 - 6x_n} = \frac{2x_n^3 - 3x_n^2 - 5}{3x_n^2 - 6x_n}$ (A1)
- (b)
- $x_2 = \frac{2(-1)^3 - 3(-1)^2 - 5}{3(-1)^2 - 6(-1)} = -\frac{10}{9}$ (B1)
 - $x_3 = -1.104$ (B1)
- (c)
- When $x = 0, f'(x) = 0$. At a stationary point the tangent will not cross the x axis. (B1)

(Total for question 9 is 6 marks)

10

- (a)
- $y = 2x^{\frac{3}{2}} - 5x^{\frac{1}{2}} + e^x + 4$ (M1)
 - $\frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}} + e^x$ (M1)
 - $\frac{dy}{dx} = 3\sqrt{x} + e^x - \frac{5}{2\sqrt{x}}$ (A1)
- (b)
- $3\sqrt{x} + e^x - \frac{5}{2\sqrt{x}} = 0 \implies 6x + 2\sqrt{x}e^x - 5 = 0$ (M1)
 - $6x = 5 - 2\sqrt{x}e^x \implies x = \frac{5 - 2\sqrt{x}e^x}{6}$ (A1)
- (c)
- (i) $x_2 = 0.44473$ (B1)
 - (ii) $x_3 = 0.48654, x_4 = 0.45512, x_5 = 0.47885 \implies x = 0.46869$ (B2)

(Total for question 10 is 8 marks)

11

- (a)
- Let $f(x) = \ln(5 - x) - x$ (M1)
 - $f(1) = \ln(4) - 1 = 0.386$ (M1)
 - $f(2) = \ln(3) - 2 = -0.901$ (M1)
 - Change of sign and continuous function $\therefore 1 < \alpha < 2$ (A1)
- (b)
- Yes. The cobweb diagram converges to the point of intersection. (B2)

(Total for question 11 is 4 marks)

12

- The staircase diagram converges to the root. Yes the iteration formula can be used. (B2)

(Total for question 12 is 2 marks)

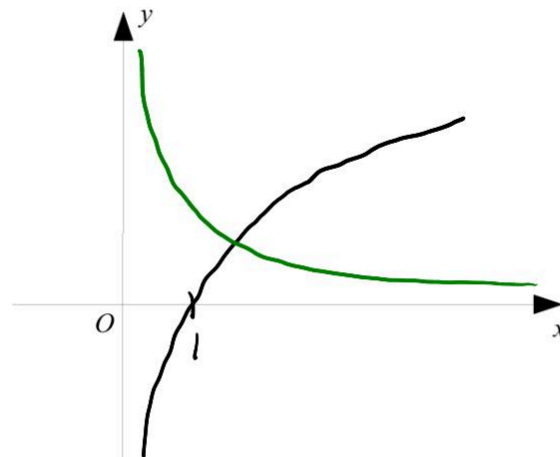
13

- (a)
- Sector area = $\frac{1}{2}r^2\theta$, Triangle area = $\frac{1}{2}r^2 \sin \theta$ (M1)
 - $\frac{1}{2}r^2 \sin \theta = \frac{3}{4} \cdot \frac{1}{2}r^2\theta$ (M1)
 - $\sin \theta = \frac{3}{4}\theta \implies \sin \theta - \frac{3}{4}\theta = 0$ (A1)
- (b)
- $f(\theta) = \sin \theta - \frac{3}{4}\theta$, $f'(\theta) = \cos \theta - \frac{3}{4}$ (M1)
 - $\theta_1 = 2 - \frac{\sin(2)-1.5}{\cos(2)-0.75} = 1.4935$ (M1)
 - $\theta_2 = 1.3105, \theta_3 = 1.2769, \theta_4 = 1.2757$ (A1)

(Total for question 13 is 6 marks)

14

(a)



$$y = \ln x$$

$$y = \frac{1}{x}$$

(2)

