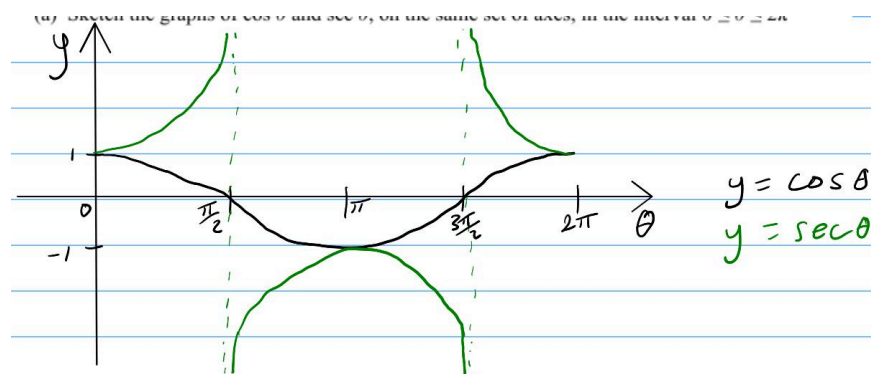


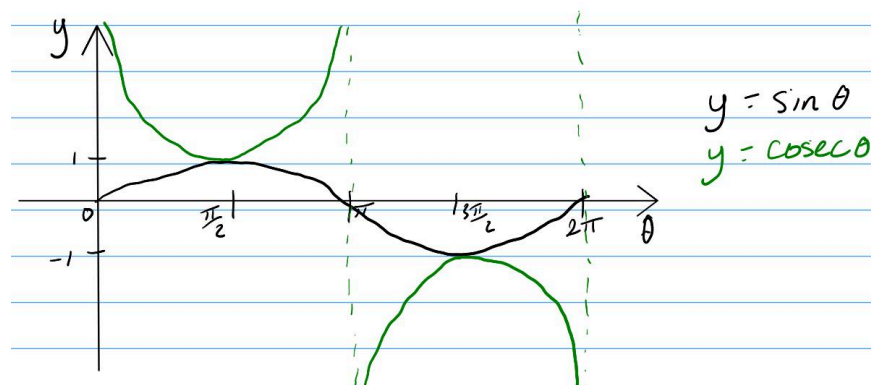
1

(a)



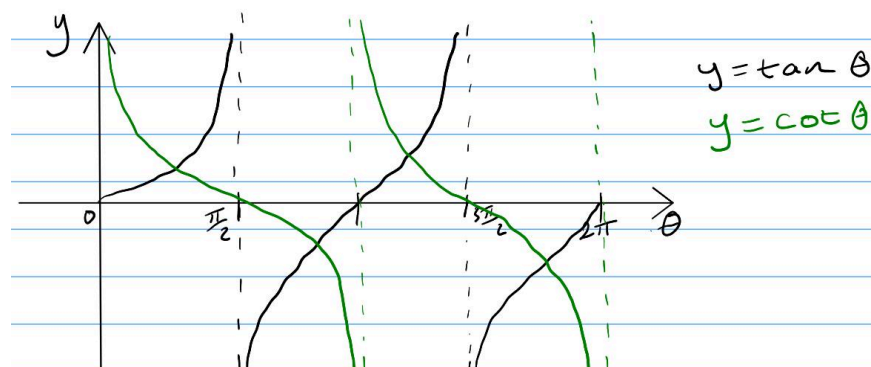
- Correct shape for $\cos \theta$ with intercepts at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ (B1)
- Correct shape for $\sec \theta$ with asymptotes at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ (B1)
- Correct relative positions (touching at peaks/troughs) (B1)
- Correct interval and labels (B1)

(b)



- Correct shape for $\sin \theta$ (B1)
- Correct shape for $\csc \theta$ with asymptotes at $0, \pi, 2\pi$ (B1)
- Correct relative positions (B1)
- Correct interval and labels (B1)

(c)



- Correct shape for $\tan \theta$ (B1)
- Correct shape for $\cot \theta$ (B1)
- Correct asymptotes and intercepts (B1)
- Correct intersection points (B1)

(Total for question 1 is 12 marks)

- 2
- Rearranges to $\csc \theta = \frac{5}{2}$ (M1)
 - Converts to $\sin \theta = \frac{2}{5}$ (M1)
 - Finds first solution $\theta = 0.412$ (A1)
 - Finds second solution $\theta = 2.73$ (A1)

(Total for question 2 is 4 marks)

- 3
- Substitutes $\cot x = \frac{\cos x}{\sin x}$ and $\sec x = \frac{1}{\cos x}$ (M1)
 - Forms quadratic $2(1 - \sin^2 x) = 3 \sin x$ (M1)
 - Solves quadratic $2 \sin^2 x + 3 \sin x - 2 = 0$ to get $\sin x = \frac{1}{2}$ (M1)
 - Identifies $\sin x = -2$ has no solution (B1)
 - Final answers $x = \frac{1}{6}\pi, \frac{5}{6}\pi$ (A1)

