

1 $f(x) = \ln x + e^x$

(a) Find $f'(x)$

(2)

(b) Find $f''(x)$

(2)

(Total for question 1 is 4 marks)

2 Differentiate with respect to x ,

(a) $2x^3 + e^{4x}$

(2)

(b) $(x^2 + 5)^3$

(2)

(Total for question 2 is 4 marks)

3 Differentiate with respect to x ,

(a) $x + \frac{5}{x^2+4}$

(2)

(b) $8 + e^{x^2}$

(2)

(Total for question 3 is 4 marks)

- 4** The point P lies on the curve with equation $y = 2 + \ln(3 - 2x)$ with x coordinate 1
Find an equation to the tangent to the curve at the point P.

(Total for question 4 is 5 marks)

5 The point P lies on the curve with equation $y = \frac{3}{2x+1}$ with x coordinate 1

(a) Find an equation to the normal to the curve at the point P.

(5)

(b) The normal intersects the curve again at the point Q. Find the exact coordinates of Q.

(2)

(Total for question 5 is 7 marks)

6 The point P lies on the curve with equation $y = \frac{2}{\sqrt{2x+1}}$ with x coordinate 4

(a) Find an equation to the tangent to the curve at the point P.

(5)

(b) The tangent intersects the x axis at the point A and the y axis at the point B . Find the exact area of the triangle AOB , where O is the origin.

(4)

(Total for question 6 is 9 marks)

7 Differentiate with respect to x

(a) $2xe^x$

(2)

(b) $3x^2 \ln 2x$

(3)

(Total for question 7 is 5 marks)

8 Differentiate with respect to x ,

(a) $x^2\sqrt{2x+1}$

(3)

(b) $x \ln(x+1)$

(3)

(Total for question 8 is 6 marks)

9 Differentiate with respect to x ,

(a) $(x + 5)(x + 1)^3$

(3)

(b) $2 + x^3 \ln(2x + 1)$

(3)

(Total for question 9 is 6 marks)

- 10** The point P lies on the curve with equation $y = (3x - 1) \ln(2 - x)$ with x coordinate 1.
Find an equation of the tangent to the curve at the point P .

(Total for question 10 is 5 marks)

- 11** Find the coordinates of the stationary points of the curve $y = x(x - 3)^3$ and determine the nature of the stationary points.

(Total for question 11 is 8 marks)

- 12** The point P lies on the curve with equation $y = x\sqrt{x-1}$ with x coordinate 5
Find an equation of the normal to the curve at the point P .

(Total for question 12 is 5 marks)

13 The point P lies on the curve with equation $y = xe^{x^2}$ with x coordinate 1

(a) Find an equation to the tangent to the curve at the point P .

(5)

(b) The tangent intersects the x axis at the point A and the y axis at the point B .

Find the exact area of the triangle AOB , where O is the origin.

(4)

(Total for question 13 is 9 marks)

14 Differentiate with respect to x

(a) $\frac{e^x}{2x+1}$

(4)

(b) $\frac{\ln(x^2+1)}{3x+2}$

(4)

(Total for question 14 is 8 marks)

- 15** The point P lies on the curve with equation $y = \frac{3x}{2x-1}$ with x coordinate 1.

Find an equation to the tangent to the curve at the point P .

(Total for question 15 is 5 marks)

- 16** The point P lies on the curve with equation $y = \frac{e^x+1}{e^x+3}$ with x coordinate 0.

Find an equation to the normal to the curve at the point P .

(Total for question 16 is 5 marks)

- 17** Find the coordinates of the stationary points of the curve $y = \frac{x}{9+x^2}$ and determine the nature of the stationary points.

(Total for question 17 is 7 marks)

18

$$f(x) = \frac{x-5}{2x+3} + \frac{2x+4}{2x^2+7x+6}$$

- (a) Express $f(x)$ as a fraction in its simplest form.

(3)

- (b) Hence find $f'(x)$ in its simplest form

(4)

(Total for question 18 is 7 marks)

19

$$f(x) = \frac{4x + 1}{2x + 3} - \frac{4x - 6}{4x^2 - 9}$$

- (a) Express $f(x)$ as a fraction in its simplest form.

