

1

- (a) • $f'(x) = \frac{1}{x} + e^x$ (M1A1)
- (b) • $f'(x) = x^{-1} + e^x$
• $f''(x) = -x^{-2} + e^x$ (M1A1)

(Total for question 1 is 4 marks)

2

- (a) • $6x^2 + 4e^{4x}$ (M1A1)
- (b) • $3(x^2 + 5)^2 \times 2x$
• $6x(x^2 + 5)^2$ (M1A1)

(Total for question 2 is 4 marks)

3

- (a) • $x + 5(x^2 + 4)^{-1}$
• $1 - 5(x^2 + 4)^{-2}(2x)$
• $1 - 10x(x^2 + 4)^{-2}$ (M1A1)
- (b) • $2xe^{x^2}$ (M1A1)

(Total for question 3 is 4 marks)

4

- When $x = 1$, $y = 2 + \ln 1 = 2$. Point is $(1, 2)$. (B1)
- $\frac{dy}{dx} = \frac{1}{3-2x}(-2) = \frac{-2}{3-2x}$ (M1)
- When $x = 1$, $\frac{dy}{dx} = \frac{-2}{3-2} = -2$ (A1)
- $y - 2 = -2(x - 1)$ (M1)
- $y = -2x + 4$ (A1)

(Total for question 4 is 5 marks)

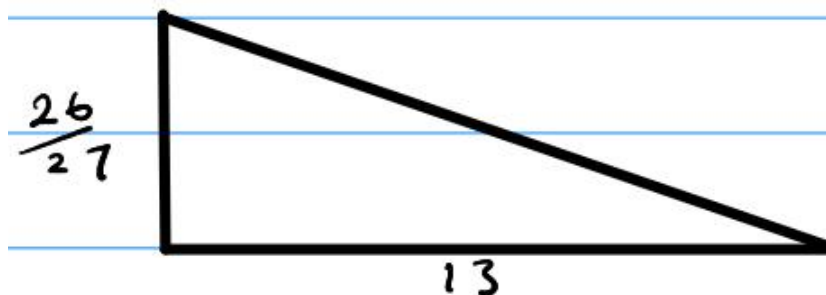
5

- (a)
- $y = 3(2x + 1)^{-1} \Rightarrow \frac{dy}{dx} = -3(2x + 1)^{-2}(2) = -6(2x + 1)^{-2}$ (M1A1)
 - When $x = 1$, $\frac{dy}{dx} = -6(3)^{-2} = -\frac{2}{3}$ (M1)
 - $y = \frac{3}{2(1)+1} = 1 \Rightarrow (1, 1)$ (B1)
 - Gradient of normal = $\frac{3}{2}$
 - $y - 1 = \frac{3}{2}(x - 1) \Rightarrow y = \frac{3}{2}x - \frac{1}{2}$ (A1)
- (b)
- $\frac{3}{2}x - \frac{1}{2} = \frac{3}{2x+1}$
 - $(3x - 1)(2x + 1) = 6 \Rightarrow 6x^2 + x - 7 = 0$ (M1)
 - $(6x + 7)(x - 1) = 0 \Rightarrow x = -\frac{7}{6}$ (since $x = 1$ is P)
 - $y = \frac{3}{2}(-\frac{7}{6}) - \frac{1}{2} = -\frac{9}{4}$
 - $Q = (-\frac{7}{6}, -\frac{9}{4})$ (A1)

(Total for question 5 is 7 marks)

6

- (a)
- When $x = 4$, $y = \frac{2}{3}$ (B1)
 - $y = 2(2x + 1)^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -2(2x + 1)^{-\frac{3}{2}}$ (M1A1)
 - When $x = 4$, $\frac{dy}{dx} = -\frac{2}{27}$ (M1)
 - $y - \frac{2}{3} = -\frac{2}{27}(x - 4) \Rightarrow y = -\frac{2}{27}x + \frac{26}{27}$ (A1)
- (b)
- Crosses y at $B(0, \frac{26}{27})$ (B1)
 - Crosses x when $y = 0 \Rightarrow 0 = -\frac{2}{27}x + \frac{26}{27} \Rightarrow x = 13$. Point $A(13, 0)$ (M1A1)
 - Area = $\frac{1}{2} \times 13 \times \frac{26}{27} = \frac{169}{27}$ (A1)



(Total for question 6 is 9 marks)

7

- (a)
- $u = 2x, v = e^x \Rightarrow \frac{du}{dx} = 2, \frac{dv}{dx} = e^x$
 - $2e^x + 2xe^x$ (M1A1)
- (b)
- $u = 3x^2, v = \ln 2x \Rightarrow \frac{du}{dx} = 6x, \frac{dv}{dx} = \frac{1}{x}$
 - $3x + 6x \ln 2x$ (M1A1A1)

(Total for question 7 is 5 marks)

8

- (a)
- $u = x^2, v = (2x + 1)^{\frac{1}{2}} \implies \frac{du}{dx} = 2x, \frac{dv}{dx} = (2x + 1)^{-\frac{1}{2}}$
 - $x^2(2x + 1)^{-\frac{1}{2}} + 2x(2x + 1)^{\frac{1}{2}}$ (M1A1A1)
- (b)
- $u = x, v = \ln(x + 1) \implies \frac{du}{dx} = 1, \frac{dv}{dx} = \frac{1}{x+1}$
 - $\frac{x}{x+1} + \ln(x + 1)$ (M1A1A1)

(Total for question 8 is 6 marks)

9

- (a)
- $u = x + 5, v = (x + 1)^3 \implies \frac{du}{dx} = 1, \frac{dv}{dx} = 3(x + 1)^2$
 - $(x + 1)^3 + 3(x + 5)(x + 1)^2$ (M1A1A1)
- (b)
- $u = x^3, v = \ln(2x + 1) \implies \frac{du}{dx} = 3x^2, \frac{dv}{dx} = \frac{2}{2x+1}$
 - $\frac{2x^3}{2x+1} + 3x^2 \ln(2x + 1)$ (M1A1A1)

(Total for question 9 is 6 marks)

10

- When $x = 1, y = (3(1) - 1) \ln(2 - 1) = 2 \ln 1 = 0$. Point is $(1, 0)$. (B1)
- $u = 3x - 1, v = \ln(2 - x) \implies \frac{du}{dx} = 3, \frac{dv}{dx} = \frac{-1}{2-x}$
- $\frac{dy}{dx} = \frac{-(3x-1)}{2-x} + 3 \ln(2 - x)$ (M1A1)
- When $x = 1, \frac{dy}{dx} = \frac{-2}{1} + 3 \ln 1 = -2$ (M1)
- $y - 0 = -2(x - 1) \implies y = -2x + 2$ (A1)

(Total for question 10 is 5 marks)

11

- Differentiates using product rule: $\frac{dy}{dx} = (x - 3)^3 + 3x(x - 3)^2$ (M1)
- Sets $\frac{dy}{dx} = 0$ and factors: $(x - 3)^2(x - 3 + 3x) = 0$ (M1)
- Finds x -coordinates $x = 3$ and $x = \frac{3}{4}$ (A1)
- Finds y -coordinates $(3, 0)$ and $(\frac{3}{4}, -\frac{2187}{256})$ (A1)
- Differentiates again or tests gradient to find $\frac{d^2y}{dx^2} = 4(x - 3)^2 + 2(x - 3)(4x - 3)$ (M1)
- Evaluates $\frac{d^2y}{dx^2}$ at $x = \frac{3}{4}$ to show it is > 0 , hence a minimum (A1)
- Evaluates gradient or second derivative around $x = 3$ to show point of inflection (M1)
- Correctly identifies $(3, 0)$ as a point of inflection and $(\frac{3}{4}, -\frac{2187}{256})$ as a minimum (A1)

