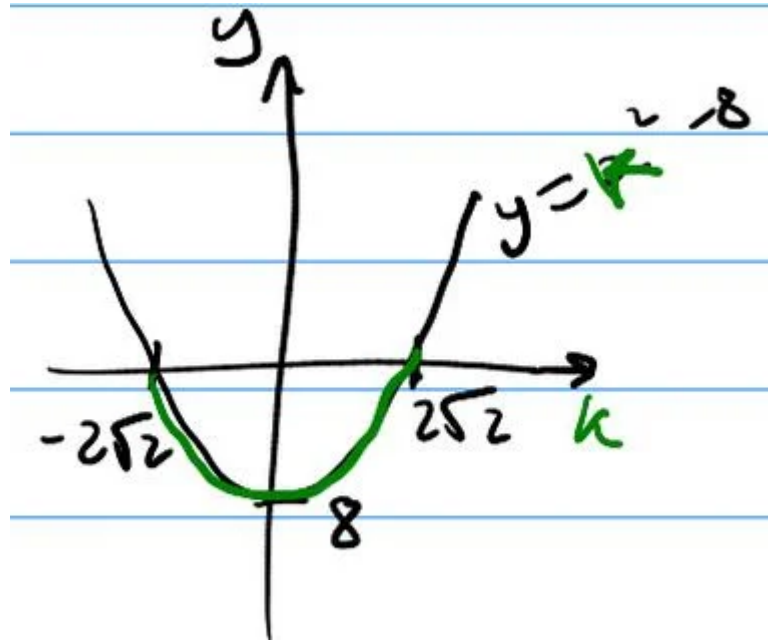


1

- Use the discriminant $b^2 - 4ac < 0$ for no real roots. (M1)
- Substitute values: $k^2 - 4(1)(2) < 0 \Rightarrow k^2 - 8 < 0$. (M1)
- Find critical values $k = \pm 2\sqrt{2}$ and identify the correct range: $-2\sqrt{2} < k < 2\sqrt{2}$. (A1)



(Total for question 1 is 3 marks)

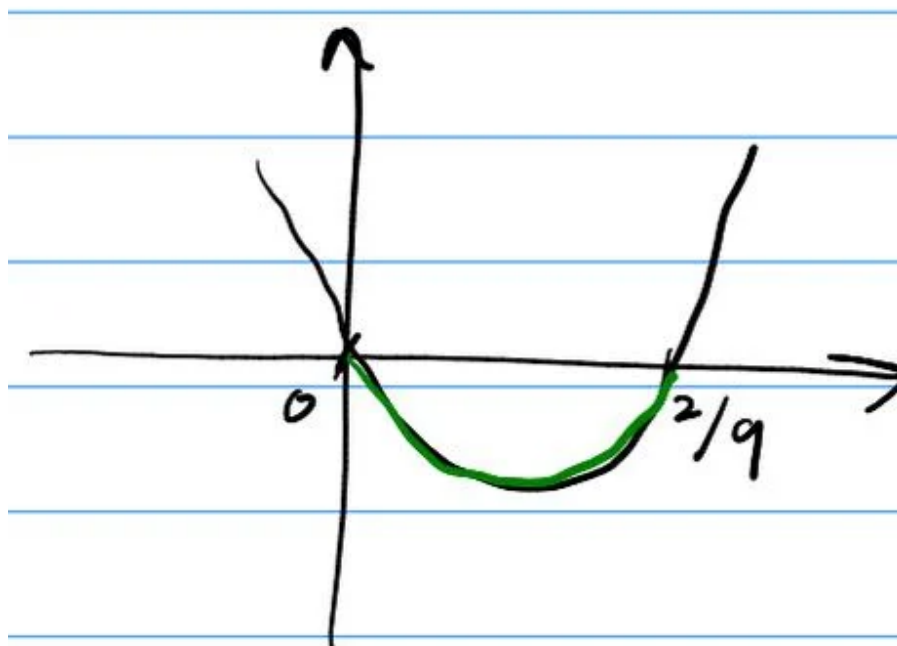
2

- Use the discriminant $b^2 - 4ac = 0$ for equal roots. (M1)
- $(5)^2 - 4(k)(k) = 0 \Rightarrow 25 - 4k^2 = 0$. (M1)
- $k^2 = \frac{25}{4} \Rightarrow k = \pm \frac{5}{2}$. Since k is positive, $k = \frac{5}{2}$. (A1)

