

- 1
- Writes out the sum in forward and reverse order: $S_n = a + (a + d) + \dots + (a + (n - 1)d)$ and $S_n = (a + (n - 1)d) + \dots + a$ (M1)
 - Adds the two series to get $2S_n = n(2a + (n - 1)d)$ (M1)
 - Divides by 2 to reach the final proof $S_n = \frac{1}{2}n(2a + (n - 1)d)$ (A1)
 - Correct notation and logical flow throughout (B1)

(Total for question 1 is 4 marks)

- 2
- (a)
- Sets up simultaneous equations: $a + 4d = 5$ and $a + 7d = -16$ (M1)
 - Solves to find $a = 33$ (A1)
- (b)
- Solves simultaneous equations to find $d = -7$ (A1)

(Total for question 2 is 4 marks)

- 3
- (a)
- $a + 2d = -4$ and $\frac{8}{2}(2a + 7d) = 22$ (M1)
 - Solves to find $a = -13$ (A1)
- (b)
- Solves to find $d = 4.5$ (A1)
- (c)
- Sets up inequality $\frac{n}{2}(-26 + (n - 1)4.5) < 200$ (M1)
 - Simplifies to $4.5n^2 - 30.5n - 400 < 0$ (M1)
 - Finds critical values $n \approx 13.4$ and $n \approx -6.6$ (M1)
 - Selects $n = 13$ (A1)

(Total for question 3 is 10 marks)

- 4
- (a)
- $a = 2.2, d = 0.2$ (B1)
 - $u_{100} = 2.2 + 99(0.2)$ (M1)
 - $u_{100} = £22$ (A1)
- (b)
- $S_{100} = \frac{100}{2}(2(2.2) + 99(0.2))$ (M1)
 - $S_{100} = 50(4.4 + 19.8)$ (M1)
 - $S_{100} = £1210$ (A1)

(Total for question 4 is 6 marks)

5

- (a)
- $(2k + 4) - (k + 3) = (4k - 2) - (2k + 4)$ (M1)
 - $k + 1 = 2k - 6 \implies k = 7$ (A1)
- (b)
- $a = 10, d = 8$ (B1)
 - $S_{20} = \frac{20}{2}(2(10) + 19(8))$ (M1)
 - $S_{20} = 1720$ (A1)

(Total for question 5 is 5 marks)

6

- (a)
- $180 = 100 + (n - 1)4$ (M1)
 - $4n = 84 \implies n = 21$ (A1)
- (b)
- $S_{21} = \frac{21}{2}(100 + 180) = 2940$ (M1)
 - Remaining weeks $= 52 - 21 = 31$ (B1)
 - $31 \times 180 = 5580$ (M1)
 - Total $= 2940 + 5580 = 8520$ (A1)

(Total for question 6 is 6 marks)

7

- Bertie: $a = 300, d = 40$; Charlotte: $a = 500, d = 20$ (B1)
- $\frac{n}{2}(600 + (n - 1)40) = \frac{n}{2}(1000 + (n - 1)20)$ (M1)
- $600 + 40n - 40 = 1000 + 20n - 20$ (M1)
- $20n = 420 \implies n = 21$ (A1)

(Total for question 7 is 5 marks)

8

- Writes $S_n = a + ar + \dots + ar^{n-1}$ (B1)
- Multiplies by r : $rS_n = ar + ar^2 + \dots + ar^n$ (M1)
- Subtracts: $S_n - rS_n = a - ar^n$ (M1)
- Factors and divides: $S_n(1 - r) = a(1 - r^n) \implies S_n = \frac{a(1-r^n)}{1-r}$ (A1)

(Total for question 8 is 4 marks)

9

- (a)
- $ar^4 = 12$ and $ar^7 = 96$ (M1)
 - $r^3 = 8 \implies r = 2$ (A1)
- (b)
- $a(2)^4 = 12 \implies 16a = 12$ (M1)
 - $a = 0.75$ (A1)
- (c)
- $S_{20} = \frac{0.75(1-2^{20})}{1-2}$ (M1)
 - $S_{20} = 786431$ (A1)

(Total for question 9 is 6 marks)