(Total for question 11 is 8 marks)

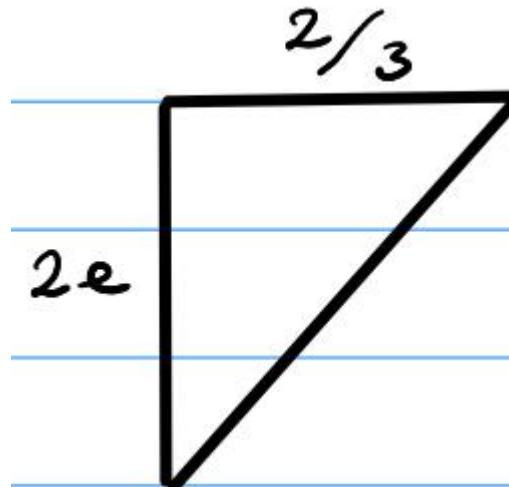
12

- Finds y -coordinate: when $x = 5$, $y = 5\sqrt{4} = 10$ (B1)
- Differentiates using product/chain rule: $\frac{dy}{dx} = (x-1)^{\frac{1}{2}} + \frac{1}{2}x(x-1)^{-\frac{1}{2}}$ (M1)
- Calculates gradient at $x = 5$: $\frac{dy}{dx} = 2 + \frac{5}{4} = \frac{13}{4}$ (A1)
- Finds gradient of normal: $m = -\frac{4}{13}$ (M1)
- Forms equation: $y - 10 = -\frac{4}{13}(x - 5)$ or $y = -\frac{4}{13}x + \frac{150}{13}$ (A1)

(Total for question 12 is 5 marks)

13

- (a)
- Finds y -coordinate: when $x = 1$, $y = e$ (B1)
 - Differentiates: $\frac{dy}{dx} = 2x^2e^{x^2} + e^{x^2}$ (M1)
 - Gradient at $x = 1$: $3e$ (A1)
 - Equation: $y - e = 3e(x - 1)$ (M1)
 - Simplifies to $y = 3ex - 2e$ (A1)
- (b)
- Finds y -intercept (B): $(0, -2e)$ (B1)
 - Finds x -intercept (A): $0 = 3ex - 2e \implies x = \frac{2}{3}$ (M1)
 - Area formula: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \frac{2}{3} \times 2e$ (M1)
 - Final area: $\frac{2}{3}e$ (A1)



(Total for question 13 is 9 marks)

14

- (a)
- Uses quotient rule: $u = e^x, v = 2x + 1$ (M1)
 - $\frac{du}{dx} = e^x, \frac{dv}{dx} = 2$ (B1)
 - Substitutes into formula: $\frac{(2x+1)e^x - 2e^x}{(2x+1)^2}$ (M1)
 - Simplifies to $\frac{e^x(2x-1)}{(2x+1)^2}$ (A1)
- (b)
- Uses quotient rule: $u = \ln(x^2 + 1), v = 3x + 2$ (M1)
 - $\frac{du}{dx} = \frac{2x}{x^2+1}, \frac{dv}{dx} = 3$ (B1)
 - Substitutes into formula: $\frac{(3x+2)(\frac{2x}{x^2+1}) - 3\ln(x^2+1)}{(3x+2)^2}$ (M1)
 - Simplifies to $\frac{2x(3x+2) - 3(x^2+1)\ln(x^2+1)}{(x^2+1)(3x+2)^2}$ (A1)

(Total for question 14 is 8 marks)

15

- Finds y -coordinate: when $x = 1, y = 3$ (B1)
- Differentiates using quotient rule: $\frac{dy}{dx} = \frac{3(2x-1) - 2(3x)}{(2x-1)^2}$ (M1)
- Simplifies derivative: $\frac{-3}{(2x-1)^2}$ (A1)
- Gradient at $x = 1$: -3 (M1)
- Equation: $y - 3 = -3(x - 1) \implies y = -3x + 6$ (A1)

(Total for question 15 is 5 marks)

16

- Finds y -coordinate: when $x = 0, y = \frac{1+1}{1+3} = \frac{1}{2}$ (B1)
- Differentiates using quotient rule: $\frac{dy}{dx} = \frac{e^x(e^x+3) - e^x(e^x+1)}{(e^x+3)^2}$ (M1)
- Gradient at $x = 0$: $\frac{1(4) - 1(2)}{16} = \frac{2}{16} = \frac{1}{8}$ (A1)
- Gradient of normal: -8 (M1)
- Equation: $y - \frac{1}{2} = -8(x - 0) \implies y = -8x + \frac{1}{2}$ (A1)

(Total for question 16 is 5 marks)

17

- Differentiates using quotient rule: $\frac{dy}{dx} = \frac{9+x^2-2x^2}{(9+x^2)^2} = \frac{9-x^2}{(9+x^2)^2}$ (M1)
- Sets $\frac{dy}{dx} = 0 \implies 9 - x^2 = 0$ (M1)
- Finds $x = \pm 3$ (A1)
- Finds y -coordinates: $(3, \frac{1}{6})$ and $(-3, -\frac{1}{6})$ (A1)
- Finds second derivative: $\frac{d^2y}{dx^2} = \frac{-2x(9+x^2)^2 - 4x(9+x^2)(9-x^2)}{(9+x^2)^4}$ (M1)
- Evaluates at $x = 3$ (Max) and $x = -3$ (Min) (A1)
- Correctly identifies Min at $(-3, -\frac{1}{6})$ and Max at $(3, \frac{1}{6})$ (A1)

(Total for question 17 is 7 marks)

18

- (a)
- Factors denominator: $2x^2 + 7x + 6 = (2x + 3)(x + 2)$ (B1)
 - Simplifies second term: $\frac{2(x+2)}{(2x+3)(x+2)} = \frac{2}{2x+3}$ (M1)
 - Combines fractions: $\frac{x-5+2}{2x+3} = \frac{x-3}{2x+3}$ (A1)
- (b)
- Uses quotient rule on $\frac{x-3}{2x+3}$ (M1)
 - $u = x - 3, v = 2x + 3 \implies u' = 1, v' = 2$ (B1)
 - $f'(x) = \frac{(2x+3) - 2(x-3)}{(2x+3)^2}$ (M1)
 - Simplifies to $\frac{9}{(2x+3)^2}$ (A1)

(Total for question 18 is 7 marks)

19

- (a)
- Factors denominator: $4x^2 - 9 = (2x + 3)(2x - 3)$ (B1)
 - Simplifies second term: $\frac{2(2x-3)}{(2x+3)(2x-3)} = \frac{2}{2x+3}$ (M1)
 - Combines: $\frac{4x+1-2}{2x+3} = \frac{4x-1}{2x+3}$ (A1)
- (b)
- Uses quotient rule on $\frac{4x-1}{2x+3}$ (M1)
 - $u = 4x - 1, v = 2x + 3 \implies u' = 4, v' = 2$ (B1)
 - $f'(x) = \frac{4(2x+3) - 2(4x-1)}{(2x+3)^2}$ (M1)
 - Simplifies to $\frac{14}{(2x+3)^2}$ (A1)

