1

(a)
- $\sin x + C$ **B1**

(b)
- $-\frac{1}{3}\operatorname{cosec} 3x + C$ **M1 A1**

(c)
- $\frac{1}{2}\tan 4x + C$ **M1 A1**

(Total for question 1 is 5 marks)

2

(a)
- $\tan^2\theta + 1 = \sec^2\theta$
- $\tan^2\theta = \sec^2\theta - 1$ **B1**

(b)
- $\int \sec^2\theta - 1\, d\theta$
- $\tan\theta - \theta + C$ **M1 A1 A1**

(Total for question 2 is 4 marks)

3

(a)
- $\frac{\sin x}{\cos x}\frac{1}{\cos x} = \tan x \sec x$
- $\int \tan x \sec x\, dx = \sec x + C$ **M1 M1 A1**

(b)
- $-2\cot x - \cos x + C$ **M1 A1 A1**

(Total for question 3 is 6 marks)

4

(a)
- $\int \sec^2(3x+1)dx$
- $\frac{1}{3}\tan(3x+1) + C$ **M1 A1 A1**

(b)
- $\int \sec^2 x + 2\sec x \tan x + \tan^2 x dx$
- Use $1 + \tan^2 x = \sec^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$
- $\int 2\sec^2 x + 2\sec x \tan x - 1 dx$
- $2\tan x + 2\sec x - x + C$ **M1 M1 A1 A1**

(Total for question 4 is 7 marks)

5

- Use $\sin 4x = 2\sin 2x \cos 2x \Rightarrow \frac{1}{2}\sin 4x = \sin 2x \cos 2x$
- $\int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 4x dx = [-\frac{1}{8}\cos 4x]_0^{\frac{\pi}{4}}$
- $-\frac{1}{8}\cos(\pi) - (-\frac{1}{8}\cos(0)) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ **M1 M1 A1 A1 A1**

(Total for question 5 is 5 marks)

6

(a)

- $\cos 2x = 1 - 2\sin^2 x$
- $2\sin^2 x = 1 - \cos 2x$
- $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ **M1 A1**

(b)

- $\int \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$
- $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ **M1 A1 A1**

(Total for question 6 is 5 marks)

7

- $\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$
- $\int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos 2x \, dx = [\frac{1}{2}x + \frac{1}{4}\sin 2x]_0^{\frac{\pi}{2}}$
- $(\frac{1}{2}(\frac{\pi}{2}) + \frac{1}{4}\sin \pi) - (0) = \frac{1}{4}\pi$ **M1 M1 A1 A1 A1 A1**

(Total for question 7 is 6 marks)

8

- $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$
- $\int \frac{2x}{u}\frac{du}{2x} = \int \frac{1}{u} du$
- $\ln u + C = \ln(x^2 + 4) + C$ **M1 M1 A1 A1 A1**

(Total for question 8 is 5 marks)

9

- $\frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$
- $\int u^3 \cos x \frac{du}{\cos x} = \int u^3 du$
- $\frac{1}{4}u^4 + C = \frac{1}{4}\sin^4 x + C$ **M1 M1 A1 A1 A1**

(Total for question 9 is 5 marks)

10

- $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$
- $\int 2x \cdot u^2 \frac{du}{2x} = \int u^2 du$
- $\frac{1}{3}u^3 + C = \frac{1}{3}(x^2 + 2)^3 + C$ **M1 M1 A1 A1 A1**

(Total for question 10 is 5 marks)

11

- Differentiates substitution: $\frac{du}{dx} = e^x \implies dx = \frac{du}{e^x}$ **M1**
- Substitutes for e^x and dx: $\int \frac{e^{3x}}{u}\frac{du}{e^x} = \int \frac{e^{2x}}{u}du$ **M1**
- Expresses e^{2x} in terms of u: $e^x = u - 1 \implies e^{2x} = (u-1)^2$ **M1**
- Expands and simplifies integrand: $\int \frac{u^2-2u+1}{u}du = \int (u - 2 + \frac{1}{u})du$ **M1**
- Integrates: $\frac{1}{2}u^2 - 2u + \ln u + C$ **A1**
- Reverses substitution: $\frac{1}{2}(1+e^x)^2 - 2(1+e^x) + \ln(1+e^x) + C$ **A1**

(Total for question 11 is 7 marks)

12

- Differentiates substitution: $\frac{du}{dx} = 3x^2 \implies dx = \frac{du}{3x^2}$ **M1**
- Changes limits: $x = 3 \implies u = 23$; $x = 2 \implies u = 4$ **B1**
- Substitutes into integral: $\int_4^{23} 2x^2u^2\frac{du}{3x^2} = \int_4^{23} \frac{2}{3}u^2du$ **M1**
- Integrates: $[\frac{2}{9}u^3]_4^{23}$ **A1**
- Evaluates: $\frac{2}{9}(23^3 - 4^3)$ **M1**
- Final answer: 2689.6 **A1**

(Total for question 12 is 6 marks)

13

- Differentiates substitution: $\frac{dx}{du} = \cos u \implies dx = \cos u du$ **M1**
- Changes limits: $x = 0.5 \implies u = \frac{\pi}{6}$; $x = 0 \implies u = 0$ **B1**
- Substitutes: $\int_0^{\pi/6} \frac{1}{\sqrt{1-\sin^2 u}}\cos u du$ **M1**
- Uses identity $\sqrt{1-\sin^2 u} = \cos u$: $\int_0^{\pi/6} \frac{\cos u}{\cos u}du = \int_0^{\pi/6} 1du$ **M1**
- Integrates: $[u]_0^{\pi/6}$ **A1**
- Final answer: $\frac{\pi}{6}$ **A1**

(Total for question 13 is 7 marks)

