

- 1
- (a) $\sin(A + B) = \sin A \cos B + \cos A \sin B$
 $\sin(A + A) = \sin A \cos A + \cos A \sin A$
 $\sin 2A = 2 \sin A \cos A$ (M1A1)
- (b) $\cos(A + B) = \cos A \cos B - \sin A \sin B$
 $\cos(A + A) = \cos A \cos A - \sin A \sin A$
 $\cos 2A = \cos^2 A - \sin^2 A$ (M1A1)
- (ci) $\sin^2 A = 1 - \cos^2 A$
 $\cos 2A = \cos^2 A - (1 - \cos^2 A)$
 $\cos 2A = 2 \cos^2 A - 1$ (M1A1)
- (cii) $\cos^2 A = 1 - \sin^2 A$
 $\cos 2A = (1 - \sin^2 A) - \sin^2 A$
 $\cos 2A = 1 - 2 \sin^2 A$ (M1A1)
- (d) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
 $\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$
 $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ (M1A1)
- (Total for question 1 is 10 marks)

- 2
- (a) $\sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$
 $= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$
 $= 2 \sin \theta \cos^2 \theta + \sin \theta - 2 \sin^3 \theta$
 $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$
 $= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$
 $= 3 \sin \theta - 4 \sin^3 \theta$ (M1A1)
- (b) $\sin 3\theta = 0.4$
 $3\theta = 23.6^\circ, 156.4^\circ, 383.6^\circ, 516.4^\circ$
 $\theta = 7.9^\circ, 52.1^\circ, 127.9^\circ, 172.1^\circ$
 (M1A1A1A1)
- (Total for question 2 is 6 marks)

- 3
- (a) $\cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$
 $= (2 \cos^2 \theta - 1) \cos \theta - 2 \sin \theta \cos \theta \sin \theta$
 $= 2 \cos^3 \theta - \cos \theta - 2 \sin^2 \theta \cos \theta$
 $= 2 \cos^3 \theta - \cos \theta - 2 \cos \theta (1 - \cos^2 \theta)$
 $= 2 \cos^3 \theta - \cos \theta - 2 \cos \theta + 2 \cos^3 \theta$
 $= 4 \cos^3 \theta - 3 \cos \theta$ (M1A1)
- (b) $\cos 3\theta = 0.5$
 $3\theta = \frac{1}{3}\pi, \frac{5}{3}\pi, \frac{7}{3}\pi$
 $\theta = \frac{1}{9}\pi, \frac{5}{9}\pi, \frac{7}{9}\pi$ (M1A1A1A1)
- (Total for question 3 is 6 marks)

- 4
- $\cos 2x = 1 - 2 \sin^2 x$
 $1 - 2 \sin^2 x - 7 \sin x + 3 = 0$
 $2 \sin^2 x + 7 \sin x - 4 = 0$
 $(2 \sin x - 1)(\sin x + 4) = 0$
 $\sin x = \frac{1}{2}$ or $\sin x = -4$ (no solutions)
 $x = 30^\circ, 150^\circ$ (M1M1A1A1A1)
- (Total for question 4 is 5 marks)

- 5
- $4 \sin 2x = 3 \sin 2x$
 $\sin 2x = \frac{3}{4}$
 $2x = 48.6^\circ, 131.4^\circ, 408.6^\circ, 491.4^\circ$
 $x = 24.3^\circ, 65.7^\circ, 204.3^\circ, 245.7^\circ$
 (M1M1A1A1A1)
- (Total for question 5 is 5 marks)

- 6
- $\cos 2x = \cos^2 x - \sin^2 x$
 $\frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x + \sin x} = \cos x - \sin x$ (M1A1)
- (Total for question 6 is 2 marks)

7

- (a) $R \cos(x + \alpha) = R \cos x \cos \alpha - R \sin x \sin \alpha$ $R \cos \alpha = 3, R \sin \alpha = 4$ $R = \sqrt{3^2 + 4^2} = 5$ $\tan \alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$ (M1A1M1A1)
- (b) $5 \cos(x + 53.1) = 1$ $\cos(x + 53.1) = 0.2$ $x + 53.1 = 78.5^\circ, 281.5^\circ$ $x = 25.4^\circ, 228.4^\circ$ (M1M1A1A1A1)
- (c) Min value is $-R = -5$ (B1)
- (d) $\cos(x + 53.1) = -1$ $x + 53.1 = 180^\circ$ $x = 126.9^\circ$ (M1A1)

(Total for question 7 is 12 marks)