(Total for question 19 is 7 marks)

20

- (a)
- Differentiates implicitly: $4x - (3y + 3x \frac{dy}{dx}) + 2y \frac{dy}{dx} = 0$ (M1) (M1)
 - Collects $\frac{dy}{dx}$ terms: $\frac{dy}{dx}(2y - 3x) = 3y - 4x$ (M1)
 - Isolates $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{3y-4x}{2y-3x}$ (A1)
- (b)
- Substitutes $(1, -2)$ into $\frac{dy}{dx}$: $\frac{3(-2)-4(1)}{2(-2)-3(1)} = \frac{-10}{-7} = \frac{10}{7}$ (M1)
 - Gradient of normal: $-\frac{7}{10}$ (B1)
 - Equation: $y + 2 = -\frac{7}{10}(x - 1) \implies y = -\frac{7}{10}x - \frac{13}{10}$ (A1)

(Total for question 20 is 8 marks)

21

- (a)
- Differentiate implicitly: $6x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$ (M2)
 - Substitute $\frac{dy}{dx} = \frac{4}{3}$: $6x + \frac{4}{3}x + y + \frac{8}{3}y = 0$ (M1)
 - Simplify: $\frac{22}{3}x + \frac{11}{3}y = 0$ (M1)
 - Divide by $\frac{11}{3}$ to reach $2x + y = 0$ (A2)
- (b)
- Substitute $y = -2x$ into original equation: $3x^2 + x(-2x) + (-2x)^2 = 20$ (M1)
 - Solve for x : $3x^2 - 2x^2 + 4x^2 = 20 \implies 5x^2 = 20 \implies x = \pm 2$ (A1)
 - Find y : $x = 2 \implies y = -4$ and $x = -2 \implies y = 4$. Coordinates are $(2, -4)$ and $(-2, 4)$ (A1)

(Total for question 21 is 9 marks)

22

- (a)
- Differentiate implicitly: $2x + 4y + 4x \frac{dy}{dx} - 1 + 2y \frac{dy}{dx} = 0$ (M2)
 - Factorize $\frac{dy}{dx}$: $\frac{dy}{dx}(4x + 2y) = 1 - 2x - 4y$ (M1)
 - Rearrange: $\frac{dy}{dx} = \frac{1-2x-4y}{4x+2y}$ (A2)
- (b)
- Substitute $(2, 3)$ into $\frac{dy}{dx} \cdot \frac{1-2(2)-4(3)}{4(2)+2(3)} = \frac{-15}{14}$ (M1)
 - Use $y - 3 = -\frac{15}{14}(x - 2)$ (M1)
 - Final answer: $y = -\frac{15}{14}x + \frac{36}{7}$ (A1)

(Total for question 22 is 8 marks)

23

- (a)
- Differentiate implicitly: $2 \cos x - 2 \sin y \frac{dy}{dx} = 0$ (M2)
 - Rearrange: $\frac{dy}{dx} = \frac{\cos x}{\sin y}$ (A1)
- (b)
- Set $\frac{\cos x}{\sin y} = 0 \implies \cos x = 0$ (M1)
 - Solve for x : $x = \frac{\pi}{2}$ (A1)
 - Substitute $x = \frac{\pi}{2}$ into original equation: $2(1) + 2 \cos y = 3 \implies \cos y = \frac{1}{2}$ (M1)
 - Solve for y : $y = \frac{\pi}{3}$. Coordinates are $(\frac{\pi}{2}, \frac{\pi}{3})$ (A1)

(Total for question 23 is 7 marks)

24

- (a)
- Take logs: $\ln y = x \ln 2$ (M1)
 - Differentiate: $\frac{1}{y} \frac{dy}{dx} = \ln 2$ (M1)
 - Substitute $y = 2^x$ to reach $\frac{dy}{dx} = 2^x \ln 2$ (A1)
- (b)
- Differentiate using chain rule: $\frac{dy}{dx} = 3^{x^2} \ln 3 \times 2x$ (M2)
 - Evaluate at $x = 2$: $m = 3^4 \ln 3 \times 4 = 81 \times 4 \ln 3 = 324 \ln 3$ (M1)
 - Use $y - 81 = 324 \ln 3(x - 2)$ (A1)

(Total for question 24 is 6 marks)

25

- (a)
- Write $\tan x = \frac{\sin x}{\cos x}$ and apply quotient rule: $u = \sin x, v = \cos x \implies \frac{du}{dx} = \cos x, \frac{dv}{dx} = -\sin x$ (M1)
 - $\frac{d \tan x}{dx} = \frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ (M1)
 - Use identity $\cos^2 x + \sin^2 x = 1$ to reach $\frac{1}{\cos^2 x} = \sec^2 x$ (A1)
- (b)
- Write $\sec x = \frac{1}{\cos x}$ and apply quotient rule or chain rule (M1)
 - Obtain $\frac{\sin x}{\cos^2 x}$ (A1)
 - Split fraction into $\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \tan x$ (A1)
- (c)
- Write $\cot x = \frac{\cos x}{\sin x}$ and apply quotient rule (M1)
 - Obtain $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ (M1)
 - Reach $-\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$ (A1)
- (d)
- Write $\operatorname{cosec} x = \frac{1}{\sin x}$ and apply quotient rule or chain rule (M1)
 - Obtain $\frac{-\cos x}{\sin^2 x}$ (A1)
 - Reach $-\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\operatorname{cosec} x \cot x$ (A1)

(Total for question 25 is 12 marks)

26

- (a)
- Use product rule: $u = x^2, v = \cos 2x \implies \frac{du}{dx} = 2x, \frac{dv}{dx} = -2 \sin 2x$ (M1)
 - Apply formula: $v \frac{du}{dx} + u \frac{dv}{dx} = 2x \cos 2x + x^2(-2 \sin 2x)$ (M1)
 - Final answer: $2x \cos 2x - 2x^2 \sin 2x$ (A1)
- (b)
- Apply chain rule: $3 \cos(2x + 1) \times 2$ (M1)
 - Final answer: $6 \cos(2x + 1)$ (A1)

(Total for question 26 is 5 marks)

27

- (a)
- Use product rule: $u = e^{3x}, v = \cos 2x + \sin x \implies \frac{du}{dx} = 3e^{3x}, \frac{dv}{dx} = -2 \sin 2x + \cos x$ (M1)
 - Apply formula: $3e^{3x}(\cos 2x + \sin x) + e^{3x}(-2 \sin 2x + \cos x)$ (M1)
 - Final answer: $e^{3x}(3 \cos 2x + 3 \sin x - 2 \sin 2x + \cos x)$ (A1)
- (b)
- Apply chain rule: $\frac{1}{\sin x} \times \cos x$ (M1)
 - Simplify to $\frac{\cos x}{\sin x}$ (M1)
 - Final answer: $\cot x$ (A1)

(Total for question 27 is 6 marks)

28

- (a)
- Differentiate x with respect to y : $\frac{dx}{dy} = 2 \sec^2 y$ (B2)
- (b)
- Use $\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{1}{2 \sec^2 y}$ (M1)
 - Use identity $\sec^2 y = 1 + \tan^2 y$ (M1)
 - Substitute $\tan y = \frac{x}{2}$ to get $\frac{1}{2(1+(x/2)^2)}$ (M1)
 - Simplify to $\frac{2}{4+x^2}$ (A1)

(Total for question 28 is 6 marks)