14

- Differentiates substitution: $\frac{du}{dx} = -\sin x \implies dx = -\frac{du}{\sin x}$ **M1**
- Changes limits: $x = \frac{\pi}{2} \implies u = 1$; $x = 0 \implies u = 2$ **B1**
- Substitutes: $\int_2^1 \frac{\sin x}{u}(-\frac{du}{\sin x}) = \int_2^1 -\frac{1}{u}du$ **M1**
- Reverses limits to remove negative: $\int_1^2 \frac{1}{u}du$ **M1**
- Integrates: $[\ln u]_1^2$ **A1**
- Evaluates: $\ln 2 - \ln 1$ **M1**
- Final answer: $\ln 2$ **A1**

(Total for question 14 is 7 marks)

15

(a)

- Sets $y = 0$: $3\sin 2t = 0 \implies 2t = 0, \pi$ **M1**
- Finds t values: $t = 0, \frac{\pi}{2}$ **A1**
- Finds x values: $x = 5\cos 0 = 5$ and $x = 5\cos\frac{\pi}{2} = 0$ **A1**
- Points are $(0, 0)$ and $(5, 0)$.

(b)

- Uses Area $= \int y\frac{dx}{dt}dt$ **M1**
- Differentiates x: $\frac{dx}{dt} = -5\sin t$ **B1**
- Sets up integral: $\int_{\pi/2}^{0}(3\sin 2t)(-5\sin t)dt$ **M1**
- Uses identity $\sin 2t = 2\sin t\cos t$: $\int_0^{\pi/2} 30\sin^2 t\cos t dt$ **M1**
- Integrates: $[10\sin^3 t]_0^{\pi/2}$ **M1**
- Evaluates: $10(1)^3 - 10(0)^3$ **A1**
- Final answer: 10 **A1**

(Total for question 15 is 10 marks)

16

(a)

- Differentiates x: $\frac{dx}{dt} = \frac{1}{t+2}$ **B1**
- Finds t limits: $\ln 4 = \ln(t+2) \implies t = 2$; $\ln 2 = \ln(t+2) \implies t = 0$ **B1**
- Substitutes into Area $= \int y\frac{dx}{dt}dt$: $\int_0^2 \frac{1}{t+1}\cdot\frac{1}{t+2}dt$ **M1**
- Result shown: $\int_0^2 \frac{1}{(t+1)(t+2)}dt$ **A1**

(b)

- Uses partial fractions: $\frac{1}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$ **M1**
- Finds $A = 1, B = -1$ **A1**
- Integrates: $[\ln(t+1) - \ln(t+2)]_0^2$ **M1**
- Substitutes limits: $(\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ **M1**
- Simplifies: $\ln 3 - \ln 4 + \ln 2 = \ln 3 - 2\ln 2 + \ln 2$ **M1**
- Final answer: $\ln 3 - \ln 2$ (or $\ln 1.5$) **A1**

(Total for question 16 is 10 marks)

17

(a)
- Sets $y = 0$: $3\sin t = 0 \implies t = 0$ **M1**
- Finds x: $x = 3\cos(0) = 3$ **A1**
- Coordinates are $(3, 0)$.

(b)
- Differentiates x: $\frac{dx}{dt} = -6\sin 2t$ **B1**
- Finds t limits: $x = 3 \implies t = 0$; $x = 0 \implies t = \frac{\pi}{4}$ **B1**
- Substitutes into Area $= \int y\frac{dx}{dt}dt$: $\int_{\pi/4}^{0}(3\sin t)(-6\sin 2t)dt$ **M1**
- Uses $\sin 2t = 2\sin t\cos t$: $\int_0^{\pi/4} 18\sin t(2\sin t\cos t)dt$ **M1**
- Result shown: $\int_0^{\pi/4} 36\sin^2 t\cos t dt$ **A1**

(c)
- Integrates: $[12\sin^3 t]_0^{\pi/4}$ **M1**
- Evaluates: $12(\sin\frac{\pi}{4})^3 - 12(0)$ **M1**
- Uses $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$: $12(\frac{1}{\sqrt{2}})^3 = 12(\frac{1}{2\sqrt{2}})$ **M1**
- Simplifies: $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ **A1**
- Final answer: $3\sqrt{2}$ **A1**

(Total for question 17 is 12 marks)

18
- Identifies $u = x$ and $\frac{dv}{dx} = \sin x$ **M1**
- Finds $\frac{du}{dx} = 1$ and $v = -\cos x$ **A1**
- Applies formula: $-x\cos x - \int -\cos x dx$ **M1**
- Final answer: $\sin x - x\cos x + C$ **A1**

(Total for question 18 is 4 marks)

19
- Identifies $u = 2x$ and $\frac{dv}{dx} = e^x$ **M1**
- Finds $\frac{du}{dx} = 2$ and $v = e^x$ **A1**
- Applies formula: $2xe^x - \int 2e^x dx$ **M1**
- Final answer: $2xe^x - 2e^x + C$ **A1**

(Total for question 19 is 4 marks)

20
- Identifies $u = x$ and $\frac{dv}{dx} = \sec^2 x$ **M1**
- Finds $\frac{du}{dx} = 1$ and $v = \tan x$ **A1**
- Applies formula: $x\tan x - \int \tan x dx$ **M1**
- Final answer: $x\tan x - \ln|\sec x| + C$ **A1**

(Total for question 20 is 4 marks)

21

- Sets $u = x$ and $\frac{dv}{dx} = e^{3x}$ **M1**
- Differentiates and integrates to find $\frac{du}{dx} = 1$ and $v = \frac{1}{3}e^{3x}$ **A1**
- Applies the integration by parts formula: $\frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx$ **M1**
- Final answer: $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$ **A1**