(3)

- (b) Hence find $f'(x)$ in its simplest form

(4)

(Total for question 19 is 7 marks)

20 A curve has the equation $2x^2 - 3xy + y^2 = 12$:

(a) Find an expression for $\frac{dy}{dx}$

(5)

(b) Find an equation for the normal to the curve at the point (1, -2)

(3)

(Total for question 20 is 8 marks)

21 A curve has the equation $3x^2 + xy + y^2 = 20$

The gradient of the tangent to the curve is $\frac{4}{3}$ at the points P and Q .

(a) Show that $2x + y = 0$ at P and Q .

(6)

(b) Find the coordinates of P and Q .

(3)

(Total for question 21 is 9 marks)

22 A curve has the equation $x^2 + 4xy - x + y^2 = 35$:

(a) Find an expression for $\frac{dy}{dx}$

(5)

(b) Find an equation for the tangent to the curve at the point P (2, 3)

(3)

(Total for question 22 is 8 marks)

23 A curve has the equation $2 \sin x + 2 \cos y = 3$ for $0 \leq x \leq \pi$ and $0 \leq y \leq \pi$

(a) Find an expression for $\frac{dy}{dx}$

(3)

(b) find the coordinates of the point where $\frac{dy}{dx} = 0$

(4)

(Total for question 23 is 7 marks)

24

(a) Given that $y = 2^x$ show that $\frac{dy}{dx} = 2^x \ln 2$

(2)

(b) Find the equation to the tangent of the curve $y = 3^{(x^2)}$ at the point (2, 81)

(4)

(Total for question 24 is 6 marks)

25 Use the derivatives of $\sin(x)$ and $\cos(x)$ to show that:

(a) $\frac{d}{dx}(\tan x) = \sec^2 x$

(3)

(b) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(3)

(c) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(3)

(d) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(3)

(Total for question 25 is 12 marks)

26 Differentiate with respect to x ,

(a) $x^2 \cos 2x$

(3)

(b) $3 \sin(2x + 1)$

(2)

(Total for question 26 is 5 marks)

27 Differentiate with respect to x ,

(a) $e^{3x}(\cos 2x + \sin x)$

(3)

(b) $\ln(\sin x)$

(3)

(Total for question 27 is 6 marks)

28 The curve C has the equation $x = 2 \tan y$

(a) Find $\frac{dx}{dy}$ in terms of y

(2)

(b) Hence find $\frac{dy}{dx}$ in terms of x

(4)

(Total for question 28 is 6 marks)

- 29** The point P lies on the curve with equation $y = \operatorname{cosec} x + \cos 2x$ with x coordinate $\frac{\pi}{4}$. Find an equation to the tangent to the curve at the point P .

(Total for question 29 is 6 marks)

- 30** The point P lies on the curve with equation $y = \sec 2x$ with x coordinate $\frac{\pi}{6}$
Find an equation to the normal to the curve at the point P .

(Total for question 30 is 6 marks)

- 31** Given that x is measured in radians, prove, from first principles, that the derivative of $\sin(x)$ is $\cos(x)$

You may assume the formula for $\sin(A \pm B)$ and that as $h \rightarrow 0$ $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(Total for question 31 is 5 marks)

- 32** Given that x is measured in radians, prove, from first principles, that the derivative of $\cos(x)$ is $-\sin(x)$

You may assume the formula for $\cos(A \pm B)$ and that as $h \rightarrow 0$ $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(Total for question 32 is 5 marks)

33 A curve has the parametric equations

$$x = 2t + 1, \quad y = t^2 - 1$$

- (a) Find the points where the curve crosses the coordinate axes.

(2)

- (b) Find an expression for $\frac{dy}{dx}$ in terms of x .

(3)

(Total for question 33 is 5 marks)

34 A curve has the parametric equations

$$x = \tan^2 t, \quad y = \cos t, \quad 0 < t < \frac{\pi}{2}$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(3)

- (b) Find an equation of the tangent to the curve when $t = \frac{\pi}{4}$

(5)

(c) Find a cartesian equation for the curve.