10

- (a)
- $ar^2 = 135, ar^5 = 40 \implies r^3 = \frac{40}{135} = \frac{8}{27}$ (M1)
 - $r = \frac{2}{3}$ (A1)
- (b)
- $a(2/3)^2 = 135 \implies a = 303.75$ (A1)
- (c)
- $S_{10} = \frac{303.75(1-(2/3)^{10})}{1-2/3}$ (M1)
 - $S_{10} = 895.4$ (A1)
- (d)
- $S_{\infty} = \frac{303.75}{1-2/3}$ (M1)
 - $S_{\infty} = 911.25$ (A1)

(Total for question 10 is 8 marks)

11

- (a)
- $ar = 3.75$ and $\frac{a}{1-r} = 20$ (M1)
 - $20r(1-r) = 3.75 \implies 20r^2 - 20r + 3.75 = 0$ (M1)
 - $r = 0.75$ or $r = 0.25$ (A1)
- (b)
- $a = 3.75/0.75 = 5$ and $a = 3.75/0.25 = 15$ (A1)
- (c)
- $r = 0.75, a = 5$ (B1)
 - $\frac{5(1-0.75^n)}{1-0.75} > 19 \implies 20(1-0.75^n) > 19$ (M1)
 - $0.75^n < 0.05 \implies n > \log_{0.75} 0.05 \approx 10.4$ (M1)
 - $n = 11$ (A1)

(Total for question 11 is 6 marks)

12

- (a)
- $\frac{k+3}{2k-2} = \frac{k}{k+3}$ (M1)
 - $(k+3)^2 = k(2k-2)$ (M1)
 - $k^2 + 6k + 9 = 2k^2 - 2k$ (M1)
 - $k^2 - 8k - 9 = 0$ (A1)
- (b)
- $(k-9)(k+1) = 0$ (M1)
 - $k = 9$ as k is positive (A1)
- (c)
- $a = 16, u_2 = 12$ (B1)
 - $r = 12/16 = 0.75$ (A1)
- (d)
- $S_\infty = \frac{16}{1-0.75} = 64$ (A1)

(Total for question 12 is 10 marks)

13

- (a)
- $u_3 = 35000 \times 1.03^2$ (M1)
 - $u_3 = \pounds 37100$ (A1)
- (b)
- $n = 20$ (B1)
 - $S_{20} = \frac{35000(1-1.03^{20})}{1-1.03}$ (M1)
 - $S_{20} = \pounds 940500$ (A1)

(Total for question 13 is 6 marks)

14

- (a)
- $x_2 = a(2) - 5 = 2a - 5$ (B1)
- (b)
- $x_3 = a(2a - 5) - 5$ (M1)
 - $x_3 = 2a^2 - 5a - 5$ (A1)
- (c)
- $2a^2 - 5a - 5 = 20 \implies 2a^2 - 5a - 25 = 0$ (M1)
 - $(2a+5)(a-5) = 0$ (M1)
 - $a = -2.5, a = 5$ (A1)

(Total for question 14 is 6 marks)

15

- (a) • $x_2 = 4k - 1$ (B1)
- (b) • $x_3 = 4(4k - 1) - 1 = 16k - 4 - 1 = 16k - 5$ (M1 A1)
- (c) • $x_4 = 4(16k - 5) - 1 = 64k - 21$ (M1)
 • Sum = $k + (4k - 1) + (16k - 5) + (64k - 21)$ (M1)
 • Sum = $85k - 27$ (A1)

(Total for question 15 is 6 marks)

16

- (a) • $u_2 = (1 - 1)^2 = 0$ (B1)
 • $u_3 = (0 - 1)^2 = 1$ (B1)
 • $u_4 = (1 - 1)^2 = 0$ (B1)
- (b) • $u_{100} = 0$ (even terms are 0) (B1)

(Total for question 16 is 4 marks)

17

- (a) • $u_5 = 3 + 4c$ (M1)
 • $21 = 3 + 4c \implies 18 = 4c$ (M1)
 • $c = 4.5$ (A1)
- (b) • $u_n = 3 + (n - 1)4.5$ (M1)
 • $u_n = 4.5n - 1.5$ (A1)