(Total for question 2 is 3 marks)

3

- $b^2 - 4ac < 0 \Rightarrow (6k)^2 - 4(k)(2) < 0$. (M1)
- $36k^2 - 8k < 0 \Rightarrow 4k(9k - 2) < 0$. (M1)
- Critical values $k = 0$ and $k = \frac{2}{9}$. (A1)
- Correct range $0 < k < \frac{2}{9}$. (A1)



(Total for question 3 is 4 marks)

4

- $b^2 - 4ac < 0 \Rightarrow (7)^2 - 4(a)(4) < 0$. (M1)
- $49 - 16a < 0 \Rightarrow 49 < 16a$. (M1)
- $a > \frac{49}{16}$. (A1)

(Total for question 4 is 3 marks)

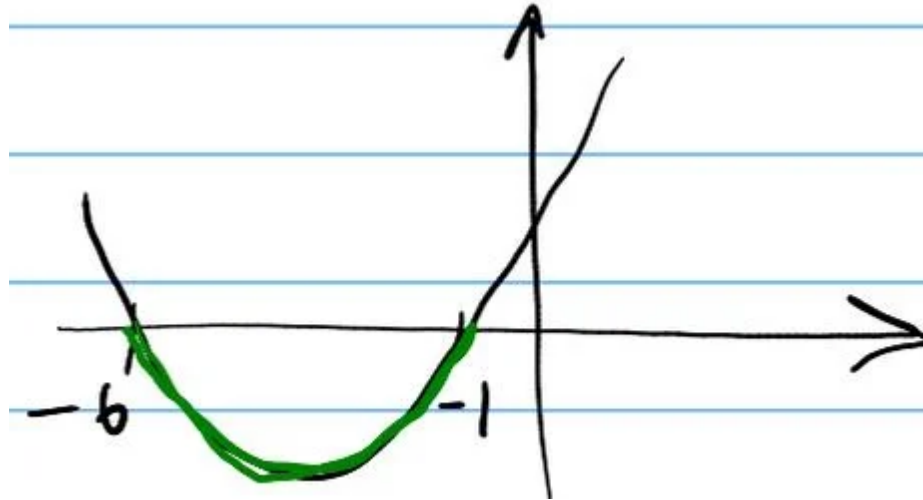
5

- Rearrange to $2x^2 - 6x + 5 = 0$. (M1)
- Calculate discriminant: $b^2 - 4ac = (-6)^2 - 4(2)(5) = 36 - 40 = -4$. (M1)
- Conclude that since $b^2 - 4ac < 0$, there are no real roots. (A1)

(Total for question 5 is 3 marks)

6

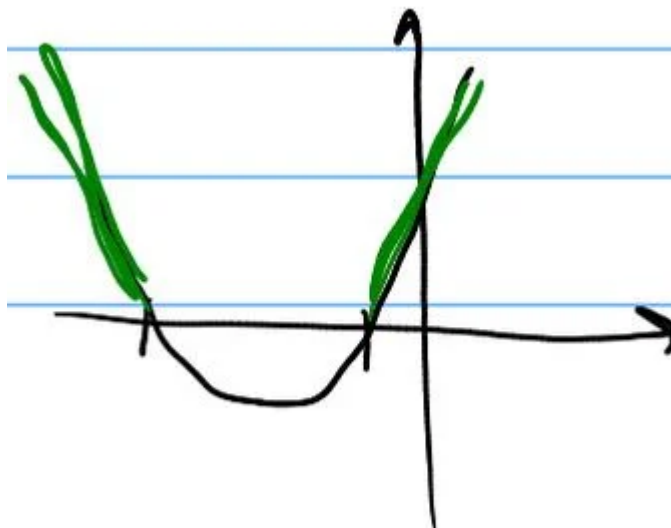
- $b^2 - 4ac > 0 \Rightarrow (4)^2 - 4(k+5)(k+2) > 0$. (M1)
- $16 - 4(k^2 + 7k + 10) > 0 \Rightarrow 16 - 4k^2 - 28k - 40 > 0$. (M1)
- $-4k^2 - 28k - 24 > 0 \Rightarrow k^2 + 7k + 6 < 0$. (M1)
- $(k+6)(k+1) < 0$. (M1)
- Critical values $k = -6, k = -1$. (A1)
- Correct range $-6 < k < -1$. (A1)