29

- Find y -coordinate: $y = \operatorname{cosec}(\frac{\pi}{4}) + \cos(\frac{\pi}{2}) = \sqrt{2} + 0 = \sqrt{2}$ (B1)
- Differentiate: $\frac{dy}{dx} = -\operatorname{cosec} x \cot x - 2 \sin 2x$ (M1)
- Evaluate gradient at $x = \frac{\pi}{4}$: $m = -\sqrt{2}(1) - 2(1) = -2 - \sqrt{2}$ (M1)
- Use $y - y_1 = m(x - x_1)$: $y - \sqrt{2} = (-2 - \sqrt{2})(x - \frac{\pi}{4})$ (M1)
- Final equation: $y = (-2 - \sqrt{2})x + \frac{\pi(2+\sqrt{2})}{4} + \sqrt{2}$ (A1)

(Total for question 29 is 6 marks)

30

- Find y -coordinate: $y = \sec(\frac{\pi}{3}) = 2$ (B1)
- Differentiate: $\frac{dy}{dx} = 2 \sec 2x \tan 2x$ (M1)
- Evaluate gradient of tangent at $x = \frac{\pi}{6}$: $m_t = 2(2)(\sqrt{3}) = 4\sqrt{3}$ (M1)
- Gradient of normal: $m_n = -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}$ (M1)
- Use $y - y_1 = m(x - x_1)$: $y - 2 = -\frac{\sqrt{3}}{12}(x - \frac{\pi}{6})$ (A1)

(Total for question 30 is 6 marks)

- 31
- $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ (M1)
 - $\frac{d \sin x}{dx} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ (M1)
 - $= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$ (M1)
 - $= \lim_{h \rightarrow 0} \frac{\sin x (\cos h - 1) + \cos x \sin h}{h}$ (M1)
 - $= \lim_{h \rightarrow 0} \frac{(\cos h - 1)}{h} \sin x + \frac{\sin h}{h} \cos x = 0(\sin x) + 1 \cos x = \cos x$ (A1)

(Total for question 31 is 5 marks)

- 32
- $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ (M1)
 - $\frac{d \cos x}{dx} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h}$ (M1)
 - $= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$ (M1)
 - $= \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1) - \sin x \sin h}{h}$ (M1)
 - $= \cos x(0) - \sin x(1) = -\sin x$ (A1)

(Total for question 32 is 5 marks)

- 33
- (a)
- For y -axis, $x = 0 \Rightarrow 2t + 1 = 0 \Rightarrow t = -0.5$. $y = (-0.5)^2 - 1 = -0.75$. Point is $(0, -0.75)$. (B1)
 - For x -axis, $y = 0 \Rightarrow t^2 - 1 = 0 \Rightarrow t = \pm 1$. $x = 2(1) + 1 = 3$ and $x = 2(-1) + 1 = -1$. Points are $(3, 0)$ and $(-1, 0)$. (B1)
- (b)
- $\frac{dx}{dt} = 2, \frac{dy}{dt} = 2t$ (M1)
 - $\frac{dy}{dx} = \frac{2t}{2} = t$ (M1)
 - Substitute $t = \frac{x-1}{2}$ to get $\frac{dy}{dx} = \frac{x-1}{2}$ (A1)

(Total for question 33 is 5 marks)

34

- (a)
- $\frac{dy}{dt} = -\sin t$ (B1)
 - $\frac{dx}{dt} = 2 \tan t \sec^2 t$ (M1)
 - $\frac{dy}{dx} = \frac{-\sin t}{2 \tan t \sec^2 t} = -\frac{1}{2} \cos^3 t$ (A1)
- (b)
- At $t = \frac{\pi}{4}$, $x = 1$, $y = \frac{\sqrt{2}}{2}$ (B1)
 - Gradient $m = -\frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^3 = -\frac{\sqrt{2}}{8}$ (M1)
 - $y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{8}(x - 1)$ (M1)
 - $y = -\frac{\sqrt{2}}{8}x + \frac{5\sqrt{2}}{8}$ (A1)
- (c)
- Use $1 + \tan^2 t = \sec^2 t$ (M1)
 - $\tan^2 t = x$ and $\sec^2 t = \frac{1}{\cos^2 t} = \frac{1}{y^2}$ (M1)
 - $1 + x = \frac{1}{y^2}$ (A1)
 - $y^2 = \frac{1}{1+x}$ (A1)

(Total for question 34 is 12 marks)

35

- (a)
- $\frac{dx}{dt} = 2 \sin t \cos t$ (B1)
 - $\frac{dy}{dt} = 2 \cos 2t$ (B1)
 - $\frac{dy}{dx} = \frac{2 \cos 2t}{2 \sin t \cos t} = \frac{\cos 2t}{\sin t \cos t}$ (A1)
- (b)
- At $t = \frac{\pi}{6}$, $x = \frac{1}{4}$, $y = \frac{\sqrt{3}}{2}$ (B1)
 - $\frac{dy}{dx} = \frac{\cos(\pi/3)}{\sin(\pi/6) \cos(\pi/6)} = \frac{0.5}{0.5 \times \sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ (M1)
 - Gradient of normal $m = -\frac{\sqrt{3}}{2}$ (M1)
 - $y - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}(x - \frac{1}{4})$ (M1)
 - $y = -\frac{\sqrt{3}}{2}x + \frac{5\sqrt{3}}{8}$ (A1)
- (c)
- $y = 2 \sin t \cos t \Rightarrow y^2 = 4 \sin^2 t \cos^2 t$ (M1)
 - $y^2 = 4 \sin^2 t (1 - \sin^2 t)$ (M1)
 - $y^2 = 4x(1 - x)$ (A1)
 - $y^2 = 4x - 4x^2$ (A1)

(Total for question 35 is 12 marks)

36

- $u = \sin \theta, v = \sin \theta + \cos \theta$ (M1)
- $\frac{du}{d\theta} = \cos \theta, \frac{dv}{d\theta} = \cos \theta - \sin \theta$ (M1)
- $\frac{dy}{d\theta} = \frac{\cos \theta (\sin \theta + \cos \theta) - \sin \theta (\cos \theta - \sin \theta)}{(\sin \theta + \cos \theta)^2}$ (M1)
- $= \frac{\cos \theta \sin \theta + \cos^2 \theta - \sin \theta \cos \theta + \sin^2 \theta}{\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta}$ (M1)
- $= \frac{1}{1 + \sin 2\theta}$ (A1)

(Total for question 36 is 5 marks)

37

- (a)
- Differentiate implicitly: $2x - (x \frac{dy}{dx} + y) + 4y \frac{dy}{dx} = 0$ (M1) (M1)
 - $2x - y = x \frac{dy}{dx} - 4y \frac{dy}{dx}$ (M1)
 - $\frac{dy}{dx}(x - 4y) = 2x - y \Rightarrow \frac{dy}{dx} = \frac{2x - y}{x - 4y}$ (A1)
- (b)
- $\frac{dy}{dx} = 0 \Rightarrow 2x - y = 0 \Rightarrow y = 2x$ (M1)
 - Substitute into curve: $x^2 - x(2x) + 2(2x)^2 = 20$ (M1)
 - $x^2 - 2x^2 + 8x^2 = 20 \Rightarrow 7x^2 = 20$ (M1)
 - $x = \pm \sqrt{\frac{20}{7}} = \pm \frac{2\sqrt{35}}{7}$ (A1)
 - Points are $(\frac{2\sqrt{35}}{7}, \frac{4\sqrt{35}}{7})$ and $(-\frac{2\sqrt{35}}{7}, -\frac{4\sqrt{35}}{7})$ (A1)