(4)

(Total for question 34 is 12 marks)

35 A curve has the parametric equations

$$x = \sin^2 t, \quad y = \sin 2t, \quad 0 < t < \pi$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(3)

- (b) Find an equation of the normal to the curve when $t = \frac{\pi}{6}$

(5)

- (c) Find a cartesian equation for the curve.

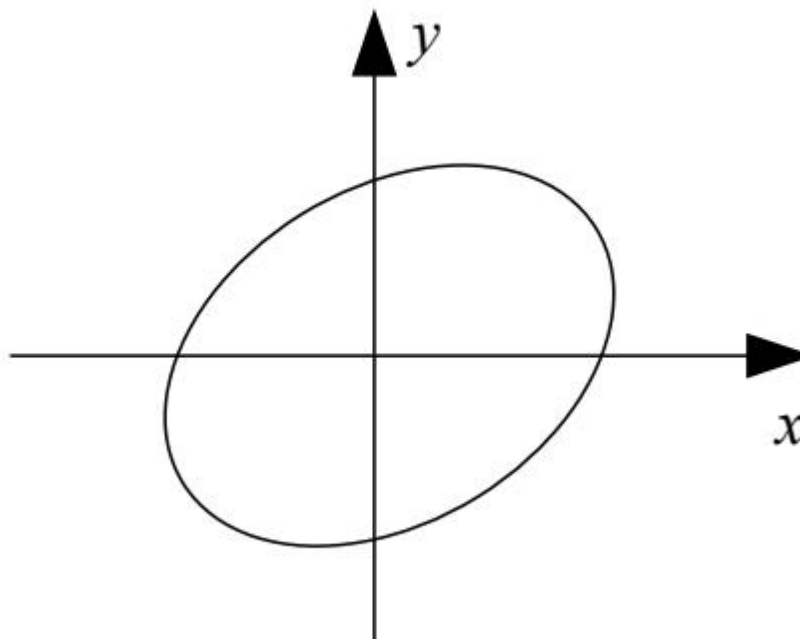
(4)

(Total for question 35 is 12 marks)

- 36** Given that $y = \frac{\sin \theta}{\sin \theta + \cos \theta}$
Show that $\frac{dy}{d\theta} = \frac{1}{\sin 2\theta + 1}$

(Total for question 36 is 5 marks)

- 37 The diagram shows a sketch of the curve with equation $x^2 - xy + 2y^2 = 20$



- (a) Show that $\frac{dy}{dx} = \frac{2x-y}{x-4y}$

(4)

(b) Find the points on the curve where $\frac{dy}{dx} = 0$

(5)

(Total for question 37 is 9 marks)

38

- (a) Sketch the graph of $y = (3 - x)^2 + 5$, $0 \leq x \leq 6$

(3)

- (b) The line with equation $x + y = k$, where k is a constant, intersects the curve at two distinct points. State the range of values of k , writing your answer in set notation.

(5)

(Total for question 38 is 8 marks)

39

$$\frac{10x + 3 - 4x^2}{(1-x)(2x+1)} = A + \frac{B}{1-x} + \frac{C}{2x+1} \quad x > 1$$

- (a) Find the values of A , B and C .

(4)

- (b) Prove that $f(x) = \frac{10x+3-4x^2}{(1-x)(2x+1)}$ is an increasing function.

(3)

(Total for question 39 is 7 marks)

40 The temperature, $K^{\circ}\text{C}$, of water in a kettle is modelled by the formula $K = 20 + 75e^{-0.2t}$ where t is the time in minutes since measurement began.

(a) State the starting temperature of the kettle.

(1)

(b) Find $\frac{dK}{dt}$

(3)

(Total for question 40 is 4 marks)

41

$$y = \frac{x^2 + 4x}{(x + 2)^2} \quad x \neq -2$$

- (a) Show that $\frac{dy}{dx} = \frac{A}{(x+2)^n}$ where A and n are constants to be found.