8

- (a) $R = \sqrt{5^2 + 12^2} = 13$ $\tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.4^\circ$ $13 \sin(x + 67.4^\circ)$ (M1A1M1A1)
- (b) Max value = 13 $\sin(x + 67.4) = 1$ $x + 67.4 = 90^\circ$ $x = 22.6^\circ$ (B1M1A1)

(Total for question 8 is 7 marks)

9

- (a) $R = \sqrt{5^2 + 1^2} = \sqrt{26}$ $\tan \alpha = \frac{1}{5} \Rightarrow \alpha = 0.197$ (M1A1M1A1)
- (b) $\sqrt{26} \cos(x - 0.197) = 2$ $\cos(x - 0.197) = \frac{2}{\sqrt{26}}$ $x - 0.197 = 1.17, 5.12$ $x = 1.37, 5.31$ (M1A1A1)
- (c) Min value = $15 - \sqrt{26} \approx 9.90$ (M1A1)
- (d) $\cos(4x - 0.197) = -1$ $4x - 0.197 = \pi$ $4x = 3.338$ $x = 0.835$ (M1M1A1)

(Total for question 9 is 12 marks)

10

- (a) $R = \sqrt{2^2 + 3^2} = \sqrt{13}$ $\tan \alpha = \frac{3}{2} \Rightarrow \alpha = 0.983$ $\sqrt{13} \sin(x - 0.983)$ (M1A1M1A1)
- (b) Greatest value = $(\sqrt{13})^2 = 13$ $\sin(x - 0.983) = 1$ $x - 0.983 = \frac{\pi}{2}$ $x = 2.55$ (B1A1)



- (c) $\sqrt{13} \sin(x - 0.983) = 1$ $\sin(x - 0.983) = \frac{1}{\sqrt{13}}$ $x - 0.983 = 0.281, 2.861$ $x = 1.264, 3.843$ (M1M1A1A1A1)

(Total for question 10 is 11 marks)

11

(a) Max temp = $16 + 4 = 20^{\circ}\text{C}$ Min temp = $16 - 4 = 12^{\circ}\text{C}$ (B1)

(b) $16 + 4 \sin 15t = 17.4 \sin 15t = 1 \implies \sin 15t = 0.25$
 $15t = 14.48, 165.52 \implies t = 0.965, 11.03$
 $0.965 \times 60 = 58 \text{ mins} \implies 12:58$
 $11.03 \implies 11 \text{ hrs } 2 \text{ mins} \implies 23:02$

(M1A1A1A1)

(Total for question 11 is 5 marks)

12

$$4 \cos \theta = \frac{1}{\sin \theta} \implies 4 \sin \theta \cos \theta = 1 \implies 2 \sin 2\theta = 1 \implies \sin 2\theta = 0.5$$

 $2\theta = \frac{\pi}{6}, \frac{5\pi}{6} \implies \theta = \frac{\pi}{12}, \frac{5\pi}{12}$ (M1M1A1A1A1)

(Total for question 12 is 5 marks)

13

$$R = 5, \alpha = 53.1^{\circ} \implies 5 \cos(\theta + 53.1) = 2 \cos(\theta + 53.1) = 0.4$$

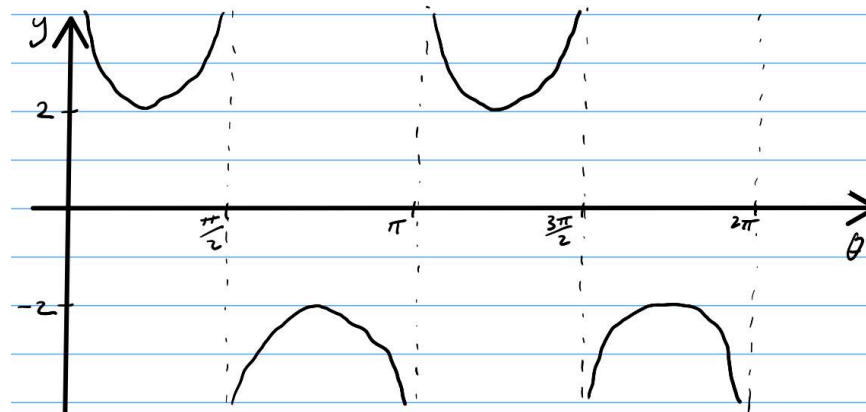
 $\theta + 53.1 = 66.4^{\circ}, 293.6^{\circ} \implies \theta = 13.3^{\circ}, 240.5^{\circ}$ (M1A1M1A1A1)

(Total for question 13 is 5 marks)

14

(a) $\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$ Using $\sin 2\theta = 2 \sin \theta \cos \theta \implies \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$
 $\frac{1}{\frac{1}{2} \sin 2\theta} = 2 \operatorname{cosec} 2\theta$ (M1M1A1A1)