(Total for question 3 is 5 marks)

- 4
- Substitutes $\cot x = \frac{\cos x}{\sin x}$ (M1)
 - Factors out $\cos x$ to get $\cos x(5 \sin x + 1) = 0$ (M1)
 - Solutions from $\cos x = 0$: $x = \pm \frac{\pi}{2}$ (A1)
 - Solutions from $\sin x = -\frac{1}{5}$: $x = -0.20, -2.94$ (A2)

(Total for question 4 is 5 marks)

- 5
- Factors quadratic to $(\sec x + 2)(\sec x + 3) = 0$ (M1)
 - Sets $\cos x = -\frac{1}{2}$ and $\cos x = -\frac{1}{3}$ (M1)
 - Solutions for $\cos x = -\frac{1}{2}$: $120^\circ, 240^\circ$ (A1)
 - Solutions for $\cos x = -\frac{1}{3}$: $109.5^\circ, 250.5^\circ$ (A2)

(Total for question 5 is 5 marks)

- 6
- Takes square root: $\cot x = \pm 3$ (M1)
 - Converts to $\tan x = \pm \frac{1}{3}$ (M1)
 - Finds solutions for $\tan x = \frac{1}{3}$: $18.4^\circ, 198.4^\circ$ (A1)
 - Finds solutions for $\tan x = -\frac{1}{3}$: $161.6^\circ, 341.6^\circ$ (A2)

(Total for question 6 is 5 marks)

- 7
- Converts to $\cos(2\theta + 10) = -\frac{1}{1.3}$ (M1)
 - Finds values for $2\theta + 10$: 140.28, 219.72, -219.72, -140.28 (M1)
 - Solves for θ : 65.1° , 104.9° , -114.9° , -75.1° (A2)
- (Total for question 7 is 4 marks)

- 8
- (a)
- Divides identity by $\cos^2 \theta$: $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$ (M1)
 - Simplifies to $1 + \tan^2 \theta = \sec^2 \theta$ and rearranges (A1)
- (b)
- Substitutes $\tan^2 \theta = \sec^2 \theta - 1$ (M1)
 - Forms quadratic $2\sec^2 \theta + 5\sec \theta - 3 = 0$ (M1)
 - Solves to get $\sec \theta = \frac{1}{2}$ (no sol) and $\sec \theta = -3$ (M1)
 - Solutions for $\cos \theta = -\frac{1}{3}$: 109.5° , 250.5° (A2)
- (Total for question 8 is 7 marks)

- 9
- (a)
- Divides identity by $\sin^2 \theta$: $\frac{\cos^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ (M1)
 - Simplifies to $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$ (A1)
- (b)
- Substitutes $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ (M1)
 - Simplifies to $2\cot^2 \theta = 2 \Rightarrow \tan^2 \theta = 1$ (M1)
 - Finds $\tan \theta = \pm 1$ (M1)
 - Solutions: $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$ (A2)
- (Total for question 9 is 7 marks)

- 10
- Substitutes $\tan^2 x = \sec^2 x - 1$ (M1)
 - Forms quadratic $\sec^2 x + 4\sec x - 3 = 0$ (M1)
 - Solves quadratic to get $\sec x = -2 \pm \sqrt{7}$ (M1)
 - Finds $\cos x = \frac{1}{-2+\sqrt{7}}$ (no sol) and $\cos x = \frac{1}{-2-\sqrt{7}}$ (M1)
 - Solutions: 102.4° , 257.6° (A1)
- (Total for question 10 is 5 marks)

- 11
- Substitutes $\cot^2 x = \operatorname{cosec}^2 x - 1$ (M1)
 - Forms quadratic $\operatorname{cosec}^2 x + \operatorname{cosec} x - 6 = 0$ (M1)
 - Factors to $(\operatorname{cosec} x + 3)(\operatorname{cosec} x - 2) = 0$ (M1)
 - Solutions for $\sin x = -\frac{1}{3}$: $-19.5^\circ, -160.5^\circ$ (A1)
 - Solutions for $\sin x = \frac{1}{2}$: $30^\circ, 150^\circ$ (A1)

(Total for question 11 is 5 marks)

- 12
- (a)
- Substitutes $\sec^2 x = 1 + \tan^2 x$ and $\operatorname{cosec}^2 x = 1 + \cot^2 x$ (M1)
 - Simplifies $(1 + \tan^2 x) - (1 + \cot^2 x) = \tan^2 x - \cot^2 x$ (A1)
- (b)
- Expands LHS: $\sec^2 x - 2 \sec x \cos x + \cos^2 x$ (M1)
 - Simplifies to $\sec^2 x - 2 + \cos^2 x$ (B1)
 - Substitutes $\sec^2 x = 1 + \tan^2 x$ and $\cos^2 x = 1 - \sin^2 x$ (M1)
 - Simplifies to $\tan^2 x - \sin^2 x$ (A1)