(Total for question 37 is 9 marks)

38

- (a)
- Parabola opening upwards with vertex at (3, 5). (B1)
 - End points at (0, 14) and (6, 14). (B1)
 - Correct domain shown. (B1)
- (b)
- Line is $y = -x + k$, gradient is -1. (B1)
 - Tangent when $\frac{dy}{dx} = -1 \Rightarrow -2(3 - x) = -1 \Rightarrow 6 - 2x = 1 \Rightarrow x = 2.5$. (M1)
 - At $x = 2.5, y = (3 - 2.5)^2 + 5 = 5.25. k = 2.5 + 5.25 = 7.75$. (M1)
 - Upper limit when line passes through (0, 14) or (6, 14): $k = 0 + 14 = 14$. (M1)
 - Range: $\{k : 7.75 < k \leq 14\}$. (A1)

(Total for question 38 is 8 marks)

39

- (a)
- $10x + 3 - 4x^2 = A(1 - x)(2x + 1) + B(2x + 1) + C(1 - x)$ (M1)
 - Let $x = 1 : 9 = 3B \Rightarrow B = 3$ (B1)
 - Let $x = -0.5 : -3 = 1.5C \Rightarrow C = -2$ (B1)
 - Compare x^2 terms: $-4 = -2A \Rightarrow A = 2$ (B1)
- (b)
- $f(x) = 2 + 3(1 - x)^{-1} - 2(2x + 1)^{-1}$ (M1)
 - $f'(x) = 3(1 - x)^{-2}(-1)(-1) - 2(2x + 1)^{-2}(-1)(2) = \frac{3}{(1-x)^2} + \frac{4}{(2x+1)^2}$ (M1)
 - Since $(1 - x)^2 > 0$ and $(2x + 1)^2 > 0$ for $x > 1$, $f'(x) > 0$, therefore $f(x)$ is increasing. (A1)

(Total for question 39 is 7 marks)

40

- (a)
- When $t = 0$, $K = 20 + 75e^0 = 20 + 75 = 95^\circ\text{C}$ (B1)
- (b)
- $\frac{dK}{dt} = 75 \times (-0.2)e^{-0.2t}$ (M1)
 - $= -15e^{-0.2t}$ (A1)

(Total for question 40 is 4 marks)

41

- (a)
- Let $u = x^2 + 4x$ and $v = (x + 2)^2$. (M1)
 - $\frac{du}{dx} = 2x + 4$ and $\frac{dv}{dx} = 2(x + 2)$. (A1)
 - Apply quotient rule: $\frac{dy}{dx} = \frac{(2x+4)(x+2)^2 - 2(x+2)(x^2+4x)}{(x+2)^4}$. (M1)
 - Simplify numerator: $2x^3 + 12x^2 + 24x + 16 - 2x^3 - 12x^2 - 16x = 8x + 16$. (A1)
 - Final form: $\frac{8(x+2)}{(x+2)^4} = \frac{8}{(x+2)^3}$. (A1)
- (b)
- $\frac{8}{(x+2)^3} < 0$ implies $(x + 2)^3 < 0$. (M1)
 - $x + 2 < 0 \Rightarrow x < -2$. (A1)

(Total for question 41 is 5 marks)

42

- (a)
- $u = \sin 2x, v = e^x \Rightarrow \frac{du}{dx} = 2 \cos 2x, \frac{dv}{dx} = e^x$. (M1)
 - $f'(x) = \frac{2e^x \cos 2x - e^x \sin 2x}{e^{2x}}$. (A1)
 - Set $f'(x) = 0$: $e^x(2 \cos 2x - \sin 2x) = 0$. (M1)
 - Since $e^x \neq 0$, $2 \cos 2x = \sin 2x$. (M1)
 - Divide by $\cos 2x$: $\tan 2x = 2$. (A1)
- (b)
- $2x = \arctan(2) \Rightarrow 2x = 1.107, 4.249, -2.034, -5.176$. (M1)
 - $x = -2.59, -1.02, 0.554, 2.12$. (A1)
 - Substitute x into $f(x)$ to find y :
 $(-2.59, 11.9), (-1.02, -2.47), (0.554, 0.514), (2.12, -0.107)$. (A1)

(Total for question 42 is 9 marks)

43

- $\frac{dx}{dy} = 4 \sec^2 2y$. (M1)
- $\frac{dy}{dx} = \frac{1}{4 \sec^2 2y}$. (M1)
- Use identity $\sec^2 2y = 1 + \tan^2 2y$: $\frac{dy}{dx} = \frac{1}{4(1+\tan^2 2y)}$. (M1)
- Substitute $\tan 2y = \frac{x}{2}$: $\frac{dy}{dx} = \frac{1}{4(1+(\frac{x}{2})^2)}$. (M1)
- Simplify: $\frac{1}{4+x^2}$. (A1)
- $a = 1, b = 4$.

(Total for question 43 is 5 marks)

44

- $\ln y = \ln x^x = x \ln x$. (M1)
- Differentiate implicitly: $\frac{1}{y} \frac{dy}{dx} = \ln x + x(\frac{1}{x}) = 1 + \ln x$. (M1)
- $\frac{dy}{dx} = y(1 + \ln x)$. (A1)
- Set $\frac{dy}{dx} = 0$: $1 + \ln x = 0$. (M1)
- $\ln x = -1 \Rightarrow x = e^{-1}$. (A1)

(Total for question 44 is 5 marks)

45

- (a)
- $V = \pi r^2 h = 12 \Rightarrow h = \frac{12}{\pi r^2}$. (M1)
 - $S = 2\pi r^2 + 2\pi r h$. (M1)
 - $S = 2\pi r^2 + 2\pi r \left(\frac{12}{\pi r^2}\right)$. (M1)
 - $S = 2\pi r^2 + \frac{24}{r}$. (A1)
- (b)
- $\frac{dS}{dr} = 4\pi r - 24r^{-2}$. (M1)
 - Set $\frac{dS}{dr} = 0$: $4\pi r = \frac{24}{r^2}$. (M1)
 - $r^3 = \frac{24}{4\pi} = \frac{6}{\pi}$. (M1)
 - $r = \sqrt[3]{\frac{6}{\pi}} \approx 1.24$ m. (A1)
- (c)
- $S = 2\pi(1.24)^2 + \frac{24}{1.24}$. (M1)
 - $S = 29.02... \approx 29$ m². (A1)

(Total for question 45 is 10 marks)

46

- (a)
- $u = 2x^2 + 3x, v = e^{-x} \Rightarrow \frac{du}{dx} = 4x + 3, \frac{dv}{dx} = -e^{-x}$. (M1)
 - $f'(x) = (4x + 3)e^{-x} - (2x^2 + 3x)e^{-x}$. (M1)
 - $f'(x) = e^{-x}(4x + 3 - 2x^2 - 3x) = e^{-x}(x + 3 - 2x^2)$. (A1)
- (b)
- $x + 3 - 2x^2 = 0 \Rightarrow 2x^2 - x - 3 = 0$. (M1)
 - $(2x - 3)(x + 1) = 0 \Rightarrow x = \frac{3}{2}, x = -1$. (A1)
 - $f\left(\frac{3}{2}\right) = \left(2\left(\frac{9}{4}\right) + 3\left(\frac{3}{2}\right)\right)e^{-3/2} = 9e^{-3/2}$.
 - $f(-1) = (2(1) - 3)e^1 = -e$.
 - Coordinates: $\left(\frac{3}{2}, 9e^{-3/2}\right)$ and $(-1, -e)$. (A1)
- (c)
- (i) Min value of $f(x)$ is $-e$, so $g(x) \geq -2e$. (B1)
 - (ii) For $x > 0$, $f(x)$ has max $9e^{-3/2}$ and approaches 0. Range: $2 < h(x) \leq 9e^{-3/2} + 2$. (B2)