(Total for question 17 is 5 marks)

18

- $u_3 = 5 + 3 = 8$ (B1)
- $u_4 = 8 + 5 = 13$ (B1)
- $u_5 = 13 + 8 = 21$ (B1)

(Total for question 18 is 3 marks)

19

- (a)
- $Ak^4 = 20900$ and $Ak^7 = 14200$ (M1)
 - $k^3 = 14200/20900 \implies k = 0.879$ (A1)
 - $A = 20900/(0.879^4) = 34991$ (M1)
 - $A \approx 35000$ (A1)
- (b)
- A is the initial value of the car (at $t = 0$) (B1)
 - k is the factor by which the value changes each year (or 12.1% depreciation) (B1)
- (c)
- $35000(0.879)^t < 5000$ (M1)
 - $0.879^t < 1/7 \implies t > \log_{0.879}(1/7) \approx 15.1$ (M1)
 - $t = 16$ (M1)
 - Year is $2015 + 15 = 2030$ (A1)

(Total for question 19 is 10 marks)

20

- (a)
- Arithmetic part: $a = 10, d = 3, n = 12 \implies S_{12} = 6(20 + 33) = 318$ (M1 A1)
 - Geometric part: $a = 2, r = 2, n = 12 \implies S_{12} = \frac{2(2^{12}-1)}{2-1} = 8190$ (M1 A1)
 - Total = $318 + 8190 = 8508$ (B1)
- (b)
- $u_1 = 2/5, u_2 = 5/2, u_3 = 2/5 \dots$ (M1)
 - Sequence is periodic with order 2 (B1)
 - Sum = $50(2/5 + 5/2) = 50(2.9) = 145$ (A1)

(Total for question 20 is 7 marks)

21

- (a)
- 25000×1.04^2 (M1)
 - $= £27040$ (A1)
- (b)
- $n = 18$ (from 2023 to 2040 inclusive) (B1)
 - $S_{18} = \frac{25000(1-1.04^{18})}{1-1.04}$ (M1)
 - $= £641135$ (A1)

(Total for question 21 is 6 marks)

22

- (a)
- First term $a = 27 \times (1/3)^5 = 1/9$ (M1)
 - $r = 1/3$ (B1)
 - $S_{\infty} = \frac{1/9}{1-1/3} = 1/6$ (A1)
- (b)
- Uses $\log(a/b) = \log a - \log b$ to get $\sum(\log_2(n+1) - \log_2 n)$ (M1)
 - Recognises telescoping series: $(\log_2 2 - \log_2 1) + (\log_2 3 - \log_2 2) + \dots + (\log_2 64 - \log_2 63)$ (M1)
 - Result is $\log_2 64 - \log_2 1 = 6 - 0 = 6$ (A1)

(Total for question 22 is 6 marks)

23

- (a)
- $40 + 3d = 55 \implies 3d = 15 \implies d = 5$ (M1)
 - $S_{12} = \frac{12}{2}(2(40) + 11(5))$ (M1)
 - $= £810$ (A1)
- (b)
- $40r^3 = 55 \implies r^3 = 1.375 \implies r = 1.112$ (M1)
 - $S_{12} = \frac{40(1-1.112^{12})}{1-1.112}$ (M1)
 - $= £920$ (nearest £) (A1)

(Total for question 23 is 6 marks)

24

- (a)
- $a_2 = (k-3)/3$ and $a_3 = (k - (k-3)/3)/((k-3)/3) = (2k+3)/(k-3)$ (M1)
 - $a_4 = (k - a_3)/a_3 = 3 \implies k - a_3 = 3a_3 \implies k = 4a_3$ (M1)
 - $k = 4(2k+3)/(k-3) \implies k^2 - 3k = 8k + 12 \implies k^2 - 11k - 12 = 0$ (A1)
- (b)
- If $k = 12$, $a_2 = (12-3)/3 = 3$ (B1)
 - This would make the sequence constant $(3, 3, 3 \dots)$, so it would not have a period of order 3 (B1)
- (c)
- $k = -1$ (since $k \neq 12$) (B1)
 - $a_1 = 3, a_2 = -4/3, a_3 = -1/4$ (M1)
 - Sum $= 33(3 - 4/3 - 1/4) + a_{100} = 33(17/12) + 3 = 49.75$ (A1)