(B2)

- (b)
- There is one intersection $\therefore$ one solution for $\ln x = \frac{1}{x}$ (B1)
- (c)
- Let $f(x) = \ln x - \frac{1}{x}$ (M1)
 - $f(1.7) = \ln(1.7) - \frac{1}{1.7} = -0.057$ (M1)
 - $f(1.8) = \ln(1.8) - \frac{1}{1.8} = 0.032$ (M1)
 - Change of sign and continuous function $\therefore$ solution lies between 1.7 and 1.8. (A1)
- (d)
- $x_2 = e^{\frac{1}{1.8}} = 1.7429$ (B1)
 - $x_3 = 1.7749$ (B1)
 - $x_4 = 1.7566$ (B1)

(Total for question 14 is 7 marks)

15

- (a)
- min point = $(\frac{\pi}{2}, 1)$ (B1)
 - asymptotes $x = 0$ and $x = \pi$ (B1)
- (b)
- $f(x) = x - \operatorname{cosec} x$ (M1)
 - $f(2.5) = 2.5 - \frac{1}{\sin 2.5} = 0.829$ (M1)
 - $f(3) = 3 - \frac{1}{\sin 3} = -4.086$ (M1)
 - Change of sign and continuous $\therefore$ root between 2.5 and 3. (A1)
- (c)
- The iteration formula cannot be used to find this root. It converges to a different root. (B3)

(Total for question 15 is 6 marks)

16

- (a)
- $f(0) = \ln(3 - 2(0)) + (0)^2 - 2(0) = \ln 3$ (M1)
 - $f(1) = \ln(3 - 2(1)) + (1)^2 - 2(1) = -1$ (M1)
 - Change of sign and continuous in the interval $\therefore$ x -coordinate lies between 0 and 1 (A1)
- (b)
- $\frac{dy}{dx} = -\frac{2}{3-2x} + 2x - 2$ (M1)
 - $x_1 = 1 - \frac{-1}{-2} = 0.5$ (M1)
 - $x_2 = 0.4716, x_3 = 0.4718, x_4 = 0.47178$ (A4)

(Total for question 16 is 8 marks)

17

- (a)
- $u = 2x, v = \sin x \implies \frac{dy}{dx} = 2 \sin x + 2x \cos x$ (M1)
 - Max point where $\frac{dy}{dx} = 0 \implies 2 \sin x + 2x \cos x = 0$ (M1)
 - Divide by $2 \cos x$: $\tan x + x = 0$ (A1)
- (b)
- $f(x) = \tan x + x, f'(x) = \sec^2 x + 1$ (M1)
 - Let $x_0 = 2$ (B1)
 - $x_1 = 2 - \frac{\tan 2 + 2}{(\sec 2)^2 + 1} = 2.0273$ (M1)
 - $x_2 = 2.0288, x_3 = 2.0288$ (A1)

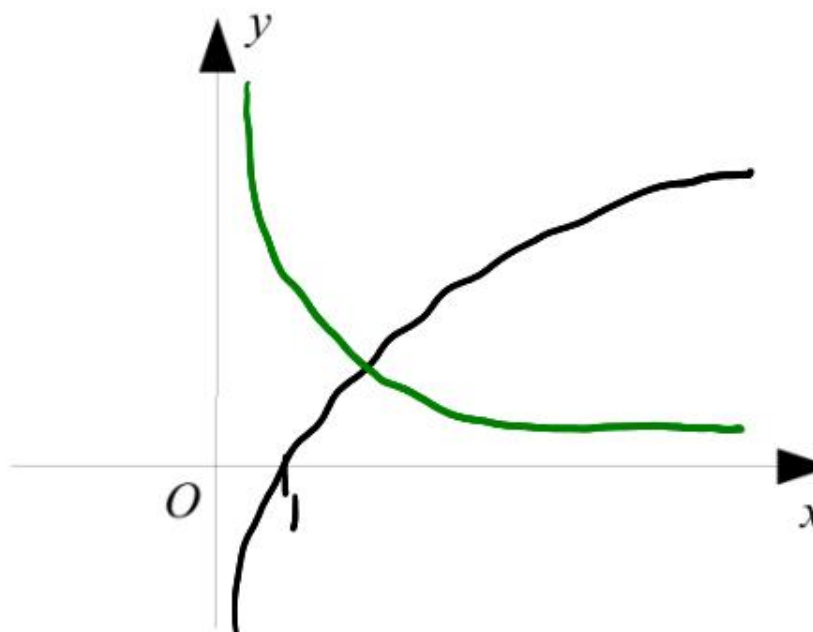
(Total for question 17 is 7 marks)