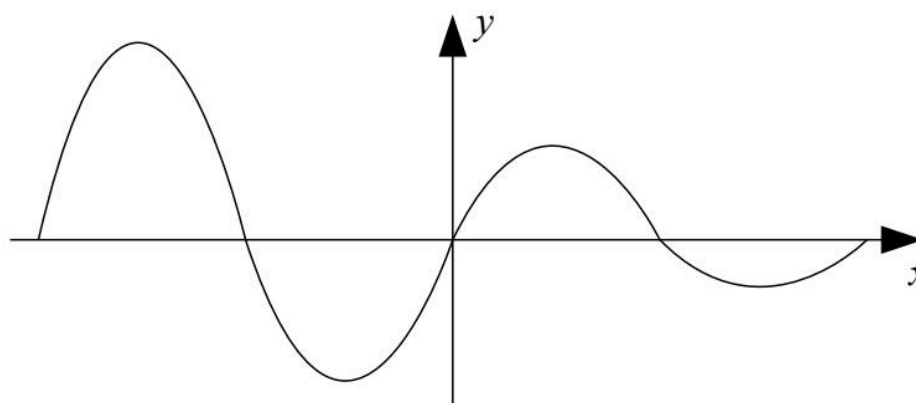
(4)

- (b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$

(1)

(Total for question 41 is 5 marks)

- 42 The curve C with equation $f(x) = \frac{\sin 2x}{e^x}$ $-\pi < x < \pi$ is shown in the diagram.



- (a) Show that the x -coordinates of the turning points of C satisfy the equation $\tan 2x = 2$

(6)

- (b) Hence find the coordinates of the turning points

(3)

(Total for question 42 is 9 marks)

- 43** The curve C has the equation $x = 2 \tan 2y - \frac{\pi}{4} < y < \frac{\pi}{4}$
Show that, for all points (x, y) lying on C ,

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are constants to be found

(Total for question 43 is 5 marks)

- 44** The curve C has the equation $y = x^x \quad x > 0$
Find, by firstly taking logarithms, the x -coordinate of the turning point of C .

(Total for question 44 is 5 marks)

- 45** A storage tank is modelled in the shape of a hollow cylinder. The walls of the tank are assumed to have negligible thickness. The cylinder has radius r metres and height h metres. The volume of the tank is 12 m^3

(a) Show that the surface area of the tank, in m^2 , is $2\pi r^2 + \frac{24}{r}$

(4)

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

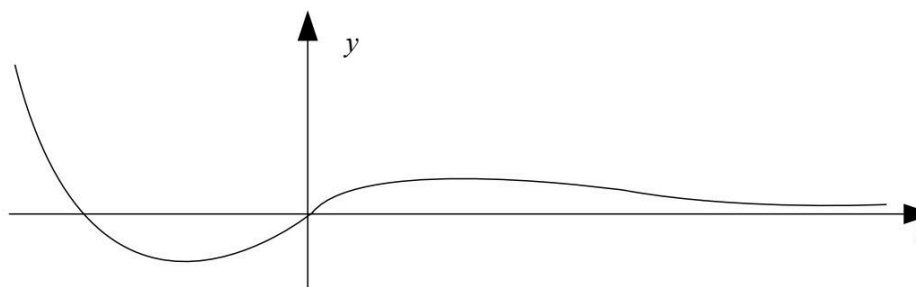
(4)

- (c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

(Total for question 45 is 10 marks)

- 46 The diagram shows the graph with equation $y = f(x)$ where $f(x) = (2x^2 + 3x)e^{-x}$ $x \in \mathbb{R}$



- (a) Show that $f'(x) = e^{-x}(x + 3 - 2x^2)$

(3)

- (b) Hence, find, in simplest form, the exact coordinates of the stationary points of C

(3)

(c) Find (i) the range of g where $g(x) = 2f(x)$ and (ii) the range of h where $h(x) = f(x) + 2, x > 0$

(3)

(Total for question 46 is 9 marks)

47 Curve C is given by $x + 5 = \sin x \cos x + 2y$ $0 < x < \pi$

(a) Show that $\frac{dy}{dx} = \sin^2 x$

(4)

(b) Find the coordinates of the point of inflection of C .

(3)

(Total for question 47 is 7 marks)

- 48** A function f is given by $f(x) = \frac{5+\ln x}{3+2\ln x} \quad x > 0$
Prove that f is a decreasing function.