(Total for question 21 is 4 marks)

22

- Sets $u = 2x$ and $\frac{dv}{dx} = \cos x$ **M1**
- Finds $\frac{du}{dx} = 2$ and $v = \sin x$ **A1**
- Applies parts formula: $2x\sin x - \int 2\sin x\,dx = 2x\sin x + 2\cos x$ **M1**
- Substitutes limits: $\left(2\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right) + 2\cos\left(\frac{\pi}{6}\right)\right) - (0 + 2\cos 0)$ **M1**
- Simplifies to $\frac{\pi}{6} + \sqrt{3} - 2$ **A2**

(Total for question 22 is 6 marks)

23

- First application: $u = x^2$, $\frac{dv}{dx} = e^x \Rightarrow x^2e^x - \int 2xe^x\,dx$ **M1** **A1**
- Second application: $u = 2x$, $\frac{dv}{dx} = e^x \Rightarrow 2xe^x - \int 2e^x\,dx$ **M1** **A1**
- Combines results: $x^2e^x - (2xe^x - 2e^x) + C$ **M1**
- Final answer: $x^2e^x - 2xe^x + 2e^x + C$ **A1**

(Total for question 23 is 6 marks)

24

- Sets $u = \ln x$ and $\frac{dv}{dx} = 1$ **M1**
- Finds $\frac{du}{dx} = \frac{1}{x}$ and $v = x$ **A1**
- Applies formula: $x\ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int 1\,dx$ **M1**
- Final answer: $x\ln x - x + C$ **A1**

(Total for question 24 is 4 marks)

25

- Sets $u = \ln x$ and $\frac{dv}{dx} = x^2$ **M1**
- Finds $\frac{du}{dx} = \frac{1}{x}$ and $v = \frac{1}{3}x^3$ **A1**
- Applies formula: $\frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2\,dx = \left[\frac{1}{3}x^3\ln x - \frac{1}{9}x^3\right]_1^2$ **M1**
- Substitutes limits: $\left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$ **M1**
- Final answer: $\frac{8}{3}\ln 2 - \frac{7}{9}$ **A2**

(Total for question 25 is 6 marks)

26

(a)
- Applies reverse chain rule or expands and integrates **M1**
- Correct integration: $\frac{1}{2} \cdot \frac{1}{3}(2x+3)^3$ **A1**
- Final answer: $\frac{1}{6}(2x+3)^3 + C$ **A1**

(b)
- Rewrites as $2(5x-1)^{-3}$ **B1**
- Integrates: $2 \cdot \frac{1}{5} \cdot \frac{(5x-1)^{-2}}{-2}$ **M1**
- Final answer: $-\frac{1}{5}(5x-1)^{-2} + C$ **A1**

(Total for question 26 is 6 marks)

27

(a)
- Rewrites as $(5x-3)^{\frac{1}{2}}$ **B1**
- Integrates: $\frac{1}{5} \cdot \frac{2}{3}(5x-3)^{\frac{3}{2}}$ **M1**
- Final answer: $\frac{2}{15}(5x-3)^{\frac{3}{2}} + C$ **A1**

(b)
- Recognises the form e^{ax+b} **M1**
- Integrates to $\frac{1}{2}e^{2x+3}$ **A1**
- Final answer: $\frac{1}{2}e^{2x+3} + C$ **A1**

(Total for question 27 is 6 marks)

28

(a)
- Recognises the form e^{ax+b} **M1**
- Integrates to $\frac{1}{-1}e^{3-x}$ **A1**
- Final answer: $-e^{3-x} + C$ **A1**

(b)
- Recognises the form $\frac{1}{ax+b}$ integrates to a natural log **M1**
- Integrates to $\frac{1}{2}\ln|2x+1|$ **A1**
- Final answer: $\frac{1}{2}\ln|2x+1| + C$ **A1**

(Total for question 28 is 6 marks)

29
- Integrates to $\frac{2}{3}\ln(3x+5)$ **M1** **A1**
- Substitutes limits: $\frac{2}{3}\ln(3(2)+5) - \frac{2}{3}\ln(3(1)+5)$ **M1**
- Simplifies to $\frac{2}{3}\ln 11 - \frac{2}{3}\ln 8$ **A1**
- Final answer: $\frac{2}{3}\ln(\frac{11}{8})$ **A1**

(Total for question 29 is 5 marks)

30

- Rewrites as $10(2x+1)^{-3}$ **B1**
- Integrates to $[-\frac{5}{2}(2x+1)^{-2}]$ **M1** **A1**
- Substitutes limits: $-\frac{5}{2}(3)^{-2} - (-\frac{5}{2}(1)^{-2})$ **M1**
- Final answer: $\frac{20}{9}$ **A1**

(Total for question 30 is 5 marks)

31

(a)

- Sets up partial fraction form: $\frac{3x}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ **M1**
- Substitutes $x = 2$ to find $B = 2$ and $x = -1$ to find $A = 1$ **M1**
- Final expression: $\frac{1}{x+1} + \frac{2}{x-2}$ **A1**

(b)

- Integrates partial fractions to get $\ln|x+1| + 2\ln|x-2|$ **M1**
- Includes constant of integration $+C$ **A1**

(Total for question 31 is 5 marks)

32

- Partial fractions: $\frac{2x^2+3x+7}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$ **M1**
- Finds $B = 3, C = 4, A = -2$ **A1**
- Integrates to $[-3(x+1)^{-1} + 4\ln(x+3) - 2\ln(x+1)]_0^5$ **M1**
- Substitutes limits: $(-3(6)^{-1} + 4\ln 8 - 2\ln 6) - (-3 + 4\ln 3 - 2\ln 1)$ **M1**
- Simplifies to $\frac{5}{2} + 4\ln 8 - 2\ln 6 - 4\ln 3$ **A1**