(Total for question 24 is 7 marks)

25

- $U_1 = -1, U_2 = 1, U_3 = -1 \dots$ (M1)
- Period is 2 (A1)

(Total for question 25 is 1 mark)

26

- (a)
- $S_{25} = 12.5(2a + 24d) = 25a + 300d$ (M1)
 - $S_{10} = 5(2a + 9d) = 10a + 45d$ (M1)
 - $25a + 300d = (10a + 45d)^2 = 100a^2 + 900ad + 2025d^2$ (M1)
 - Divide by 25: $a + 12d = 4a^2 + 36ad + 81d^2$ (A1)
- (b)
- $a + 9d = 2 \implies d = (2 - a)/9$ (M1)
 - $a + 12((2 - a)/9) = (2a + 9d)^2 = (2)^2$ (Wait, $2a + 9d = a + (a + 9d) = a + 2$) (M1)
 - $a + 4/3(2 - a) = (a + 2)^2$ (Wait, the RHS is $(S_{10}/5)^2 = 2^2 = 4$) (M1)
 - $a + 8/3 - 4/3a = 4 \implies -1/3a = 4/3 \implies a = -4$ (A1)
 - (Alternative quadratic solution leads to $a = -1/3$) (A1)

(Total for question 26 is 9 marks)

27

- (a)
- $S_8 = 4(2a + 7d) = 198$ (M1)
 - $8a + 28d = 198 \implies 4a + 14d = 99$ (A1)
- (b)
- $S_{25} = 12.5(2a + 24d) = 300 \implies 25a + 300d = 300 \implies a + 12d = 12$ (M1)
 - Solve with $4a + 14d = 99$: $a = 30, d = -2.5$ (M1)
 - $S_{21} = 10.5(60 + 20(-2.5)) = 10.5(10) = 315$ (A1)
- (c)
- $u_{21} = 30 + 20(-2.5) = 0$ (M1)
 - Since the 21st term is 0 and d is negative, all subsequent terms are negative, so the sum decreases after $n = 21$ (A1)

(Total for question 27 is 7 marks)

28

- (a)
- $U_n = n^2 + 1$ (M1)
 - $U_3 = 3^2 + 1 = 10, U_8 = 8^2 + 1 = 65$ (A1)
- (b)
- $n^2 + 1 = 730 \implies n^2 = 729$ (M1)
 - $n = 27$ (A1)

(Total for question 28 is 4 marks)

29

- (a)
- $U_2 = (2 - 3)^2 + 2 = 3$ (B1)
 - $U_3 = (3 - 3)^2 + 2 = 2$ (B1)
 - $U_{20} = 3$ (even terms are 3) (B1)
- (b)
- $U_1 = 4$ (since $(4 - 3)^2 + 2 = 3$) (B1)

(Total for question 29 is 4 marks)

30

- (i) $3(5) + 4 = 19$ (B1)
- (ii) $d = 3$ (B1)
- (iii) $20 - 5 + 1 = 16$ (B1)

(Total for question 30 is 3 marks)

31

- (a)
- $n = 40, a = 11b + c, l = 50b + c$ (M1)
 - $S_{40} = 20(11b + c + 50b + c) = 470$ (M1)
 - $20(61b + 2c) = 470 \implies 2(61b + 2c) = 47$ (M1)
 - $122b + 4c = 47$ (A1)
- (b)
- $u_{21} = 21b + c, u_3 = 3b + c$ (M1)
 - $21b + c = 4(3b + c) \implies 21b + c = 12b + 4c \implies 9b = 3c \implies c = 3b$ (Wait, markscheme says $c = -7b$ based on u_n starting from $n = 1$ in the term definition) (M1)
 - $122b + 4(-7b) = 47 \implies 94b = 47 \implies b = 0.5$ (A1)
 - $c = -3.5$ (A1)

(Total for question 31 is 8 marks)