(Total for question 6 is 6 marks)

7

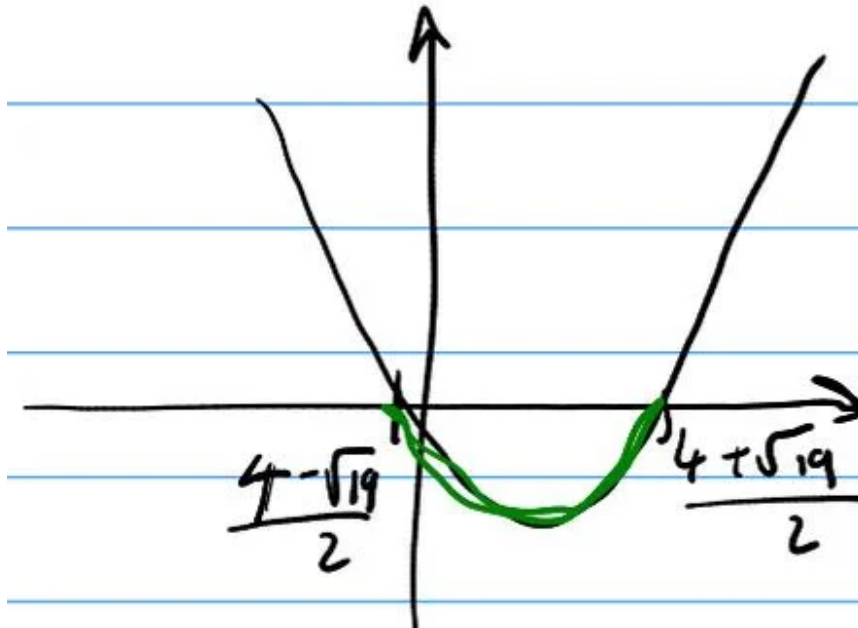
- $b^2 - 4ac > 0 \Rightarrow (n+1)^2 - 4(1)(3-3n) > 0$. (M1)
- $n^2 + 2n + 1 - 12 + 12n > 0 \Rightarrow n^2 + 14n - 11 > 0$. (M1)
- Solve $n^2 + 14n - 11 = 0$ using quadratic formula: $n = \frac{-14 \pm \sqrt{14^2 - 4(1)(-11)}}{2} = \frac{-14 \pm \sqrt{240}}{2} = -7 \pm 2\sqrt{15}$. (M1)
- Identify correct regions: $n < -7 - 2\sqrt{15}$ or $n > -7 + 2\sqrt{15}$. (A1)



(Total for question 7 is 4 marks)