(Total for question 32 is 8 marks)

33

(a)

- $\frac{dy}{dt} = 0.02y$ **B2**

(b)

- Separates variables: $\int \frac{1}{y}dy = \int 0.02dt$ **M1**
- Integrates: $\ln y = 0.02t + c$ **A1**
- Exponentiates: $y = e^{0.02t+c}$ **M1**
- Uses $e^c = A$ to show $y = Ae^{0.02t}$ **A1**

(c)

- Sets $A = 2.5$ and $t = 10$ **M1**
- Calculates $y = 2.5e^{0.02(10)} = 3.05$ million **A1**

(Total for question 33 is 8 marks)

34

(a)
- $\frac{dy}{dt} = k(y - 20)$ **B3**

(b)
- Separates variables: $\int \frac{1}{y-20} dy = \int k dt$ **M1**
- Integrates: $\ln(y - 20) = kt + c$ **A1**
- Exponentiates: $y - 20 = e^{kt+c}$ **M1**
- Rearranges to $y = 20 + Ae^{kt}$ **A1**

(c)
- $100 = 20 + Ae^0 \implies A = 80$ **B1**

(d)
- Substitutes $t = 8, y = 60, A = 80$: $60 = 20 + 80e^{8k}$ **M1**
- Simplifies: $40 = 80e^{8k} \implies \frac{1}{2} = e^{8k}$ **M1**
- Takes logs: $8k = \ln(\frac{1}{2}) \implies k = -\frac{1}{8}\ln 2$ **A1**

(Total for question 34 is 11 marks)

35

(a)
- Sets up $\frac{3x-3}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$ **M1**
- Finds $A = 2$ and $B = -1$ **A2**

(b)
- Separates variables: $\int \frac{1}{y} dy = \int \frac{3x-3}{(x+1)(2x-1)} dx$ **M1**
- Uses partial fractions from (a): $\int \frac{1}{y} dy = \int \frac{2}{x+1} - \frac{1}{2x-1} dx$ **M1**
- Integrates: $\ln y = 2\ln(x+1) - \frac{1}{2}\ln(2x-1) + C$ **A3**

(c)
- Substitutes $(5, 6)$ to find C: $\ln 6 = 2\ln 6 - \frac{1}{2}\ln 9 + C$ **M1**
- Finds $C = \ln 3 - \ln 6$ **A1**
- Combines logs: $\ln y = \ln\left(\frac{3(x+1)^2}{6(2x-1)^{\frac{1}{2}}}\right)$ **M1**
- Final form: $y = \frac{(x+1)^2}{2(2x-1)^{\frac{1}{2}}}$ **A1**

(Total for question 35 is 12 marks)

36
- Separates variables: $\int \frac{1}{y} dy = \int x \sin x dx$ **M1**
- Integrates LHS: $\ln y$ **B1**
- Integrates RHS by parts: $u = x, v' = \sin x \implies -x\cos x + \sin x$ **M2**
- Final solution: $\ln y = -x\cos x + \sin x + c$ **A2**

(Total for question 36 is 6 marks)

37

- Separates variables: $\int \frac{1}{y^2}dy = \int \ln x dx$ **M1**
- Integrates LHS: $-y^{-1}$ **B1**
- Integrates RHS by parts: $u = \ln x, v' = 1 \implies x\ln x - x$ **M2**
- Final solution: $-\frac{1}{y} = x\ln x - x + c$ **A2**

(Total for question 37 is 6 marks)

38

- Separates variables: $\int (y+1)^{-2}dy = \int 1dx$ **M1**
- Integrates: $-(y+1)^{-1} = x + c$ **A1**
- Substitutes $(2, 0)$: $-1 = 2 + c \implies c = -3$ **M1**
- Rearranges: $-\frac{1}{y+1} = x - 3$ **M1**
- Final form: $y = -\frac{1}{x-3} - 1$ or $y = \frac{x-2}{3-x}$ **A2**

(Total for question 38 is 6 marks)

39

(a)

- Separates variables: $\int \frac{1}{H}dH = \int \frac{1}{5}\sin(0.2t)dt$ **M1**
- Integrates: $\ln H = -\cos(0.2t) + C$ **A1**
- Substitutes $t = 0, H = 48/e$: $\ln(48/e) = -1 + C \implies C = \ln 48$ **M1**
- Exponentiates: $H = e^{-\cos(0.2t) + \ln 48}$ **M1**
- Simplifies to $H = 48e^{-\cos(0.2t)}$ **A1**

(b)

- Max height when $\cos(0.2t) = -1$ **M1**
- $0.2t = \pi \implies t = 5\pi \approx 16$ minutes **A1**

(Total for question 39 is 7 marks)

40

- Uses integration by parts: $u = 5x, v' = (x+1)^{-1/2}$ **M1**
- Finds $u' = 5, v = 2(x+1)^{1/2}$ **A1**
- Applies formula: $[10x(x+1)^{1/2}]_0^3 - \int_0^3 10(x+1)^{1/2}dx$ **M1**
- Integrates second term: $[10x(x+1)^{1/2} - \frac{20}{3}(x+1)^{3/2}]_0^3$ **A1**
- Substitutes limits: $(60 - 160/3) - (0 - 20/3)$ **M1**
- Simplifies to $40/3$ **A1**

(Total for question 40 is 6 marks)