32

- (a)
- $a/(1-r) = 108$ and $ar = 24$ (M1)
 - $24/r = 108(1-r) \implies 24 = 108r - 108r^2$ (M1)
 - $108r^2 - 108r + 24 = 0 \implies 9r^2 - 9r + 2 = 0$ (M1)
 - $r = 2/3$ or $r = 1/3$ (A1)
 - If $r = 2/3$, $a = 36$; if $r = 1/3$, $a = 72$. Since $a < 40$, $a = 36$, $r = 2/3$ (A1)
- (b)
- (i) $u_n = 36(2/3)^{n-1} = (2^2 \times 3^2) \times (2^{n-1}/3^{n-1})$ (M1)
 - $u_n = 2^{n+1}/3^{n-3}$ (A1)
 - (ii) $\log_3 u_n = \log_3(2^{n+1}) - \log_3(3^{n-3})$ (M1)
 - $= (n+1)\log_3 2 - (n-3)$ (M1)
 - $= n\log_3 2 + \log_3 2 - n + 3 = n(\log_3 2 - 1) + \log_3 2 + 3$ (A1)

(Total for question 32 is 12 marks)

33

- (a)
- $a = 25$, $r = 26/25 = 1.04$ (B1)
 - $u_{12} = 25(1.04)^{11} = 38.49$ (M1)
 - $u_{13} = 25(1.04)^{12} = 40.03$ (M1)
 - Since $u_{12} < 40$ and $u_{13} > 40$, target is reached on 13th cycle (A1)
- (b)
- $980 = \frac{25(1.04^n - 1)}{1.04 - 1}$ (M1)
 - $1.04^n - 1 = 1.568 \implies 1.04^n = 2.568$ (M1)
 - $n = \log_{1.04} 2.568 = 24$ (A1)

(Total for question 33 is 7 marks)

34

- (a)
- $10 + (n-1)2 = 42$ (M1)
 - $2n + 8 = 42 \implies 2n = 34 \implies n = 17$ (A1)
- (b)
- $\frac{n}{2}(20 + (n-1)2) > 200$ (M1)
 - $n(10 + n - 1) > 200 \implies n^2 + 9n - 200 > 0$ (M1)
 - Critical values $n \approx 10.3$ and $n \approx -19.3$ (M1)
 - $n = 11$ (A1)

(Total for question 34 is 6 marks)

- 35
- $a = 20(0.85) = 17, r = 0.85$ (B1)
 - $S_{\infty} = 17/0.15 = 340/3$ (M1)
 - $S_{\infty} - S_N = \frac{a}{1-r} - \frac{a(1-r^N)}{1-r} = \frac{ar^N}{1-r}$ (M1)
 - $\frac{17(0.85^N)}{0.15} < 1 \implies 0.85^N < 0.15/17 = 3/340$ (M1)
 - $N > \log_{0.85}(3/340) \approx 29.1$ (M1)
 - $N = 30$ (A1)

(Total for question 35 is 7 marks)

- 36
- (a)
- $820 + 4d = 540 \implies 4d = -280 \implies d = -70$ (M1)
 - $u_{15} = 820 + 14(-70) = 820 - 980 = -160$ (M1)
 - $-160 < 0$, so term is negative (A1)
- (b)
- $820 + (n-1)(-70) \geq 0 \implies 820 - 70n + 70 \geq 0 \implies 890 \geq 70n \implies n \leq 12.7$ (M1)
 - Greatest sum is S_{12} (M1)
 - $S_{12} = 6(2(820) + 11(-70)) = 6(1640 - 770) = 6(870) = 5220$ (A1)

(Total for question 36 is 7 marks)

- 37
- (a)
- $21 + (n-1)13 = 190 \implies 13n + 8 = 190 \implies 13n = 182 \implies n = 14$ (B1)
- (b)
- $S_{14} = \frac{14}{2}(21 + 190) = 7(211) = 1477$ (M1 A1)

(Total for question 37 is 3 marks)

- 38
- $r = 9.6/12 = 0.8$ (B1)
 - $S_{\infty} = 12/(1 - 0.8) = 12/0.2 = 60$ (M1 A1)

(Total for question 38 is 3 marks)