(Total for question 46 is 9 marks)

47

- (a)
- Differentiate implicitly: $1 = \cos^2 x - \sin^2 x + 2 \frac{dy}{dx}$. (M1)
 - Use $\cos^2 x = 1 - \sin^2 x$: $1 = 1 - 2 \sin^2 x + 2 \frac{dy}{dx}$. (M1)
 - $0 = -2 \sin^2 x + 2 \frac{dy}{dx}$. (M1)
 - $\frac{dy}{dx} = \sin^2 x$. (A1)
- (b)
- $\frac{d^2y}{dx^2} = 2 \sin x \cos x = \sin 2x$. (M1)
 - Set $\sin 2x = 0 \Rightarrow 2x = \pi \Rightarrow x = \frac{\pi}{2}$. (A1)
 - Substitute $x = \frac{\pi}{2}$ into original: $\frac{\pi}{2} + 5 = 0 + 2y \Rightarrow y = \frac{\pi+10}{4}$. (A1)

(Total for question 47 is 7 marks)

48

- $u = 5 + \ln x, v = 3 + 2 \ln x \Rightarrow u' = \frac{1}{x}, v' = \frac{2}{x}$. (M1)
- $f'(x) = \frac{\frac{1}{x}(3+2 \ln x) - \frac{2}{x}(5+\ln x)}{(3+2 \ln x)^2}$. (M1)
- $f'(x) = \frac{1}{x} \frac{3+2 \ln x - 10 - 2 \ln x}{(3+2 \ln x)^2} = \frac{-7}{x(3+2 \ln x)^2}$.
- Since $x > 0$ and $(3 + 2 \ln x)^2 > 0$, $f'(x) < 0$ for all x . (A1)

(Total for question 48 is 3 marks)

49

- (a)
- $y = x + 5x^{-1} + \ln x$. (M1)
 - $\frac{dy}{dx} = 1 - 5x^{-2} + \frac{1}{x}$. (A1)
 - $\frac{dy}{dx} = 1 - \frac{5}{x^2} + \frac{1}{x} = \frac{x^2 - 5 + x}{x^2}$. (M1)
 - $\frac{dy}{dx} = \frac{x^2 + x - 5}{x^2}$. (A1)
- (b)
- $x^2 + x - 5 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1)(-5)}}{2} = \frac{-1 \pm \sqrt{21}}{2}$. (M1)
 - Since $x > 0$, $x = \frac{-1 + \sqrt{21}}{2} \approx 1.79$. (A1)
 - $y = \frac{1.79^2 + 5}{1.79} + \ln 1.79 \approx 5.17$. (A1)

(Total for question 49 is 7 marks)

50

- (a)
- $\frac{dx}{dt} = -3 \cdot 2^{-t} \ln 2$. (M1)
 - $\frac{dy}{dt} = 5 \cdot 2^t \ln 2$. (M1)
 - $\frac{dy}{dx} = \frac{5 \cdot 2^t \ln 2}{-3 \cdot 2^{-t} \ln 2} = -\frac{5 \cdot 2^t}{3 \cdot 2^{-t}} = -\frac{5}{3} \cdot 2^{2t}$. (A1)
- (b)
- $x - 2 = \frac{3}{2^t} \Rightarrow 2^t = \frac{3}{x-2}$. (M1)
 - $y = 5\left(\frac{3}{x-2}\right) - 3 = \frac{15}{x-2} - 3$. (A1)

(Total for question 50 is 5 marks)

51

- (a)
- $V = x^2y = 5000$ (M1)
 - $y = \frac{5000}{x^2}$ (A1)
 - $A = 4xy + 2x^2$ (M1)
 - $A = 4x \left(\frac{5000}{x^2} \right) + 2x^2 = \frac{20000}{x} + 2x^2$ (A1)
- (b)
- $\frac{dA}{dx} = -20000x^{-2} + 4x$ (M1) (A1)
 - Stationary when $\frac{dA}{dx} = 0$ (M1)
 - $4x^3 = 20000 \Rightarrow x^3 = 5000 \Rightarrow x = 17.1$ (A1)
- (c)
- $\frac{d^2A}{dx^2} = 40000x^{-3} + 4$ (M1)
 - When $x = 17.1$, $\frac{d^2A}{dx^2} = 12 > 0$, therefore minimum. (A1)
- (d)
- $A = \frac{20000}{17.1} + 2(17.1)^2$ (M1)
 - $A = 1754 \text{ cm}^2$ (A1)

(Total for question 51 is 12 marks)

52

- $2x + 2 = e^y \frac{dy}{dx}$ (M1) (A1)
- Stationary point where $\frac{dy}{dx} = 0 \Rightarrow 2x + 2 = 0$ (M1)
- $x = -1$ (A1)
- When $x = -1$, $1 - 2 = e^y - 3 \Rightarrow 2 = e^y$ (M1)
- $y = \ln 2$ (A1)
- $(-1, \ln 2)$ (B1)

(Total for question 52 is 7 marks)

53

- (a) • $x > \frac{3}{2}$ (B2)
- (b) • $u = x, v = (2x - 3)^{\frac{1}{2}}$
 • $\frac{du}{dx} = 1, \frac{dv}{dx} = (2x - 3)^{-\frac{1}{2}}$ (M1)
 • $f'(x) = \frac{(2x-3)^{\frac{1}{2}} - x(2x-3)^{-\frac{1}{2}}}{(2x-3)}$ (M1)
 • Multiply numerator and denominator by $(2x - 3)^{\frac{1}{2}}$ to get $\frac{2x-3-x}{(2x-3)^{\frac{3}{2}}} = \frac{x-3}{(2x-3)^{\frac{3}{2}}}$ (A1)
- (c) • $u = x - 3, v = (2x - 3)^{\frac{3}{2}}$
 • $\frac{du}{dx} = 1, \frac{dv}{dx} = 3(2x - 3)^{\frac{1}{2}}$ (M1)
 • $f''(x) = \frac{(2x-3)^{\frac{3}{2}} - 3(x-3)(2x-3)^{\frac{1}{2}}}{(2x-3)^3}$ (M1)
 • $f''(x) = 0 \Rightarrow (2x - 3)^{\frac{1}{2}}(2x - 3 - 3x + 9) = 0$ (M1)
 • $(2x - 3)^{\frac{1}{2}}(6 - x) = 0$ (A1)
 • $x = \frac{3}{2}$ (not in domain) or $x = 6$ (A1)
 • Check change of sign in $f''(x)$ around $x = 6$ (M1)
 • Conclude exactly one point of inflection at $x = 6$ (A1)

(Total for question 53 is 12 marks)

54

- $\frac{dy}{dx} = ke^{kx}$ (B2)

(Total for question 54 is 2 marks)

