

1

- (a) Use the identity for $\sin(A + B)$ to express $\sin 2A$ in terms of $\sin A$ and $\cos A$.

(2)

- (b) Use the identity for $\cos(A + B)$ to express $\cos 2A$ in terms of $\sin A$ and $\cos A$.

(2)

- (ci) Hence, express $\cos 2A$ in terms of $\cos A$.

(2)

(cii) Hence, express $\cos 2A$ in terms of $\sin A$.

(2)

(d) Use the identity for $\tan(A + B)$ to express $\tan 2A$ in terms of $\tan A$.

(2)

(Total for question 1 is 10 marks)

2

- (a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$ show that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

(2)

- (b) Solve, for $0 \leq \theta \leq 180$, the equation,

$$3 \sin \theta - 4 \sin^3 \theta = 0.4$$

Give your answers to 1 decimal place.

(4)

(Total for question 2 is 6 marks)

3

- (a) By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$ show that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

(2)

- (b) Solve, for $0 \leq \theta \leq \pi$, the equation,

$$4 \cos^3 \theta - 3 \cos \theta = 0.5$$

Give your answers in terms of π .

(4)

(Total for question 3 is 6 marks)

4 Solve, for $-180 \leq x \leq 180$, the equation,

$$\cos 2x - 7 \sin x + 3 = 0$$

(Total for question 4 is 5 marks)

5 Solve, for $0 \leq x \leq 360$, the equation,

$$8 \sin x \cos x = 3$$

Give your answers to 1 decimal place.

(Total for question 5 is 5 marks)

6 Prove the identity:

$$\frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x$$

(Total for question 6 is 2 marks)

$$f(x) = 3 \cos x - 4 \sin x$$

Given that $f(x) = R \cos(x + \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 90$,

- (a) Find the value of R and the value of α

(4)

- (b) Hence solve, for $0 \leq \theta \leq 360$, the equation

$$3 \cos x - 4 \sin x = 1$$

Give your answers to 1 decimal place.

(5)

(c) Write down the minimum value of $3 \cos x - 4 \sin x$

(1)

(d) Find, to 1 decimal place, the smallest positive value of x for which this minimum occurs

(2)

(Total for question 7 is 12 marks)

8

- (a) Express $5 \sin x + 12 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 90$

(4)

- (b) Hence find the maximum value of $5 \sin x + 12 \cos x$ and find, the smallest positive value of x for which this maximum occurs

(3)

(Total for question 8 is 7 marks)

$$f(x) = 5 \cos \theta + \sin \theta$$

Given that $f(x) = R \cos(\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$

- (a) Find the value of R and the value of α to 3 decimal places

(4)

- (b) Hence, solve for $0 \leq \theta \leq 2\pi$, the equation

$$5 \cos \theta + \sin \theta = 2$$

(3)

- (c) Calculate the minimum value of

$$5 \cos 4x + \sin 4x + 15$$

(2)

- (d) Find the smallest positive value of x for which this minimum occurs

(3)

(Total for question 9 is 12 marks)

10

- (a) Express $2 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$

(4)

- (b) Hence find the greatest value of $(2 \sin x - 3 \cos x)^2$ and find, the smallest positive value of x for which this maximum occurs

(2)

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$2 \sin x - 3 \cos x = 1$$

Give your answers to 3 decimal places.

(5)

(Total for question 10 is 11 marks)

- 11** The temperature, $f(t)$ degrees Celsius, of a building is modelled by the formula

$$f(t) = 16 + 4 \sin(15t)^\circ \quad 0 \leq t < 24$$

where t is the number of hours after midday.

- (a) State the maximum and minimum temperature of the building according to the model.

(1)

- (b) Find the times, to the nearest minute, when the temperature is equal to 17 Degrees Celsius.

(4)

(Total for question 11 is 5 marks)

12 Solve, for $0 \leq \theta < \pi$,

$$4 \cos \theta = \operatorname{cosec} \theta$$

(Total for question 12 is 5 marks)

13 Solve, for $0 \leq \theta < 360^\circ$,

$$3 \cos \theta - 4 \sin \theta = 2$$

giving your answers to 1 decimal place.

(Total for question 13 is 5 marks)

14

(a) Prove that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta$$

(4)

(b) Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$.