18

- (a)
- $f(0) = e^0 \sqrt{0} - 1 = -1$ (M1)
 - $f(1) = e^1 \sqrt{1} - 1 = 1.718$ (M1)
 - Change of sign and continuous function $\therefore \alpha$ lies between 0 and 1. (A1)
- (b)
- $u = e^x, v = x^{\frac{1}{2}} \implies f'(x) = e^x x^{\frac{1}{2}} + e^x \frac{1}{2} x^{-\frac{1}{2}}$ (M1)
 - $f'(x) = e^x \left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right) = e^x \left(\frac{2x+1}{2\sqrt{x}} \right)$ (A2)
- (c)
- $x_2 = 1 - \frac{e-1}{1.5e} = 0.57859$ (M1)
 - $x_3 = 0.43757$ (Note: markscheme says x_3 is 0.42636 in later steps) (A1)
- (d)
- When $x = 0$ the tangent is vertical / $f'(0)$ is undefined. (B2)

(Total for question 18 is 9 marks)

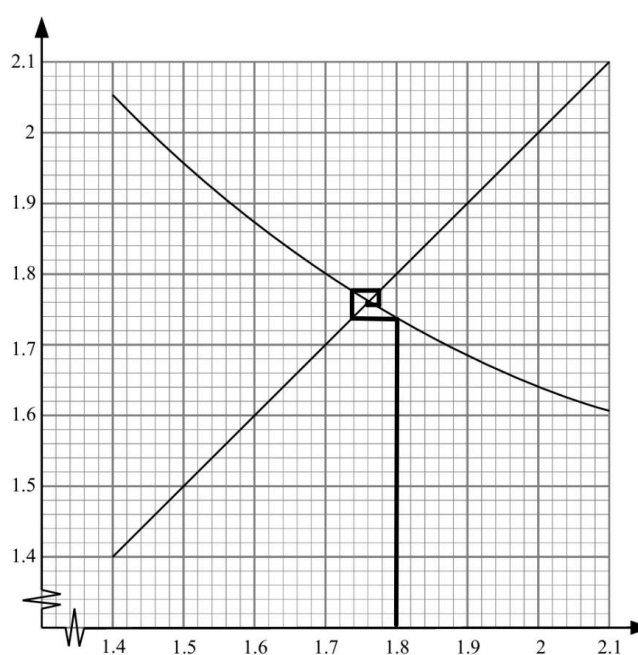
(a)



(B2)

- (b) • One intersection of $y = \frac{1}{x}$ and $y = \ln x$ $\therefore$ one solution. (B1)
- (c) • $\ln 1.7 - \frac{1}{1.7} = -0.0576$ (M1)
 • $\ln 1.8 - \frac{1}{1.8} = 0.0322$ (M1)
 • Change of sign and continuous function $\therefore$ solution lies between 1.7 and 1.8. (A1)
- (d) • $x_2 = e^{\frac{1}{1.8}} = 1.7429$ (B1)
 • $x_3 = 1.7749$ (B1)
 • $x_4 = 1.7566$ (B1)

(e)



(B2)

(Total for question 19 is 9 marks)
