1 Work out the first four terms, in ascending powers of x, in the binomial expansion of $\frac{1}{(1+x)^2}$

(Total for question 1 is 3 marks)

2 (a) Expand $(4 + 6x)^{-\frac{1}{2}}$ in ascending powers of x up to and including x^3

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(Total for question 2 is 5 marks)

3 (a) Expand $(8 + 12x)^{\frac{1}{3}}$ in ascending powers of x up to and including x^3

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(Total for question 3 is 5 marks)

4 (a) Expand $\sqrt[3]{1+2x}$ in ascending powers of x up to and including x^3 and state the set of values of x for which the expansion is valid.

(5)

(b) Use your expansion to find an approximation for $\sqrt[3]{1.1}$ to 5 decimal places.

(2)

(Total for question 4 is 7 marks)

5 (a) Work out the first three terms in the binomial expansion of $\frac{1}{(2+3x)^2}$

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(Total for question 5 is 5 marks)

6 (a) Expand $(3 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including x^3

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(c) Use your expansion, with $x = 0.1$, to find an approximation of the value of $\sqrt{5}$. Give your answer to 3 decimal places.

(3)

(Total for question 6 is 8 marks)

7 (a) Expand $(1 + 3x)^{-3}$ in ascending powers of x up to and including x^3

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(c) Use your expansion to find the expansion of $\frac{2x+1}{(1+3x)^3}$ up to and including x^3

(3)

(Total for question 7 is 8 marks)

8 (a) Expand $(9 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including x^3

(4)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(c) Use your expansion, with a suitable value of x, to find the value of $\sqrt{8.9}$ correct to 5 significant figures

(2)

(Total for question 8 is 7 marks)

9 (a) Use binomial expansion to show that $\sqrt{\frac{1+5x}{1-x}} \approx 1 + 3x - \frac{3}{2}x^2$

(6)

(b) A student substitutes $x = \frac{1}{2}$ into the expansion to find an estimate for $\sqrt{7}$. Give a reason why the student should not use $x = \frac{1}{2}$

(1)

(c) Substitute $x = \frac{1}{9}$ into $\sqrt{\frac{1+5x}{1-x}} = 1 + 3x - \frac{3}{2}x^2$ to obtain an approximation for $\sqrt{7}$. Give your answer as a fraction in its simplest form.

(3)

(Total for question 9 is 10 marks)

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10 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{9+x}}$$

giving each coefficient in its simplest form.

(4)

(bi) Without evaluating your expansion, state, giving a reason, which of the three values of x should not be used.

(1)

(bii) Without evaluating your expansion, state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{3}$

(1)

(Total for question 10 is 6 marks)

11 (a) Find the first four terms, in ascending powers of x, of the binomial expansion of $\sqrt{1+6x}$ giving each term in its simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{12}$ in the expansion to find an approximation for $\sqrt{6}$. There is no need to carry out the calculation.

(2)

(Total for question 11 is 5 marks)

12 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient in its simplest form.

(3)

(b) Hence find first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x^2}}$$

(2)

(Total for question 12 is 5 marks)

13 (a) Find the first three terms of the expansion of $(9 + 2x)^{-\frac{1}{2}}$ in ascending powers of x.

(4)

(b) The expansion of $\frac{a+bx}{\sqrt{9+2x}}$ is $3 + x \ldots$ Find the values of the constants a and b.

(3)

(Total for question 13 is 7 marks)

14 (a) Find the coefficient of x^5 in the expansion of $(2 + 3x)^7$

(1)

(bi) Expand $\sqrt{1 - 2x}$ as far as the term x^3.

(3)

(bii) State the range of the values for which your expansion is valid.

(1)

(biii) Use your expansion to find an estimate for $\sqrt{98}$, correct to five decimal places, and compare this with the value of $\sqrt{98}$ given by your calculator.

(3)

(Total for question 14 is 8 marks)

15 In this question you should assume $-\frac{1}{2} < x < \frac{1}{2}$

(a) For the binomial expansion of $(1 - 2x)^{-2}$ find and simplify the first four terms.

(2)

(b) Write down the sum to infinity of the series $1 + 2x + 4x^2 + 8x^3 + \ldots$

(1)

(c) Hence or otherwise find and simplify an expression for $2 + 6x + 16x^2 + 40x^3 + \ldots$

(3)

(Total for question 15 is 6 marks)

16 Find the first three terms of the expansion of $(8 - 4x^3)^{\frac{1}{3}}$ in ascending powers of x.

(Total for question 16 is 3 marks)

17 The function $f(x)$ is given by $f(x) = \sqrt{1 + ax}$ where a is a non-zero constant. In the binomial expansion of $f(x)$, the coefficients of x and x^2 are equal.

(a) Find the value of a.

(3)

(bi) Using this value of a, state the set of values for which the binomial expansion is valid.

(1)

(bii) Using this value of a, write down the quadratic function that approximates $f(x)$ when x is small.

(1)

(Total for question 17 is 5 marks)

18 (a) Work out the first three terms in the binomial expansion of $(1 + 3x)^{\frac{1}{2}}$

(3)

(b) Write down the range of values of x for which the expansion is valid.

(1)

(Total for question 18 is 4 marks)