41

- Separates variables and integrates: $\int \frac{1}{\theta-20}d\theta = \int -k dt$ **M1**
- Obtains $\ln(\theta - 20) = -kt + c$ **A1**
- Uses $t = 0, \theta = 98$ to find $c = \ln 78$ **M1**
- Uses $t = 5, \theta = 59$ to find k: $\ln 39 = -5k + \ln 78 \Rightarrow \ln \frac{1}{2} = -5k \Rightarrow k = \frac{1}{5}\ln 2$ **M1**
- Identifies $B = 20$ from the differential equation/asymptote **B1**
- States final values: $A = 78$, $B = 20$, $k = \frac{1}{5}\ln 2$ **A1**

(Total for question 41 is 6 marks)

42

- Separates variables: $\int \frac{1}{P}dP = \int \frac{1}{5}dt$ **M1**
- Integrates to $\ln P = \frac{1}{5}t + C$ **A1**
- Uses $t = 0, P = 50$ to find $C = \ln 50$ **M1**
- Rearranges to exponential form: $P = e^{\frac{1}{5}t + \ln 50}$ **M1**
- Final answer: $P = 50e^{\frac{1}{5}t}$ **A1**

(Total for question 42 is 5 marks)

43

(a)

- Sets up differential equation: $\frac{dv}{dt} = -\frac{k}{v^2}$ **M1**
- Integrates: $\frac{1}{3}v^3 = -kt + c$ **M1**
- Uses $t = 0, v = 40$ to find $c = \frac{64000}{3}$ **A1**
- Uses $t = 2, v = 30$ to find k: $9000 = -2k + \frac{64000}{3} \Rightarrow k = \frac{18500}{3}$ **M1**
- Final equation: $v^3 = -18500t + 64000$ **A1**

(b)

- Sets $v = 0$ in their equation: $0 = -18500t + 64000$ **M1**
- Solves for t: $t = \frac{64000}{18500} \approx 3.459...$ **A1**
- Converts to hours and minutes: 3 hours 28 mins **A1**

(c)

- Suggests weather conditions (sun/rain) would change the rate of evaporation. **B1**

(Total for question 43 is 9 marks)

44

- Identifies lower limit by solving $x \ln x = 0 \Rightarrow x = 1$ **B1**
- Sets up integral $\int_1^4 x \ln x \, dx$ **M1**
- Uses integration by parts: $u = \ln x, v' = x \Rightarrow u' = 1/x, v = x^2/2$ **M1**
- Obtains $[\frac{1}{2}x^2 \ln x] - \int \frac{1}{2}x \, dx$ **A1**
- Integrates to $[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^4$ **A1**
- Substitutes limits: $(8 \ln 4 - 4) - (0 - 1/4)$ **M1**
- Simplifies to $16 \ln 2 - 15/4$ **A1**

(Total for question 44 is 7 marks)

45

(a)

- Applies chain rule: $5(x^2 - 3)^4 \times 2x$ **M1**
- Final answer: $10x(x^2 - 3)^4$ **A1**

(b)

- Recognises integral is $\frac{1}{5}$ of the derivative found in (a) **M1**
- Final answer: $\frac{1}{5}(x^2 - 3)^5 + C$ **A1**

(Total for question 45 is 4 marks)

46

- Recognises the limit of the sum as the integral $\int_2^5 e^{2x} dx$ **B1**
- Integrates to $[\frac{1}{2}e^{2x}]_2^5$ **M1**
- Substitutes limits: $\frac{1}{2}e^{10} - \frac{1}{2}e^4$ **A1**

(Total for question 46 is 3 marks)

47

- Differentiates substitution: $\frac{du}{dx} = -\frac{1}{2\sqrt{x}} \Rightarrow dx = -2\sqrt{x}du$ **M1**
- Substitutes $\sqrt{x} = 2 - u$: $dx = -2(2 - u)du$ **M1**
- Rewrites integral in terms of u: $\int \frac{-2(2-u)}{u} du$ **A1**
- Splits fraction: $\int (\frac{-4}{u} + 2)du$ **M1**
- Integrates: $-4 \ln u + 2u + c$ **A1**
- Substitutes u back and simplifies constant: $-4 \ln |2 - \sqrt{x}| + 2(2 - \sqrt{x}) + c = -4 \ln |2 - \sqrt{x}| - 2\sqrt{x} + k$ **A1**

(Total for question 47 is 6 marks)

48

(a)

- Differentiates: $dx = 2u\,du$ **B1**
- Changes limits: $x = 10 \Rightarrow u = 3, x = 2 \Rightarrow u = 1$ **B1**
- Substitutes $x - 1 = u^2$: $\int_1^3 \frac{2}{u^2(2+u)} \cdot 2u\,du$ **M1**
- Simplifies to $\int_1^3 \frac{4}{u(u+2)}\,du$ with $p = 3, q = 1$ **A1**

(b)

- Partial fractions: $\frac{4}{u(u+2)} = \frac{2}{u} - \frac{2}{u+2}$ **M1**
- Integrates: $[2\ln u - 2\ln(u+2)]_1^3$ **A1**
- Substitutes limits: $(2\ln 3 - 2\ln 5) - (2\ln 1 - 2\ln 3)$ **M1**
- Combines logs: $4\ln 3 - 2\ln 5$ **M1**
- Uses power law: $\ln 81 - \ln 25$ **M1**
- Final answer: $\ln(\frac{81}{25})$ so $a = \frac{81}{25}$ **A1**

(Total for question 48 is 10 marks)