(2)

(c) Solve, $0 \leq \theta < 360^\circ$,

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 5$$

giving your answers to 1 decimal place.

(6)

(Total for question 14 is 12 marks)

15 Solve, for $-180 \leq \theta < 180^\circ$,

$$4 \sin 2\theta = 3 \tan \theta$$

giving your answers, where necessary, to 1 decimal place.

(Total for question 15 is 6 marks)

16

(a) Show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x$$

(4)

(b) Hence, solve, for $0 \leq \theta < \pi$,

$$8 \sin^3 x - 6 \sin x + 1 = 0$$

(5)

(Total for question 16 is 9 marks)

- 17 The temperature, $\theta^\circ\text{C}$, inside a warehouse is modelled by the equation

$$\theta = 8 + 2 \sin(15t - 160) + 3 \cos(15t - 160)$$

where t is the number of hours after midnight.

- (a) Express $2 \sin x + 3 \cos x$ in the form $R \sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$

(3)

- (b) Deduce the maximum temperature of the room during this day,

(1)

- (c) Find the time of day when the maximum temperature occurs, giving your answer to the nearest minute

(3)

(Total for question 17 is 7 marks)

18

(a) Show that $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$

(4)

(b) Hence, solve, for $-\pi \leq \theta \leq \pi$,

$$1 - \cos 3x = \sin^2 x$$

(5)

(Total for question 18 is 9 marks)

19

- (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3} \sin x - \cos x + 4$

(7)

- (b) Calculate the minimum value of $\frac{1}{\sqrt{3} \sin x - \cos x + 4}$

(2)

(Total for question 19 is 9 marks)

20

(a) Show that $\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x$

(4)

(b) Hence solve, for $-\pi \leq \theta \leq \pi$,

$$\frac{1 - \cos 2x}{1 + \cos 2x} = 3$$

(2)

(Total for question 20 is 6 marks)

21

(a) Prove the identity:

$$\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x$$

(4)

(b) Hence, solve, for $0 \leq \theta < 2\pi$,

$$1 - \cos 2\theta = \sin 2\theta$$

(3)

(Total for question 21 is 7 marks)

22 Prove the identity:

$$\frac{2 \sin x}{2 \cos x - \sec x} \equiv \tan 2x$$

(Total for question 22 is 4 marks)

23 A curve has the equation

$$y = a \sin x + b \cos x$$

where a and b are constants.

The maximum value of y is 6 and the curve passes through the point $(\frac{\pi}{4}, 3\sqrt{3})$

Find, to 3 decimal places, the values of a and b .

(Total for question 23 is 6 marks)

24 Solve, for $0 \leq \theta < 360^\circ$,

$$\sin(\theta - 45) = \cos(\theta + 30)$$

(Total for question 24 is 6 marks)

25 Given that θ satisfies the equation $\sin(2\theta - 45) = 3 \cos(2\theta - 45)$.

(a) Show that $\tan 2\theta = -2$

(3)

(b) Hence find, in surd form, the exact value of $\tan \theta$, given that θ is an obtuse angle.

(5)

(Total for question 25 is 8 marks)

26

- (a) Express $5 \cos 2x - 3 \sin 2x$ in the form $R \cos(2x + a)$

(3)

- (b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ onto the curve $y = 5 \cos 2x - 3 \sin 2x$

(4)

(Total for question 26 is 7 marks)

27 Solve, for $0 \leq \theta < 2\pi$,

$$\cos\left(\theta + \frac{\pi}{3}\right) = \sin \theta$$

(Total for question 27 is 4 marks)

28

- (a) Express $5 \cos x + 12 \sin x$ in the form $R \cos(x - a)$ where $R > 0$ and $0 \leq a \leq \frac{\pi}{2}$

(3)

- (b) Write down the range of the function

$$f(x) = 2 + 5 \cos x + 12 \sin x \quad 0 \leq x < 2\pi$$

(2)

(Total for question 28 is 5 marks)

29

- (a) Express $7 \cos x - 4 \sin x$ in the form $R \cos(x + a)$ where $R > 0$ and $0 \leq a \leq \frac{\pi}{2}$

(4)

- (b) Give full details of a sequence of two transformations needed to transform the curve

$$y = \sec x \text{ onto the curve } y = \frac{1}{7 \cos x - 4 \sin x}$$

(3)

(Total for question 29 is 7 marks)