(Total for question 48 is 3 marks)

49 A curve C has the equation $y = \frac{x^2+5}{x} + \ln x \quad x > 0$

(a) Show that $\frac{dy}{dx} = \frac{x^2+x-5}{x^2}$

(4)

(b) Hence find the coordinates of the turning point of C .

(3)

(Total for question 49 is 7 marks)

50 A curve is defined by the parametric equations

$$x = 3 \times 2^{-t} + 2$$

$$y = 5 \times 2^t - 3$$

(a) Show that $\frac{dy}{dx} = -\frac{5}{3} \times 2^{2t}$

(3)

(b) Find a Cartesian equation for the curve.

(2)

(Total for question 50 is 5 marks)

- 51** A six-sided box, in the shape of a cuboid, is made from a sheet metal. The base of the box is x cm by y cm and the height of the box is x cm.

The volume of the box is 5000 cm^3 .

- (a) Show that the area of sheet metal, $A \text{ cm}^2$, is given by $A = \frac{20000}{x} + 2x^2$

(4)

- (b) Use calculus to show to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A .

(2)

(d) Calculate the minimum area of sheet metal needed to make the box.

(2)

(Total for question 51 is 12 marks)

52 Find the coordinates of the stationary point of the curve with equation

$$x^2 + 2x = e^y - 3$$

(Total for question 52 is 7 marks)

53 A function f is defined by $f(x) = \frac{x}{\sqrt{2x-3}}$

(a) State the maximum possible domain of f .

(2)

(b) Use the quotient rule to show that $f'(x) = \frac{x-3}{(2x-3)^{\frac{3}{2}}}$

(3)

- (c) Show that the graph of $y = f(x)$ has exactly one point of inflection.

(7)

(Total for question 53 is 12 marks)

- 54 Given $y = e^{kx}$, where k is a constant, find $\frac{dy}{dx}$ in terms of k .

(Total for question 54 is 2 marks)

- 55** The volume of a sphere is increasing at a constant rate of $2 \text{ cm}^3 \text{ s}^{-1}$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

Show that the rate of increase of the radius when $r = 2 \text{ cm}$ is $\frac{a}{\pi}$, where a is a constant.

(Total for question 55 is 4 marks)

- 56** Given $y = 3^x$, show that $\frac{dy}{dx} = 3^x \ln 3$

(Total for question 56 is 2 marks)

57 A curve, C , has the equation $y = \frac{e^{2x+1}}{x^2}$

Show that C has exactly one stationary point.

Fully justify your answer.

(Total for question 57 is 7 marks)

58 A curve, C , has the equation $xy^2 + x^2y = 10$

(a) Prove that the curve does not intersect the coordinate axes.

(2)

(b) Show that $\frac{dy}{dx} = -\frac{y(2x+y)}{x(x+2y)}$

(5)

(c) Find the exact coordinates of the stationary point.

(5)

(Total for question 58 is 12 marks)

- 59** A curve, C , has the equation $x^2y + xy = 6$
When $x > 0$, find the equation of the tangent to C when $y = 1$.

(Total for question 59 is 7 marks)

60 A curve, C , has the equation $5 \sin y + x \cos y = Ax$ where A is a constant. C passes through the point $P \left(\sqrt{3}, \frac{\pi}{3} \right)$

(a) Show that $A = 3$

(2)

(b) Show that $\frac{dy}{dx} = \frac{3 - \cos y}{5 \cos y - x \sin y}$

(5)

(c) Hence find the equation of the tangent to C at P .

(4)

(Total for question 60 is 11 marks)

61 A curve has parametric equations $x = 3t + \frac{1}{t}$ and $y = 3t - \frac{1}{t}$ for $t \neq 0$

(a) Find $\frac{dy}{dx}$ in terms of t , giving your answer in its simplest form.

(4)

(b) Explain why the curve has no stationary points.

(2)

- (c) By considering $x + y$, or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets.

(4)

(Total for question 61 is 10 marks)

62 A curve has the equation $y = x \ln x$

(a) Find the coordinates of the turning point of the curve.

(4)

(b) Determine whether this turning point is a maximum or a minimum.