49

- Finds x-intercept: $15 - x = 0 \Rightarrow x = 15$ **B1**
- Sets up integral $\int_3^{15} \frac{15-x}{(2x-3)(x+1)}dx$ **M1**
- Partial fractions: $\frac{15-x}{(2x-3)(x+1)} = \frac{27/5}{2x-3} - \frac{16/5}{x+1}$ **M1**
- Integrates: $[\frac{27}{10}\ln(2x-3) - \frac{16}{5}\ln(x+1)]_3^{15}$ **A1**
- Substitutes limits: $(\frac{27}{10}\ln 27 - \frac{16}{5}\ln 16) - (\frac{27}{10}\ln 3 - \frac{16}{5}\ln 4)$ **M1**
- Uses log laws: $\ln 27 = 3\ln 3$, $\ln 16 = 4\ln 2$, $\ln 4 = 2\ln 2$ **M1**
- Simplifies to $\frac{54}{10}\ln 3 - \frac{32}{5}\ln 2$ **A1**
- Final constants: $a = -32/5, b = 5.4$ **A1**

(Total for question 49 is 8 marks)

50

(a)

- Calculates $x = 2$: $\sqrt{2^2+1} = \sqrt{5} \approx 2.236$ **B1**
- Calculates $x = 2.5$: $\sqrt{2^{2.5}+1} \approx 2.580$ **B1**
- Calculates $x = 3$: $\sqrt{2^3+1} = 3$ **B1** (Note: 2 marks total for completing the table)

(b)

- Uses $h = 0.5$ **B1**
- Applies formula: $0.5 \times \frac{0.5}{2}[1.414 + 3 + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)]$ **M1**
- Correct substitution of values **M1**
- Final answer: 6.133 **A1**

(c)

- States overestimate **B1**
- Reason: The curve is convex (or the top of the trapeziums lie above the curve) **B1**

(Total for question 50 is 8 marks)

51

(a)
- States $\frac{dV}{dt} = c$ and $V = \frac{4}{3}\pi r^3$ **B1**
- Differentiates V to get $\frac{dV}{dr} = 4\pi r^2$ **M1**
- Uses chain rule $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1}{4\pi r^2} \times c$ and defines $k = \frac{c}{4\pi}$ **A1**

(b)
- Separates variables: $\int r^2 dr = \int k dt$ **M1**
- Integrates to get $\frac{1}{3}r^3 = kt + c$ **A1**
- Uses $t = 0, r = 0$ to find $c = 0$ **B1**
- Uses $t = 5, r = 4$ to find $k = \frac{64}{15}$ **M1**
- Final equation $r^3 = \frac{64}{5}t$ or equivalent **A1**

(c)
- The balloon cannot keep inflating; it will eventually pop. **B1**

(Total for question 51 is 9 marks)

52

(a)
- Applies trapezium rule: $0.5 \times \frac{0.5}{2}[1.414 + 1.155 + 2(1.291 + 1.225 + 1.183)]$ **M1**
- Correct calculation: $0.5 \times (0.707 + 1.291 + 1.225 + 1.183 + 0.5775)$ **M1**
- Final answer 2.49 **A1**

(b)
- Recognises $\sqrt{\frac{8+4x}{1+x}} = 2\sqrt{\frac{2+x}{1+x}}$ **M1**
- $2 \times 2.49 = 4.98$ **A1**

(Total for question 52 is 4 marks)

53

(a)
- Performs algebraic division or equates coefficients **M1**
- Obtains $2x - 3$ as quotient **A1**
- Obtains 4 as remainder **A1**
- $A = 2, B = -3, C = 4$

(b)
- Integrates $2x - 3 + \frac{4}{x+1}$ to get $x^2 - 3x + 4\ln(x+1)$ **M1**
- Substitutes limits 7 and 0 **M1**
- $(49 - 21 + 4\ln 8) - (0)$ **A1**
- Simplifies $4\ln 8 = 12\ln 2$ to get $28 + 12\ln 2$ (Note: markscheme says $28 - 12\ln 2$ but calculation shows $28 + 12\ln 2$) **A1**

(Total for question 53 is 7 marks)

54

- Integrates $f'(x)$ to get $f(x) = 2x^3 + \frac{a}{2}x^2 + 7x + c$ **M1**
- Uses y-intercept to find $c = -20$ **B1**
- Uses factor theorem $f(-4) = 0$ **M1**
- $2(-4)^3 + \frac{a}{2}(-4)^2 + 7(-4) - 20 = 0$ **M1**
- $-128 + 8a - 28 - 20 = 0 \Rightarrow 8a = 176 \Rightarrow a = 22$ **A1**
- Final answer $f(x) = 2x^3 + 11x^2 + 7x - 20$ **A1**

(Total for question 54 is 6 marks)

55

(a)

- Finds limits for t: $y = 0 \Rightarrow \sin 2t = 0 \Rightarrow t = 0, \frac{\pi}{2}$ **B1**
- Finds $\frac{dx}{dt} = 2\cos t$ **B1**
- Uses Area $= \int y\frac{dx}{dt}dt = \int 3\sin 2t(2\cos t)dt$ **M1**
- Uses $\sin 2t = 2\sin t\cos t$ to get $\int 12\sin t\cos^2 t dt$ **A1**

(b)

- Integrates by inspection or substitution: $\int 12\sin t\cos^2 t dt = [-4\cos^3 t]$ **M1**
- Substitutes limits: $-4\cos^3(\frac{\pi}{2}) - (-4\cos^3 0)$ **M1**
- $0 - (-4) = 4$ **A1**

(Total for question 55 is 6 marks)

56

- Uses integration by parts: $u = 2x - 2, v' = e^x$ **M1**
- $f(x) = (2x-2)e^x - \int 2e^x dx$ **A1**
- $f(x) = 2xe^x - 2e^x - 2e^x + c = 2xe^x - 4e^x + c$ **A1**
- Substitutes $(1, e)$: $e = 2(1)e^1 - 4e^1 + c$ **M1**
- $e = -2e + c \Rightarrow c = 3e$ **A1**
- $f(x) = 2xe^x - 4e^x + 3e$ **A1**

(Total for question 56 is 6 marks)

57

(a)