8

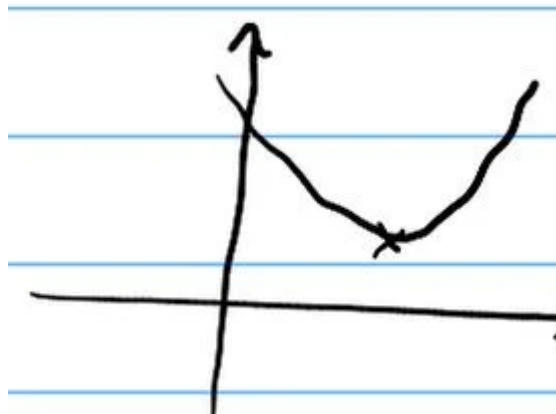
- $b^2 - 4ac < 0 \Rightarrow (2k - 3)^2 - 4(1)(k + 3) < 0$. (M1)
- $4k^2 - 12k + 9 - 4k - 12 < 0 \Rightarrow 4k^2 - 16k - 3 < 0$. (M1)
- Solve $4k^2 - 16k - 3 = 0$: $k = \frac{16 \pm \sqrt{256 - 4(4)(-3)}}{8} = \frac{16 \pm \sqrt{304}}{8} = \frac{16 \pm 4\sqrt{19}}{8} = \frac{4 \pm \sqrt{19}}{2}$. (M1)
- Correct range: $\frac{4 - \sqrt{19}}{2} < k < \frac{4 + \sqrt{19}}{2}$. (A1)



(Total for question 8 is 4 marks)

9

- Complete the square: $(x - 3)^2 - 9 + 11 = (x - 3)^2 + 2$. (M1)
- State that $(x - 3)^2 \geq 0$ for all x . (M1)
- Conclude that $(x - 3)^2 + 2 \geq 2$, which is always > 0 . (A1)



(Total for question 9 is 3 marks)

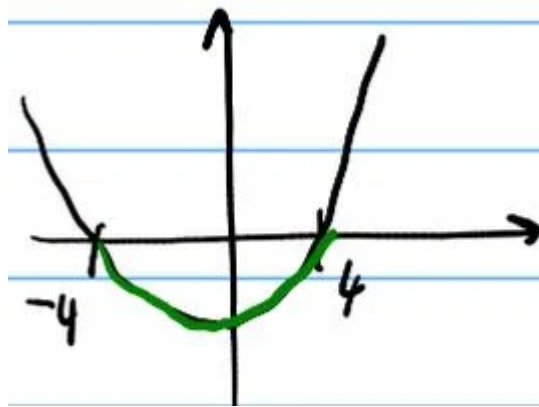
10

- Rearrange to $x^2 + 3x + 9 > 0$. (M1)
- Complete the square: $(x + \frac{3}{2})^2 - \frac{9}{4} + 9 = (x + \frac{3}{2})^2 + \frac{27}{4}$. (M1)
- State that $(x + \frac{3}{2})^2 \geq 0$, so $(x + \frac{3}{2})^2 + \frac{27}{4} \geq \frac{27}{4}$. (M1)
- Conclude that the expression is always > 0 . (A1)

(Total for question 10 is 4 marks)

11

- (a)
- $(x + \frac{p}{2})^2 - \frac{p^2}{4} + 4 = 0$. (M1)
 - $(x + \frac{p}{2})^2 = \frac{p^2}{4} - 4$. (M1)
 - $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - 4}$ or $x = \frac{-p \pm \sqrt{p^2 - 16}}{2}$. (A1)
- (b)
- No real roots when $p^2 - 16 < 0$. (M1)
 - $(p + 4)(p - 4) < 0 \Rightarrow -4 < p < 4$. (A1)



(Total for question 11 is 5 marks)

12

- Equation in completed square form is $(x - 4)^2 - 3$. (M1)
- Expand: $x^2 - 8x + 16 - 3 = x^2 - 8x + 13$. (M1)
- Compare coefficients: $a = -8$. (A1)
- $b = 13$. (A1)

(Total for question 12 is 4 marks)

13

- Factor out 2: $2(x^2 + 4x) + 1$. (M1)
- Complete the square inside: $2[(x + 2)^2 - 4] + 1$. (M1)
- Expand: $2(x + 2)^2 - 8 + 1 = 2(x + 2)^2 - 7$. (A1)
- $a = 2, b = 2, c = -7$. (A1)

(Total for question 13 is 4 marks)