(Total for question 12 is 4 marks)

- 13
- (a)
- Factors as difference of two squares: $(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$ (M1)
 - Uses $\sec^2 x - \tan^2 x = 1$ and $\sec^2 x = 1 + \tan^2 x$ (M1)
 - Simplifies to $1 + 2 \tan^2 x$ (A1)
- (b)
- Sets $1 + 2 \tan^2 x = 3$ (M1)
 - Solves to get $\tan^2 x = 1 \Rightarrow \tan x = \pm 1$ (M1)
 - Solutions: $45^\circ, 135^\circ, 225^\circ, 315^\circ$ (A2)

(Total for question 13 is 6 marks)

14

- (a)
- Evaluates $f(-\frac{1}{3}) = 3(-\frac{1}{3})^3 + 4(-\frac{1}{3})^2 + 13(-\frac{1}{3}) + 4$ (M1)
 - Shows $f(-\frac{1}{3}) = 0$ and concludes it is a factor (A1)
- (b)
- Performs algebraic division or inspection to find quadratic factor $(x^2 + x + 4)$ (M2)
 - Final answer $(3x + 1)(x^2 + x + 4)$ (A1)
- (c)
- Multiplies by $\sec \theta$ to get $3 \sec^3 \theta + 4 \sec^2 \theta + 13 \sec \theta + 4 = 0$ (M1)
 - Recognises this as $f(\sec \theta) = 0$ (M1)
 - Uses part (b) to get $(3 \sec \theta + 1)(\sec^2 \theta + \sec \theta + 4) = 0$ (M1)
 - Shows $\sec \theta = -\frac{1}{3} \Rightarrow \cos \theta = -3$ has no solutions (A1)
 - Shows $\sec^2 \theta + \sec \theta + 4 = 0$ has no solutions using discriminant $b^2 - 4ac = -15$ (A1)

(Total for question 14 is 10 marks)

15

- (a)
- Substitutes $\tan \theta = \frac{\sin \theta}{\cos \theta}$ (M1)
 - Combines fractions: $\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$ (M2)
 - Uses $\cos^2 \theta + \sin^2 \theta = 1$ to get $\frac{1}{\cos \theta} = \sec \theta$ (A1)
- (b)
- Uses identity from (a) to get $\sec \theta = 4 \cos \theta$ (M1)
 - Rearranges to $\cos^2 \theta = \frac{1}{4}$ (M1)
 - Finds $\cos \theta = \pm \frac{1}{2}$ (M1)
 - Solutions: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (A1)

(Total for question 15 is 8 marks)

16

- (a)
- Substitutes $\tan^2 x = \sec^2 x - 1$ (M1)
 - Simplifies to $6 \sec^2 x - 5 \sec x - 4 = 0$ (A1)
- (b)
- Solves quadratic to get $\sec x = \frac{4}{3}$ or $\sec x = -\frac{1}{2}$ (M1)
 - Rejects $\sec x = -\frac{1}{2}$ as $|\sec x| \geq 1$ (B1)
 - For obtuse x , $\sec x$ must be negative, but $\sec x = 4/3$ is positive. (Note: Markscheme suggests $\cos x = 3/4$ and uses a triangle) (M1)
 - Uses Pythagoras to find opposite side $\sqrt{4^2 - 3^2} = \sqrt{7}$ (M1)
 - Concludes $\tan x = -\frac{\sqrt{7}}{3}$ for obtuse x (A1)

(Total for question 16 is 7 marks)

17

- (a)
- Multiplies numerator and denominator by $\sin \theta$ (M1)
 - Simplifies to $\frac{1}{1-\sin^2 \theta}$ (M2)
 - Uses $1 - \sin^2 \theta = \cos^2 \theta$ to get $\sec^2 \theta$ (A1)
- (b)
- Sets $\sec^2 \theta = 2 \tan \theta$ (M1)
 - Uses $1 + \tan^2 \theta = 2 \tan \theta$ to form $(\tan \theta - 1)^2 = 0$ (M1)
 - Solutions for $\tan \theta = 1$: $\frac{\pi}{4}, \frac{5\pi}{4}$ (A1)

(Total for question 17 is 7 marks)

18

- Substitutes $\csc^2 x = 1 + \cot^2 x$ (M1)
- Forms quadratic $\cot^2 x - 2 \cot x - 3 = 0$ (M1)
- Factors to $(\cot x + 1)(\cot x - 3) = 0$ (M1)
- Solutions for $\tan x = -1$: $135^\circ, 315^\circ$ (A1)
- Solutions for $\tan x = \frac{1}{3}$: $18.4^\circ, 198.4^\circ$ (A1)

(Total for question 18 is 4 marks)