- Uses $1 + \cot^2 x = \text{cosec}^2 x$ **B1**
- Uses $\sin 2x = 2\sin x \cos x$ **B1**
- $\frac{2\sin x \cos x}{\text{cosec}^2 x} = 2\sin x \cos x \cdot \sin^2 x = 2\sin^3 x \cos x$ **A1**

(b)

- Uses part (a) to rewrite integrand as $4(2\sin^3 2\theta \cos 2\theta) = 8\sin^3 2\theta \cos 2\theta$ **M1**
- Recognises integral is of form $k\sin^4 2\theta$ **M1**
- Differentiates $\sin^4 2\theta$ to get $8\sin^3 2\theta \cos 2\theta$ **M1**
- Final answer $\sin^4 2\theta + C$ (Note: markscheme says $\frac{1}{4}\sin^4 2\theta$ but calculation shows $\sin^4 2\theta$) **A2**

(Total for question 57 is 8 marks)

58

- Performs algebraic division: $\frac{4x^2}{x+2} = 4x - 8 + \frac{16}{x+2}$ **M1**
- Integrates: $[2x^2 - 8x + 16\ln(x+2)]_0^6$ **A1**
- Substitutes limits: $(2(36) - 8(6) + 16\ln 8) - (0 - 0 + 16\ln 2)$ **M1**
- $(72 - 48 + 16\ln 2^3) - 16\ln 2 = 24 + 48\ln 2 - 16\ln 2$ **M1**
- Final answer $24 + 32\ln 2$ **A1**

(Total for question 58 is 5 marks)

59

(a)

- Uses product rule: $u = e^x, v = \sin x - \cos x$ **M1**
- $\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(\sin x - \cos x)$ **A1**
- Simplifies to $2e^x \sin x$ **A1**

(b)

- Recognises $\int e^x \sin x dx = \frac{1}{2}\int \frac{dy}{dx} dx$ **M1**
- Final answer $\frac{1}{2}e^x(\sin x - \cos x) + C$ **A1**

(Total for question 59 is 5 marks)

60

- Sets $e^{2x} = 12 - e^x$ to find intersection **M1**
- Solves quadratic $e^{2x} + e^x - 12 = 0 \Rightarrow (e^x + 4)(e^x - 3) = 0$ **M1**
- $e^x = 3 \Rightarrow x = \ln 3$ **A1**
- Sets up integral: $\int_0^{\ln 3}(12 - e^x - e^{2x})dx$ **M1**
- Integrates: $[12x - e^x - \frac{1}{2}e^{2x}]_0^{\ln 3}$ **A1**
- Substitutes limits: $(12\ln 3 - 3 - \frac{9}{2}) - (0 - 1 - \frac{1}{2})$ **M1**
- $12\ln 3 - 7.5 + 1.5 = 12\ln 3 - 6$ **A1**

(Total for question 60 is 10 marks)

61

- Let $u = 5x - 1$, then $\frac{du}{dx} = 5$ and $dx = \frac{1}{5}du$ **M1**
- Change limits: when $x = 2, u = 9$; when $x = 1, u = 4$ **B1**
- Substitute $x = \frac{u+1}{5}$ to get $\int_4^9 \frac{u+1}{5} \cdot u^{\frac{1}{2}} \cdot \frac{1}{5}du$ **M1**
- Simplify to $\frac{1}{25}\int_4^9 (u^{\frac{3}{2}} + u^{\frac{1}{2}})du$ **M1**
- Integrate to get $\frac{1}{25}[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}]_4^9$ **M1**
- Evaluate at limits to show $\frac{1456}{375}$ **A1**

(Total for question 61 is 6 marks)

62

(a)

- Calculate ordinates:
  - $x = 1, y = 1.649$
  - $x = 1.5, y = 3.176$
  - $x = 2, y = 5.437$
  - $x = 2.5, y = 8.726$
  - $x = 3, y = 13.445$ **M1**
- Apply formula: $0.5 \times 0.5 \times (\frac{1.649 + 13.445}{2} + 3.176 + 5.437 + 8.726)$ **M1**
- Final answer 12.4 **A1**

(b)

- State that more strips (or smaller width) could be used. **B1**

(Total for question 62 is 4 marks)

63

- Recognize integral is of the form $k\frac{f'(x)}{f(x)}$ **M1**
- Identify $\int \frac{2\sin 2x}{3 - \cos 2x}dx = \ln(3 - \cos 2x)$ **M1**
- Integral is $[2\ln(3 - \cos 2x)]_0^{\frac{\pi}{2}}$ **A1**
- Evaluate at $\frac{\pi}{2}$: $2\ln(3 - \cos \pi) = 2\ln 4$ **M1**
- Evaluate at 0: $2\ln(3 - \cos 0) = 2\ln 2$ **M1**
- $2\ln 4 - 2\ln 2 = \ln 16 - \ln 4 = \ln 4$ **A1**

(Total for question 63 is 6 marks)

64

- Separate variables: $\int 2y\,dy = \int x^{-2}\ln x\,dx$ **M1**
- Integrate LHS: y^2 **B1**
- Integrate RHS using parts: $u = \ln x, v' = x^{-2} \implies u' = \frac{1}{x}, v = -x^{-1}$ **M1**
- RHS $= -\frac{1}{x}\ln x - \int -x^{-2}dx = -\frac{1}{x}\ln x - \frac{1}{x} + C$ **A1**
- Substitute $(1, 1)$: $1^2 = -\frac{1}{1}\ln 1 - \frac{1}{1} + C \implies 1 = -1 + C \implies C = 2$ **M1**
- Final equation: $y^2 = -\frac{1}{x}\ln x - \frac{1}{x} + 2$ **A1**

(Total for question 64 is 7 marks)