(b)



(B1B1)

(c) $2 \operatorname{cosec} 2\theta = 5 \implies \sin 2\theta = 0.4$
 $2\theta = 23.6^{\circ}, 156.4^{\circ}, 383.6^{\circ}, 516.4^{\circ} \implies \theta = 11.8^{\circ}, 78.2^{\circ}, 191.8^{\circ}, 258.2^{\circ}$ (M1M1A1A1A1A1)

(Total for question 14 is 12 marks)

15

$$8 \sin \theta \cos \theta = \frac{3 \sin \theta}{\cos \theta} \implies 8 \sin \theta \cos^2 \theta - 3 \sin \theta = 0 \implies \sin \theta (8 \cos^2 \theta - 3) = 0$$

 $\sin \theta = 0 \implies \theta = 0, -180^{\circ}$
 $\cos^2 \theta = \frac{3}{8} \implies \cos \theta = \pm \sqrt{0.375} \implies \theta = \pm 52.2^{\circ}, \pm 127.8^{\circ}$ (M1M1A1A1A1A1)

(Total for question 15 is 6 marks)

16

- (a) $\sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x = (2 \sin x \cos x) \cos x + (1 - 2 \sin^2 x) \sin x$
 $= 2 \sin x \cos^2 x + \sin x - 2 \sin^3 x = 2 \sin x(1 - \sin^2 x) + \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x$ (M1M1A1A1)
- (b) $-2(3 \sin x - 4 \sin^3 x) + 1 = 0 \Rightarrow -2 \sin 3x + 1 = 0 \Rightarrow \sin 3x = 0.5 \Rightarrow 3x =$
 $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \Rightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ (M1M1A1A1A1)

(Total for question 16 is 9 marks)

17

- (a) $R = \sqrt{2^2 + 3^2} = \sqrt{13} \tan \alpha = \frac{3}{2} \Rightarrow \alpha = 56.3^\circ \Rightarrow \sqrt{13} \sin(x + 56.3^\circ)$ (M1A1A1)
- (b) Max temp $= 8 + \sqrt{13} \approx 11.6^\circ\text{C}$ (B1)
- (c) $\sin(15t - 160 + 56.3) = 1 \Rightarrow 15t - 103.7 = 90 \Rightarrow 15t = 193.7 \Rightarrow t = 12.91... \times$
 $60 = 55 \text{ mins} \Rightarrow 12:55$ (M1A1A1)

(Total for question 17 is 7 marks)

18

- (a) $\cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta = (2 \cos^2 \theta - 1) \cos \theta - (2 \sin \theta \cos \theta) \sin \theta =$
 $2 \cos^3 \theta - \cos \theta - 2 \sin^2 \theta \cos \theta = 2 \cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta) \cos \theta = 4 \cos^3 \theta - 3 \cos \theta$ (M1M1A1A1)
- (b) $1 - (4 \cos^3 x - 3 \cos x) = 1 - \cos^2 x \Rightarrow 4 \cos^3 x - \cos^2 x - 3 \cos x = 0 \Rightarrow \cos x(4 \cos^2 x -$
 $\cos x - 3) = 0 \Rightarrow \cos x(4 \cos x + 3)(\cos x - 1) = 0 \Rightarrow \cos x = 0 \Rightarrow x = \pm \frac{\pi}{2} \Rightarrow \cos x =$
 $-0.75 \Rightarrow x = \pm 2.42 \Rightarrow \cos x = 1 \Rightarrow x = 0$ (M1M1A1A1A1)

(Total for question 18 is 9 marks)

19

- (a) $R = \sqrt{(\sqrt{3})^2 + 1^2} = 2 \tan \alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{6} \text{ (or } 30^\circ) \Rightarrow y = 2 \sin(x - \frac{\pi}{6}) + 4$
- Translation $\frac{\pi}{6}$ in positive x direction
 - Stretch SF 2 in y direction
 - Translation 4 in positive y direction (M1A1A1M1A1A1A1)
- (b) Max value of denominator is $2 + 4 = 6$ Min value of fraction is $\frac{1}{6}$ (M1A1)



(Total for question 19 is 9 marks)

20

(a) $\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$ (M1M1A1A1)

(b) $\tan^2 x = 3 \implies \tan x = \pm\sqrt{3} \implies x = \frac{\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}$ (M1A1)

(Total for question 20 is 6 marks)

21

(a) $\frac{1-(1-2\sin^2 x)}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$ (M1M1A1A1)