14

- (a)
- $(x + 4.5)^2 - 20.25 + 3$. (M1)
 - $(x + 4.5)^2 - 17.25$ or $(x + \frac{9}{2})^2 - \frac{69}{4}$. (A1)
- (b)
- Minimum point is $(-4.5, -17.25)$ or $(-\frac{9}{2}, -\frac{69}{4})$. (A2)

(Total for question 14 is 4 marks)

15

- (a)
- $(x + \frac{k}{2})^2 - \frac{k^2}{4} - 6 = 0$. (M1)
 - $(x + \frac{k}{2})^2 = \frac{k^2}{4} + 6$. (M1)
 - $x + \frac{k}{2} = \pm \sqrt{\frac{k^2}{4} + 6}$. (M1)
 - $x = -\frac{k}{2} \pm \sqrt{\frac{k^2}{4} + 6}$. (A1)
- (b)
- Substitute $k = 4$ into the expression from part (a). (M1)
 - $x = -\frac{4}{2} \pm \sqrt{\frac{16}{4} + 6} = -2 \pm \sqrt{10}$. (A1)

(Total for question 15 is 6 marks)

16

- (a)
- Set equations equal: $\frac{2}{x} + k = -3x + 2$. (M1)
 - Multiply by x : $2 + kx = -3x^2 + 2x$. (M1)
 - Rearrange: $3x^2 + kx - 2x + 2 = 0 \Rightarrow 3x^2 + (k - 2)x + 2 = 0$. (A1)
- (b)
- For a tangent, the discriminant $b^2 - 4ac = 0$. (M1)
 - $(k - 2)^2 - 4(3)(2) = 0 \Rightarrow (k - 2)^2 - 24 = 0$. (M1)
 - $k - 2 = \pm \sqrt{24} = \pm 2\sqrt{6} \Rightarrow k = 2 \pm 2\sqrt{6}$. (A1)

(Total for question 16 is 6 marks)

17

- (a)
- Substitute $x = 70$: $y = 55 - 0.015(70 - 50)^2$. (M1)
 - $y = 55 - 0.015(400) = 55 - 6 = 49$. (A1)
- (b)
- The maximum value occurs at the vertex of the parabola. (M1)
 - $x = 50$ mph. (A1)
- (c)
- Fuel consumption y cannot be negative. (M1)
 - Solve $55 - 0.015(x - 50)^2 = 0 \Rightarrow (x - 50)^2 = \frac{55}{0.015} \approx 3666.7$. (M1)
 - $x \approx 50 \pm 60.5$. Range is roughly $0 < x < 111$. (A1)

(Total for question 17 is 7 marks)

18

- (a)
- $-3(x^2 - 6x) + 4$. (M1)
 - $-3[(x - 3)^2 - 9] + 4$. (M1)
 - $-3(x - 3)^2 + 27 + 4 = -3(x - 3)^2 + 31$. (A1)
- (b)
- From part (a), vertex is (3, 31). (A2)

(Total for question 18 is 5 marks)

19

- (a)
- $b = -3, c = 8$. (A2)
- (b)
- New equation: $y = a(x + 4 - 3)^2 + 8 = a(x + 1)^2 + 8$. (M1)
 - Substitute $(-6, 13)$: $13 = a(-6 + 1)^2 + 8$. (M1)
 - $5 = 25a \Rightarrow a = \frac{1}{5}$. (A1)

(Total for question 19 is 5 marks)

20

- (a)
- $2(x^2 + 4.5x) + k$. (M1)
 - $2[(x + 2.25)^2 - 5.0625] + k$. (M1)
 - $2(x + 2.25)^2 - 10.125 + k$. (A1)
- (b)
- Minimum point must be above $y = 2$: $k - 10.125 > 2$. (M1)
 - $k > 12.125$ or $k > \frac{97}{8}$. (A2)

(Total for question 20 is 6 marks)