65

(a)
- $4x + 5 = A(x + 2) + B$ **M1**
- Let $x = -2 \implies -3 = B$ **A1**
- Let $x = 0 \implies 5 = 2A - 3 \implies A = 4$ **A1**

(b)
- Integrate: $\int \frac{4}{x+2} - 3(x + 2)^{-2} dx = 4\ln(x + 2) + 3(x + 2)^{-1}$ **M1** **A1**
- Substitute limits: $(4\ln 6 + \frac{3}{6}) - (4\ln 3 + \frac{3}{3})$ **M1**
- Simplify: $4\ln 6 - 4\ln 3 + \frac{1}{2} - 1 = 4\ln(\frac{6}{3}) - \frac{1}{2}$ **M1**
- Final answer: $4\ln 2 - \frac{1}{2}$ **A1**

(Total for question 65 is 8 marks)

66
- Identify $h = 1$ (since 6 ordinates over $x \in [0, 5]$) **B1**
- Calculate y values: $\ln 6, \ln 5, \ln 4, \ln 3, \ln 2, 0$ **M1**
- Apply formula: $\frac{1}{2} \times 1 \times (\ln 6 + 0 + 2(\ln 5 + \ln 4 + \ln 3 + \ln 2))$ **M1**
- Calculation: $\frac{1}{2}(1.7917 + 2(1.6094 + 1.3863 + 1.0986 + 0.6931))$ **M1**
- Final answer: 5.68 **A1**

(Total for question 66 is 5 marks)

67
- Use integration by parts: $u = \ln x, v' = 4x^{-3}$ **M1**
- Find $u' = \frac{1}{x}, v = -2x^{-2}$ **A1**
- Apply formula: $-2x^{-2}\ln x - \int -2x^{-2} \cdot \frac{1}{x} dx = -2x^{-2}\ln x + \int 2x^{-3} dx$ **M1**
- Final answer: $-2x^{-2}\ln x - x^{-2} + C$ or $-\frac{2}{x^2}\ln x - \frac{1}{x^2} + C$ **A1**

(Total for question 67 is 4 marks)

68

(a)
- Separate variables: $\int \frac{1}{1000 - \frac{3}{5}P} dP = \int 1 dt$ **M1**
- Integrate LHS: $-\frac{5}{3}\ln(1000 - \frac{3}{5}P) = t + C$ **M1** **A1**
- Use $(0, 0)$: $-\frac{5}{3}\ln(1000) = C$ **M1**
- Rearrange: $\ln(1000 - \frac{3}{5}P) = -\frac{3}{5}t + \ln(1000)$ **M1**
- $1000 - \frac{3}{5}P = 1000e^{-\frac{3}{5}t}$ **M1**
- Final answer: $P = \frac{5000}{3}(1 - e^{-\frac{3}{5}t})$ **A1**

(b)
- Graph starting at $(0, 0)$ **B1**
- Increasing curve with horizontal asymptote at $P = \frac{5000}{3}$ **B1**

(Total for question 68 is 9 marks)

69

- $x = u^2 \implies \frac{dx}{du} = 2u$ **B1**
- Limits: $x = 4 \implies u = 2$; $x = 1 \implies u = 1$ **B1**
- Substitute: $\int_1^2 \frac{2u}{2+u} du$ **M1**
- Use division or partial fractions: $\frac{2u}{u+2} = 2 - \frac{4}{u+2}$ **M1**
- Integrate: $[2u - 4\ln(u+2)]_1^2$ **A1**
- Evaluate: $(4 - 4\ln 4) - (2 - 4\ln 3)$ **M1**
- Final answer: $2 - 4\ln 4 + 4\ln 3$ or $2 + 4\ln(\frac{3}{4})$ **A1**

(Total for question 69 is 7 marks)

70

(a)

- Separate variables: $\int y^{-2} dy = \int (2x+1)^{-2} dx$ **M1**
- Integrate: $-y^{-1} = -\frac{1}{2}(2x+1)^{-1} + C$ **M1** **A1**
- Use $(0, 1)$: $-1 = -\frac{1}{2}(1) + C \implies C = -\frac{1}{2}$ **M1**
- Rearrange: $-\frac{1}{y} = -\frac{1}{2(2x+1)} - \frac{1}{2}$ **M1**
- Final answer: $y = \frac{2x+1}{x+1}$ **A1**

(b)

- State $x \neq -1$ or $x > -0.5$ (based on the original DE context). **B1**

(Total for question 70 is 7 marks)

71

(a)

- Sets up the proportional relationship $\frac{dh}{dt} = -k\sqrt{h}$ **M1**
- Substitutes $h = 36$ and $\frac{dh}{dt} = -0.4$ to find k: $-k\sqrt{36} = -0.4 \Rightarrow -6k = -0.4 \Rightarrow k = \frac{0.4}{6} = \frac{1}{15}$ **A1**

(b)

- Separates variables: $\int h^{-\frac{1}{2}} dh = \int -\frac{1}{15} dt$ **M1**
- Integrates correctly: $2h^{\frac{1}{2}} = -\frac{1}{15}t + C$ **A1**
- Substitutes $t = 10, h = 36$ to find C: $2(6) = -\frac{10}{15} + C \Rightarrow 12 = -\frac{2}{3} + C \Rightarrow C = \frac{38}{3}$ **M1**
- Rearranges to find h: $h = \left(-\frac{1}{30}t + \frac{19}{3}\right)^2$ **A1**

(c)

- Sets $h = 0$ and solves for t: $-\frac{1}{30}t + \frac{19}{3} = 0 \Rightarrow \frac{t}{30} = \frac{19}{3} \Rightarrow t = 190$ seconds **B1**

(Total for question 71 is 7 marks)