(b) $\frac{1-\cos 2\theta}{\sin 2\theta} = 1 \implies \tan \theta = 1 \implies \theta = \frac{\pi}{4}, \frac{5\pi}{4}$ (M1A1A1)

(Total for question 21 is 7 marks)

22

$\frac{2\sin x}{2\cos x - \frac{1}{\cos x}} = \frac{2\sin x \cos x}{2\cos^2 x - 1} = \frac{\sin 2x}{\cos 2x} = \tan 2x$ (M1M1A1A1)

(Total for question 22 is 4 marks)

23

$R = 6 \implies a^2 + b^2 = 36 \implies 3\sqrt{3} = 6 \sin(\frac{\pi}{4} + \alpha) \implies \sin(\frac{\pi}{4} + \alpha) = \frac{\sqrt{3}}{2} \implies \frac{\pi}{4} + \alpha = \frac{\pi}{3} \implies \alpha = \frac{\pi}{12} \implies \tan \frac{\pi}{12} = \frac{b}{a} \implies b = (2 - \sqrt{3})a \implies a^2 + (2 - \sqrt{3})^2 a^2 = 36 \implies a^2(1 + 7 - 4\sqrt{3}) = 36 \implies a = 5.796, b = 1.553$ (M1M1A1M1A1A1)

(Total for question 23 is 6 marks)

24

$\sin \theta \cos 45 - \cos \theta \sin 45 = \cos \theta \cos 30 - \sin \theta \sin 30 \implies \frac{\sqrt{2}}{2} \sin \theta - \frac{\sqrt{2}}{2} \cos \theta = \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta$
 $(\sqrt{2} + 1) \sin \theta = (\sqrt{3} + \sqrt{2}) \cos \theta \implies \tan \theta = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} + 1} \approx 1.303 \implies \theta = 52.5^\circ, 232.5^\circ$

(M1M1A1M1A1A1)

(Total for question 24 is 6 marks)

25

(a) $\tan(2\theta - 45) = 3 \implies \frac{\tan 2\theta - \tan 45}{1 + \tan 2\theta \tan 45} = 3 \implies \frac{\tan 2\theta - 1}{1 + \tan 2\theta} = 3 \implies \tan 2\theta - 1 = 3 + 3 \tan 2\theta$
 $-2 \tan 2\theta = 4 \implies \tan 2\theta = -2$ (M1M1A1)

(b) $\frac{2 \tan \theta}{1 - \tan^2 \theta} = -2 \implies 2 \tan \theta = -2 + 2 \tan^2 \theta \implies \tan^2 \theta - \tan \theta - 1 = 0$
 $\tan \theta = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$ Since θ is obtuse, $\tan \theta$ is negative $\implies \tan \theta = \frac{1 - \sqrt{5}}{2}$

(M1M1A1A1A1)

(Total for question 25 is 8 marks)

26

(a) $R = \sqrt{5^2 + 3^2} = \sqrt{34} \implies \tan a = \frac{3}{5} \implies a = 0.540 \text{ radians}$

(M1A1A1)

- (b)
1. Translation 0.540 in negative x direction
 2. Stretch SF 0.5 in x direction
 3. Stretch SF $\sqrt{34}$ in y direction

(M1A1A1A1)

(Total for question 26 is 7 marks)

27 $\cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3} = \sin \theta \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = \sin \theta \frac{1}{2} \cos \theta = (1 + \frac{\sqrt{3}}{2}) \sin \theta \tan \theta = \frac{1}{2+\sqrt{3}} = 2 - \sqrt{3} \theta = \frac{\pi}{12}, \frac{13\pi}{12}$ **(M1M1A1A1)**

(Total for question 27 is 4 marks)

28 (a) $R = \sqrt{5^2 + 12^2} = 13 \tan a = \frac{12}{5} \implies a = 1.176 \text{ radians } 13 \cos(x - 1.176)$ **(M1A1A1)**

(b) Range is $2 \pm 13 - 11 \leq f(x) \leq 15$ **(M1A1)**

(Total for question 28 is 5 marks)

29 (a) $R = \sqrt{7^2 + 4^2} = \sqrt{65} \tan a = \frac{4}{7} \implies a = 0.519 \text{ radians } \sqrt{65} \cos(x + 0.519)$ **(M1A1A1A1)**

(b) $y = \frac{1}{\sqrt{65} \cos(x+0.519)} = \frac{1}{\sqrt{65}} \sec(x + 0.519)$

1. Translation 0.519 units in negative x direction

2. Stretch in y direction SF $\frac{1}{\sqrt{65}}$ **(M1A1A1)**

(Total for question 29 is 7 marks)