(3)

(Total for question 62 is 7 marks)

63 A curve has the equation $x^3 - x^2y - 2y - 5 = 0$

(a) Show that $\frac{dy}{dx} = \frac{3x^2 - 2xy}{x^2 + 2}$

(4)

(b) Find the equation of the normal to the curve at the point (1, 2).

(4)

(Total for question 63 is 8 marks)

64 Let $f(x) = x^3 + 5x$. Use differentiation from first principles to show that $f'(x) = 3x^2 + 5$.

(Total for question 64 is 6 marks)

65 A curve has the equation $y = a^{x^2}$, where a is a constant greater than 1.

(a) Show that $\frac{dy}{dx} = 2xa^{x^2} \ln a$

(3)

(b) The tangent at the point $(1, a)$ passes through the point $(\frac{1}{2}, 0)$. Find the value of a , giving your answer in an exact form.

(4)

(c) By considering $\frac{d^2y}{dx^2}$ show that the curve is convex for all values of x .

(5)

(Total for question 65 is 12 marks)

66 Differentiate with respect to x :

(a) $(2x + 5)^5$

(2)

(b) $3x^2 \ln x$

(3)

(Total for question 66 is 5 marks)

67 A solid right circular cylinder has radius r cm and height h cm. The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, $V \text{ cm}^3$, of the cylinder is given by $V = 400r - \pi r^3$

(4)

(b) use calculus to find the maximum value of V , to the nearest cm^3 .

(5)

- (c) Justify that the value of V you have found is a maximum.

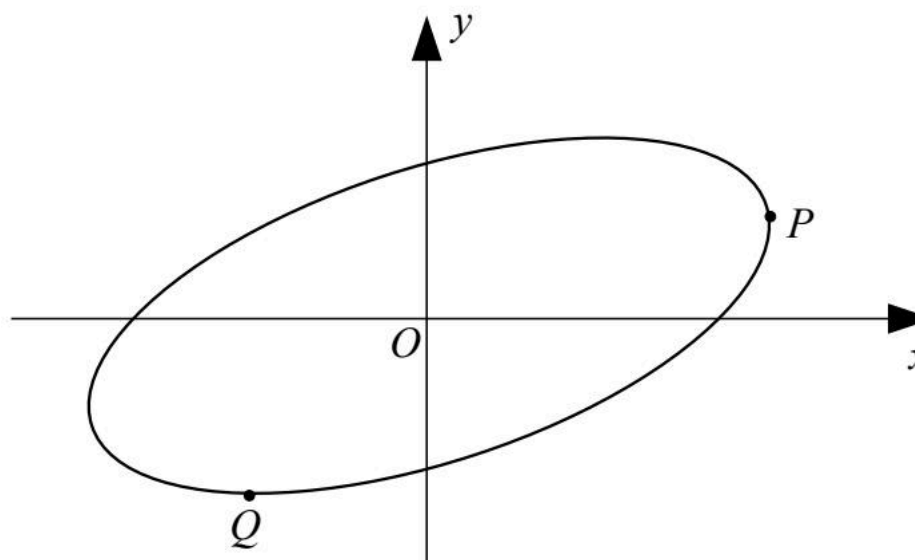
(2)

(Total for question 67 is 11 marks)

- 68** A curve has the equation $y = x^3 + kx^2 + 5$, where k is a constant.
Given that the curve has a point of inflection where $x = 2$, show that $k = -6$.

(Total for question 68 is 5 marks)

- 69 The diagram shows the curve with equation $x^2 - 2xy + 4y^2 - 12 = 0$



- (a) Show that $\frac{dy}{dx} = \frac{x-y}{x-4y}$

(3)

- (b) At the point P on the curve the tangent to the curve is parallel to the y -axis and at the point Q on the curve the tangent to the curve is parallel to the x -axis. Show that the exact distance PQ , is $k\sqrt{5}$ where k is a rational number to be determined.

