

1

- (a) Sketch the graphs of $\cos \theta$ and $\sec \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$

(4)

- (b) Sketch the graphs of $\sin \theta$ and $\csc \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$

(4)

- (c) Sketch the graphs of $\tan \theta$ and $\cot \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$

(4)

(Total for question 1 is 12 marks)

- 2 Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$2 \csc \theta = 5$$

Give your answers to 3 significant figures.

(Total for question 2 is 4 marks)

3 Solve, for $0 \leq x \leq 2\pi$, the equation,

$$2 \cot x = 3 \sec x$$

Give your answers in terms of π .

(Total for question 3 is 5 marks)

4 Solve, for $-\pi \leq x \leq \pi$, the equation,

$$5 \cos x + \cot x = 0$$

Give your answers to 2 decimal places where appropriate.

(Total for question 4 is 5 marks)

5 Solve, for $0 \leq x \leq 360$, the equation,

$$\sec^2 x + 5 \sec x + 6 = 0$$

Give your answers to 1 decimal place where appropriate.

(Total for question 5 is 5 marks)

6 Solve, for $0 \leq x \leq 360$, the equation,

$$\cot^2 x = 9$$

Give your answers to 1 decimal place.

(Total for question 6 is 5 marks)

7 Solve, for $-180 \leq \theta \leq 180$, the equation,

$$\sec(2\theta + 10) = -1.3$$

Give your answers to 1 decimal place.

(Total for question 7 is 4 marks)

8

- (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$

(2)

- (b) Solve, for $0 \leq \theta \leq 360$, the equation,

$$\tan^2 \theta + \sec^2 \theta + 5 \sec \theta = 2$$

Give your answers to 1 decimal place.

(5)

(Total for question 8 is 7 marks)

9

- (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$

(2)

- (b) Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$\operatorname{cosec}^2 \theta + \cot^2 \theta = 3$$

Give your answers in terms of π .

(5)

(Total for question 9 is 7 marks)

10 Solve, for $0 \leq x \leq 360$, the equation,

$$\tan^2 x + 4 \sec x - 2 = 0$$

Give your answers to 1 decimal place.

(Total for question 10 is 5 marks)

11 Solve, for $-180 \leq x \leq 180$, the equation,

$$2 \cot^2 x - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$$

Give your answers to 1 decimal place where appropriate.

(Total for question 11 is 5 marks)

12

(a) Prove the identity: $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$

(2)

(b) Prove the identity: $(\sec x - \cos x)^2 \equiv \tan^2 x - \sin^2 x$

(2)

(Total for question 12 is 4 marks)

13

(a) Prove that: $\sec^4 x - \tan^4 x \equiv 1 + 2 \tan^2 x$

(2)

(b) Hence solve, for $0 \leq x \leq 360$, the equation,

$$\sec^4 x - \tan^4 x = 3$$

(4)

(Total for question 13 is 6 marks)

14 $f(x) = 3x^3 + 4x^2 + 13x + 4$

(a) Show that $(3x + 1)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely

(3)

(c) Prove that there are no real solutions to $3 \sec^2 \theta + 4 \sec \theta + 13 + 4 \cos \theta = 0$

(5)

(Total for question 14 is 10 marks)

15

(a) Prove that $\cos \theta + \sin \theta \tan \theta \equiv \sec \theta$

(4)

(b) Hence find the exact roots of the equation $\cos \theta + \sin \theta \tan \theta = 4 \cos \theta$, for $0 \leq \theta \leq 2\pi$

(4)

(Total for question 15 is 8 marks)

16

- (a) Show that the equation $2 \sec^2 x + 4 \tan^2 x = 5 \sec x$ can be written in the form $a \sec^2 x + b \sec x + c = 0$

(2)

- (b) Hence, given x is obtuse find the exact value of $\tan x$.

(5)

(Total for question 16 is 7 marks)

17

(a) Prove that $\frac{\csc \theta}{\csc \theta - \sin \theta} \equiv \sec^2 \theta$

(4)

(b) Hence solve, for $0 < \theta < 2\pi$, $\frac{\csc \theta}{\csc \theta - \sin \theta} = 2 \tan \theta$

(3)

(Total for question 17 is 7 marks)

18 Solve the equation $\csc^2 x - 2 \cot x = 4$ for $0 < \theta < 360$

(Total for question 18 is 4 marks)