55

- $\frac{dV}{dt} = 2$ (B1)
- $\frac{dV}{dr} = 4\pi r^2$ (B1)
- $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1}{4\pi r^2} \times 2 = \frac{1}{2\pi r^2}$ (M1)
- When $r = 2$, $\frac{dr}{dt} = \frac{1}{8\pi}$, so $a = \frac{1}{8}$ (A1)

(Total for question 55 is 4 marks)

56

- $\ln y = x \ln 3$ (M1)
- $\frac{1}{y} \frac{dy}{dx} = \ln 3$ (M1)
- $\frac{dy}{dx} = y \ln 3 = 3^x \ln 3$ (A1)
- (Note: 2 marks total for question) (B2)

(Total for question 56 is 2 marks)

57

- $u = e^{2x+1}, v = x^2 \Rightarrow \frac{du}{dx} = 2e^{2x+1}, \frac{dv}{dx} = 2x$ (M1)
- $\frac{dy}{dx} = \frac{2x^2 e^{2x+1} - 2x e^{2x+1}}{x^4}$ (M1) (A1)
- Stationary point where $\frac{dy}{dx} = 0 \Rightarrow e^{2x+1}(2x^2 - 2x) = 0$ (M1)
- $2x(x - 1) = 0 \Rightarrow x = 0$ or $x = 1$ (A1)
- $x = 0$ is undefined for the curve (B1)
- Therefore exactly one stationary point at $x = 1$ (A1)

(Total for question 57 is 7 marks)

58

- (a)
- If $y = 0, x(0) + x^2(0) = 0 \neq 10$ (B1)
 - If $x = 0, (0)y^2 + (0)y = 0 \neq 10$ (B1)
- (b)
- Differentiate $xy^2: y^2 + 2xy \frac{dy}{dx}$ (M1)
 - Differentiate $x^2y: 2xy + x^2 \frac{dy}{dx}$ (M1)
 - $y^2 + 2xy \frac{dy}{dx} + x^2 \frac{dy}{dx} + 2xy = 0$ (A1)
 - $\frac{dy}{dx}(x^2 + 2xy) = -y^2 - 2xy$ (M1)
 - $\frac{dy}{dx} = -\frac{y(y+2x)}{x(x+2y)}$ (A1)
- (c)
- $\frac{dy}{dx} = 0 \Rightarrow -y(y + 2x) = 0$ (M1)
 - $y = 0$ (no intersection) or $y = -2x$ (A1)
 - Substitute $y = -2x$ into $xy^2 + x^2y = 10: x(-2x)^2 + x^2(-2x) = 10$ (M1)
 - $4x^3 - 2x^3 = 10 \Rightarrow 2x^3 = 10 \Rightarrow x = \sqrt[3]{5}$ (A1)
 - $y = -2\sqrt[3]{5}$ (A1)

(Total for question 58 is 12 marks)

59

- When $y = 1, x^2 + x - 6 = 0 \Rightarrow (x + 3)(x - 2) = 0 \Rightarrow x = 2$ (as $x > 0$) (M1) (A1)
- Implicit differentiation: $x^2 \frac{dy}{dx} + 2xy + x \frac{dy}{dx} + y = 0$ (M1) (A1)
- At $(2, 1): 4 \frac{dy}{dx} + 4 + 2 \frac{dy}{dx} + 1 = 0 \Rightarrow 6 \frac{dy}{dx} = -5 \Rightarrow \frac{dy}{dx} = -\frac{5}{6}$ (M1) (A1)
- $y - 1 = -\frac{5}{6}(x - 2) \Rightarrow y = -\frac{5}{6}x + \frac{8}{3}$ (A1)

(Total for question 59 is 7 marks)

60

- (a) • $5 \sin \frac{\pi}{3} + \sqrt{3} \cos \frac{\pi}{3} = \sqrt{3}A$ (M1)
 • $\frac{5\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}A \Rightarrow 3\sqrt{3} = A\sqrt{3} \Rightarrow A = 3$ (A1)
- (b) • $5 \cos y \frac{dy}{dx} + \cos y - x \sin y \frac{dy}{dx} = 3$ (M1) (M1) (A1)
 • $\frac{dy}{dx}(5 \cos y - x \sin y) = 3 - \cos y$ (M1)
 • $\frac{dy}{dx} = \frac{3 - \cos y}{5 \cos y - x \sin y}$ (A1)
- (c) • At $(\sqrt{3}, \frac{\pi}{3})$, $\frac{dy}{dx} = \frac{3 - \frac{1}{2}}{5(\frac{1}{2}) - (\sqrt{3})(\frac{\sqrt{3}}{2})}$ (M1)
 • $\frac{dy}{dx} = \frac{2.5}{2.5 - 1.5} = \frac{2.5}{1} = 2.5$ (A1)
 • $y - \frac{\pi}{3} = \frac{5}{2}(x - \sqrt{3})$ (M1) (A1)

(Total for question 60 is 11 marks)

61

- (a) $\frac{dx}{dt} = 3 - t^{-2}$ and $\frac{dy}{dt} = 3 + t^{-2}$ (M1) $\frac{dy}{dx} = \frac{3+t^{-2}}{3-t^{-2}}$ (M1) Multiply numerator and denominator by t^2 : $\frac{3t^2+1}{3t^2-1}$ (A1A1)
- (b) Stationary points occur where $\frac{dy}{dx} = 0$, so $3t^2 + 1 = 0$ (M1) $t^2 = -\frac{1}{3}$. Since t^2 is always positive for real t , there are no solutions. (A1)
- (c) $x + y = 6t \Rightarrow t = \frac{1}{6}(x + y)$ (M1) $x - y = \frac{2}{t} \Rightarrow t = \frac{2}{x-y}$ (M1) $\frac{x+y}{6} = \frac{2}{x-y} \Rightarrow (x+y)(x-y) = 12$ (M1) $x^2 - y^2 = 12$ or $x^2 = y^2 + 12$ (A1)

(Total for question 61 is 10 marks)

62

- (a) Use product rule: $u = x, v = \ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x$ (M1) Set $\frac{dy}{dx} = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$ (M1) $y = e^{-1} \ln(e^{-1}) = -e^{-1}$ (A1) Coordinates: $(e^{-1}, -e^{-1})$ (A1)
- (b) $\frac{d^2y}{dx^2} = \frac{1}{x}$ (M1) At $x = e^{-1}$, $\frac{d^2y}{dx^2} = e$ (A1) Since $e > 0$, the turning point is a minimum. (A1)

(Total for question 62 is 7 marks)

63

- (a) Differentiate implicitly: $3x^2 - (x^2 \frac{dy}{dx} + 2xy) - 2 \frac{dy}{dx} = 0$ (M1A1) $3x^2 - 2xy = \frac{dy}{dx}(x^2 + 2)$ (M1) $\frac{dy}{dx} = \frac{3x^2 - 2xy}{x^2 + 2}$ (A1)
- (b) At $(1, 2)$, $\frac{dy}{dx} = \frac{3(1)^2 - 2(1)(2)}{1^2 + 2} = \frac{3-4}{3} = -\frac{1}{3}$ (M1) Gradient of normal = 3 (M1) $y - 2 = 3(x - 1)$ (M1) $y = 3x - 1$ (A1)

(Total for question 63 is 8 marks)

64

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 + 5(x+h) - (x^3 + 5x)}{h} \quad (\text{M1}) = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 5x + 5h - x^3 - 5x}{h} \quad (\text{M1}) = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 5h}{h} \quad (\text{A1}) = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2 + 5) \quad (\text{M1})$$