21

- (a)
- Set $y = 0$: $-0.25x^2 + 2.125x - 4.125 = 0$. (M1)
 - Solve for x : $x = 3$ or $x = 5.5$. (M1)
 - From diagram, A is the first root, so $A = (3, 0)$. (A1)
- (b)
- Distance = $5.5 - 3 = 2.5$. (A1)
- (c)
- Max height at midpoint of roots: $x = \frac{3+5.5}{2} = 4.25$. (M1)
 - Substitute $x = 4.25$ into equation. (M1)
 - $y = -0.25(4.25)^2 + 2.125(4.25) - 4.125$. (M1)
 - $y = \frac{25}{64} \approx 0.39$. (A1)

(Total for question 21 is 8 marks)

- 22
- $b^2 - 4ac = 0 \Rightarrow (-5)^2 - 4(1)(k) = 0$. (M1)
 - $25 - 4k = 0 \Rightarrow k = 6.25$. (A1)

(Total for question 22 is 2 marks)

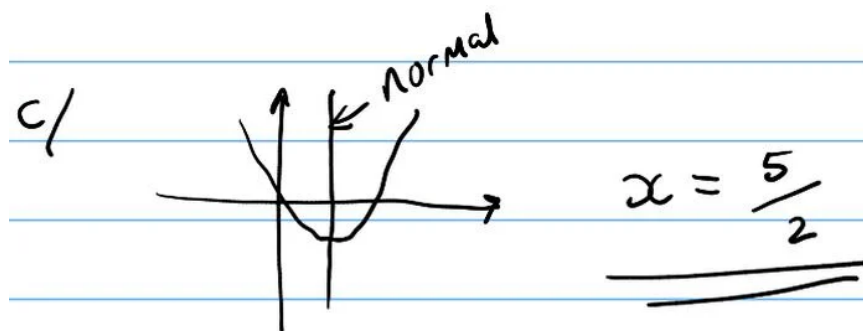
- 23
- (a)
- $4(x^2 - 2x) + 5$. (M1)
 - $4[(x - 1)^2 - 1] + 5$. (M1)
 - $4(x - 1)^2 + 1$. (A1)
- (b)
- Discriminant: $(-8)^2 - 4(4)(5) = 64 - 80 = -16$. (M1)
 - Since $b^2 - 4ac < 0$, there are 0 real roots. (A1)
- (c)
- If $r > 0$, the minimum point is above the x -axis, so 0 real roots. (C1)
 - If $r = 0$, the minimum point is on the x -axis, so 1 real root. (C1)
 - If $r < 0$, the minimum point is below the x -axis, so 2 real roots. (C1)



(Total for question 23 is 8 marks)

24

- (a)
- $3(x^2 - 5x) + 4$. (M1)
 - $3[(x - 2.5)^2 - 6.25] + 4$. (M1)
 - $3(x - 2.5)^2 - 18.75 + 4 = 3(x - 2.5)^2 - 14.75$. (A1)
- (b)
- Minimum point is $(2.5, -14.75)$. (A2)
- (c)
- The normal at the minimum point is a vertical line through the x -coordinate. (M1)
 - $x = 2.5$. (A1)



(Total for question 24 is 7 marks)

25

- (a)
- Substitute $x = 25$: $P = 35 - 1.25(25 - 15)^2$. (M1)
 - $P = 35 - 1.25(100) = 35 - 125 = -90$. (A1)
 - A profit of $-\text{£}90,000$ means they make a loss. (C1)
- (b)
- Set $P = 20$: $20 = 35 - 1.25(x - 15)^2$. (M1)
 - $1.25(x - 15)^2 = 15 \Rightarrow (x - 15)^2 = 12$. (M1)
 - $x = 15 \pm \sqrt{12} = 15 \pm 2\sqrt{3}$. (M1)
 - $x \approx 15 - 3.46 = 11.54$. Least price is $\text{£}11.54$. (A1)
- (c)
- (i) Max profit is the constant term: $\text{£}35,000$. (A1)
 - (ii) Max profit occurs when the squared term is zero: $x = \text{£}15$. (A2)

(Total for question 25 is 10 marks)