(7)

(Total for question 69 is 10 marks)

70 Differentiate with respect to x :

(a) $\sin(2x + 5)$

(2)

(b) $\frac{3x^2}{2x-1}$

(3)

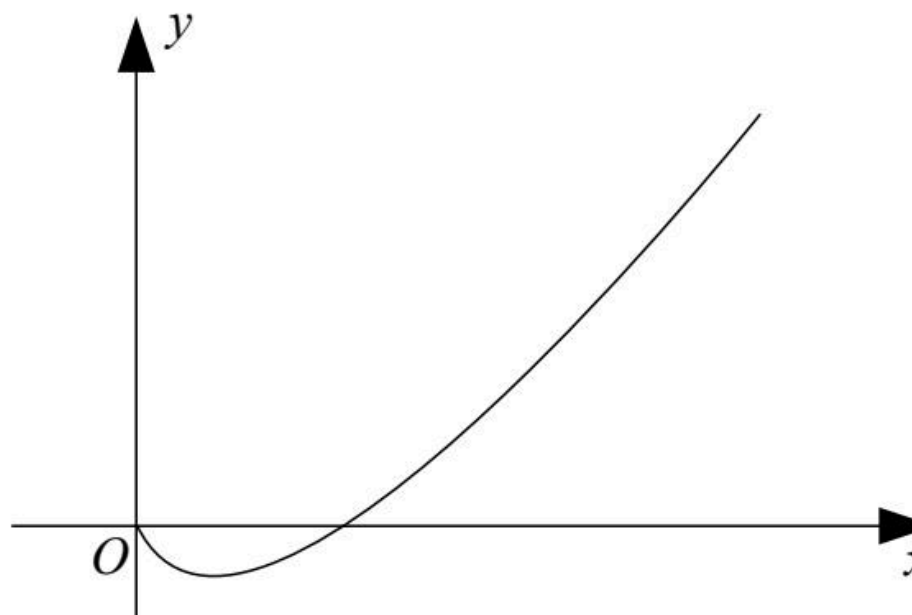
(c) $\tan(x^2)$

(2)

(Total for question 70 is 7 marks)

- 71** The curve C is defined by the parametric equations $x = 2 \cos t$, $y = \sin t$.
The line L is a tangent to C at the point given by $t = \frac{\pi}{3}$.
Find the point where L cuts the y -axis.

(Total for question 71 is 6 marks)



The diagram shows the curve given parametrically by the equations $x = 4t^2$, $y = t(t^2 - 8)$, for $t > 0$

- (a) Show that $\frac{dy}{dx} = \frac{3t^2 - 8}{8t}$

(3)

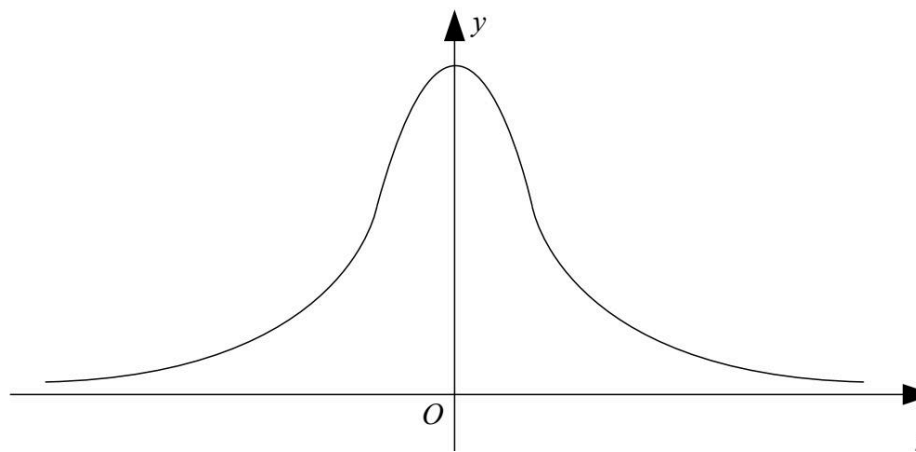
- (b) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $3x - 12y + 8 = 0$

(3)

- (c) Find the cartesian equation of the curve.

(3)

(Total for question 72 is 9 marks)



The diagram shows the curve with the equation $y = \frac{1}{1+x^2}$

- (a) Find $\frac{d^2y}{dx^2}$

(5)

- (b) Hence find the set of values of x for which the curve is concave.

(2)

(Total for question 73 is 7 marks)