As $h \rightarrow 0$, $3xh \rightarrow 0$ and $h^2 \rightarrow 0$, so $f'(x) = 3x^2 + 5$ (A1A1)

(Total for question 64 is 6 marks)

65

- (a) $\ln y = x^2 \ln a$ (M1) $\frac{1}{y} \frac{dy}{dx} = 2x \ln a$ (M1) $\frac{dy}{dx} = 2xy \ln a = 2xa^{x^2} \ln a$ (A1)
- (b) At $(1, a)$, gradient $m = 2(1)a^{1^2} \ln a = 2a \ln a$ (M1) Gradient between $(1, a)$ and $(\frac{1}{2}, 0)$ is $\frac{a-0}{1-0.5} = 2a$ (M1) $2a \ln a = 2a \implies \ln a = 1$ (M1) $a = e$ (A1)
- (c) Using $a = e$, $\frac{dy}{dx} = 2xe^{x^2}$ (M1) $\frac{d^2y}{dx^2} = 2e^{x^2} + 2x(2xe^{x^2}) = e^{x^2}(2 + 4x^2)$ (M1A1) $e^{x^2} > 0$ and $(2 + 4x^2) > 0$ for all x (M1) Since $\frac{d^2y}{dx^2} > 0$, the curve is convex. (A1)

(Total for question 65 is 12 marks)

66

- (a) Use chain rule: $5(2x + 5)^4 \times 2 = 10(2x + 5)^4$ (M1A1)
- (b) Use product rule: $u = 3x^2, v = \ln x \implies \frac{du}{dx} = 6x, \frac{dv}{dx} = \frac{1}{x}$ (M1) $6x \ln x + 3x^2(\frac{1}{x}) = 6x \ln x + 3x$ (A1A1)

(Total for question 66 is 5 marks)

67

- (a) $S = 2\pi r^2 + 2\pi rh = 800$ (M1) $400 = \pi r^2 + \pi rh \implies h = \frac{400 - \pi r^2}{\pi r}$ (M1) $V = \pi r^2 h = \pi r^2(\frac{400 - \pi r^2}{\pi r})$ (M1) $V = r(400 - \pi r^2) = 400r - \pi r^3$ (A1)
- (b) $\frac{dV}{dr} = 400 - 3\pi r^2$ (M1) Set $\frac{dV}{dr} = 0 \implies 3\pi r^2 = 400 \implies r = \sqrt{\frac{400}{3\pi}} \approx 6.51$ (M1A1) $V = 400(6.51) - \pi(6.51)^3 \approx 1737$ (M1A1)
- (c) $\frac{d^2V}{dr^2} = -6\pi r$ (M1) Since $r > 0$, $\frac{d^2V}{dr^2}$ is negative, therefore it is a maximum. (A1)

(Total for question 67 is 11 marks)

68

- $\frac{dy}{dx} = 3x^2 + 2kx$ (M1) $\frac{d^2y}{dx^2} = 6x + 2k$ (M1) At point of inflection, $\frac{d^2y}{dx^2} = 0$ (M1) $6(2) + 2k = 0 \implies 12 + 2k = 0 \implies k = -6$ (A1A1)

(Total for question 68 is 5 marks)

69

- (a) $2x - (2y + 2x \frac{dy}{dx}) + 8y \frac{dy}{dx} = 0$ (M1A1) $2x - 2y = 2x \frac{dy}{dx} - 8y \frac{dy}{dx}$ (M1) $x - y = \frac{dy}{dx}(x - 4y) \implies \frac{dy}{dx} = \frac{x-y}{x-4y}$ (A1)
- (b) At Q , $\frac{dy}{dx} = 0 \implies x = y$. Sub into curve: $x^2 - 2x^2 + 4x^2 = 12 \implies 3x^2 = 12 \implies x = \pm 2$. From diagram Q is $(-2, -2)$. (M1A1) At P , $\frac{dy}{dx}$ is undefined $\implies x = 4y$. Sub into curve: $(4y)^2 - 2(4y)y + 4y^2 = 12 \implies 12y^2 = 12 \implies y = \pm 1$. From diagram P is $(4, 1)$. (M1A1) $PQ = \sqrt{(4 - (-2))^2 + (1 - (-2))^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$. (M1A1) $k = 3$ (A1)

(Total for question 69 is 10 marks)

70

(a) $2 \cos(2x + 5)$ (M1A1)

(b) Use quotient rule: $u = 3x^2, v = 2x - 1 \implies \frac{du}{dx} = 6x, \frac{dv}{dx} = 2$ (M1) $\frac{6x(2x-1) - 2(3x^2)}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$ (A1A1)

(c) $2x \sec^2(x^2)$ (M1A1)

(Total for question 70 is 7 marks)

71

• Find derivatives: $\frac{dx}{dt} = -2 \sin t$ and $\frac{dy}{dt} = \cos t$ (M1)

• Calculate gradient $\frac{dy}{dx} = \frac{\cos t}{-2 \sin t}$ (M1)

• At $t = \frac{\pi}{3}, \frac{dy}{dx} = -\frac{\sqrt{3}}{6}$ (A1)

• Find coordinates at $t = \frac{\pi}{3}: x = 1, y = \frac{\sqrt{3}}{2}$ (B1)

• Form equation of tangent: $y - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{6}(x - 1)$ (M1)

• Find y -intercept: $y = \frac{2\sqrt{3}}{3}$ (A1)

(Total for question 71 is 6 marks)

72

(a) • $\frac{dx}{dt} = 8t$ (B1)

• $y = t^3 - 8t \implies \frac{dy}{dt} = 3t^2 - 8$ (B1)

• $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3t^2 - 8}{8t}$ (B1)

(b) • Rearrange line to $y = \frac{1}{4}x + \frac{2}{3}$ to find gradient $m = \frac{1}{4}$ (M1)

• Set $\frac{3t^2 - 8}{8t} = \frac{1}{4} \implies 3t^2 - 2t - 8 = 0$ (M1)

• Solve for t : $(3t + 4)(t - 2) = 0$, so $t = 2$ (since $t > 0$) (A1)

• Substitute $t = 2$ into x and y : $(16, -8)$ (A1)

(c) • Rearrange $x = 4t^2$ for t : $t = \sqrt{\frac{x}{4}} = \frac{x^{1/2}}{2}$ (M1)

• Substitute into $y = t^3 - 8t$: $y = (\frac{x^{1/2}}{2})^3 - 8(\frac{x^{1/2}}{2})$ (M1)

• Simplify: $y = \frac{x^{3/2}}{8} - 4x^{1/2}$ (A1)

(Total for question 72 is 9 marks)

73

- (a)
- Rewrite $y = (1 + x^2)^{-1}$ and find $\frac{dy}{dx} = -2x(1 + x^2)^{-2}$ (M1)
 - Use product or quotient rule for second derivative (M1)
 - $\frac{d^2y}{dx^2} = -2(1 + x^2)^{-2} + 8x^2(1 + x^2)^{-3}$ (A1)
 - Simplify to $\frac{8x^2}{(1+x^2)^3} - \frac{2}{(1+x^2)^2}$ or $\frac{6x^2-2}{(1+x^2)^3}$ (A2)
- (b)
- Set $\frac{d^2y}{dx^2} < 0$ (M1)
 - $6x^2 - 2 < 0 \implies 3x^2 - 1 < 0$ (M1)
 - Solve inequality: $-\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}$ (A1)

(Total for question 73 is 7 marks)
