

- 1 (a) $\frac{dy}{dx} = 6x^2 + 10x - 7$ (M1A1)
 (b) when $x = 2$ $\frac{dy}{dx} = 6(2)^2 + 10(2) - 7 = 37$ (B1)

(Total for question 1 is 3 marks)

- 2 (a) $y = 3x + x^{-1} \frac{dy}{dx} = 3 - x^{-2}$ (M1A1)
 (b) $3 - x^{-2} = 0 \Rightarrow 3 = \frac{1}{x^2} \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm \sqrt{\frac{1}{3}}$ (M1A1)

(Total for question 2 is 4 marks)

- 3 $f(x) = 3x^{\frac{3}{2}} + 3x^{-2} - 6x \Rightarrow f'(x) = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} - 6$ (M1A3)

(Total for question 3 is 4 marks)

- 4 (a) $y = 4x^{\frac{1}{2}} + \frac{1}{2}x^{-1} + 10 \Rightarrow \frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2}$ (M1A2)
 (b) $\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} + x^{-3}$ (M1A1)

(Total for question 4 is 5 marks)

- 5 (a) $y = 2x - 5 + 3x^{-1} \Rightarrow \frac{dy}{dx} = 2 - 3x^{-2}$ (M1A2)
 (b) when $x = 3$ $\frac{dy}{dx} = 2 - 3(3)^{-2} = \frac{5}{3}$ (B1)

(Total for question 5 is 4 marks)

- 6 (a) $\frac{dy}{dx} = 3x^2 - 8x - 3$ (M1A1)
 (b) $3x^2 - 8x - 3 > 0 \Rightarrow (3x + 1)(x - 3) > 0 \Rightarrow x < -\frac{1}{3} \text{ or } x > 3$ (M1M1A1)

(Total for question 6 is 5 marks)

- 7 $\frac{dy}{dx} = 6x^2 + 18x - 24 \Rightarrow 6(x^2 + 3x - 4) = 0 \Rightarrow (x + 4)(x - 1) = 0 \Rightarrow x = -4, x = 1$
 $\frac{d^2y}{dx^2} = 12x + 18$ When $x = -4$, $\frac{d^2y}{dx^2} = -30$ (Max) $y = 2(-4)^3 + 9(-4)^2 - 24(-4) + 13 = 125$
 $(-4, 125)$ (M1A1M1A1M1A1)

(Total for question 7 is 6 marks)

- 8 $\frac{dy}{dx} = 6x^2 + 10x - 7$ When $x = 1$, $m = 6(1)^2 + 10(1) - 7 = 9$ When $x = 1$, $y = 2(1)^3 + 5(1)^2 - 7(1) + 10 = 10$ $y - 10 = 9(x - 1) \Rightarrow y = 9x + 1$ (M1A1M1A1M1A1)

(Total for question 8 is 6 marks)

9 $f'(x) = 6x^2 + 2x - 18$ $6x^2 + 2x - 18 = 2 \implies 6x^2 + 2x - 20 = 0 \implies 3x^2 + x - 10 = 0$
 $(3x - 5)(x + 2) = 0 \implies x = \frac{5}{3}, x = -2$ When $x = \frac{5}{3}, y = -\frac{431}{27}$ When $x = -2, y = 26$
 $A(\frac{5}{3}, -\frac{431}{27}), B(-2, 26)$ (M1A1M1A1M1A1)

(Total for question 9 is 6 marks)

10 $y = \frac{4x^2 + 7x - 2}{2x} = 2x + \frac{7}{2} - x^{-1}$ When $x = -2, y = 0 \implies (-2, 0) \frac{dy}{dx} = 2 + x^{-2}$ When $x = -2, \frac{dy}{dx} = 2 + (-2)^{-2} = \frac{9}{4}$ Normal gradient $m = -\frac{4}{9}$ $y - 0 = -\frac{4}{9}(x + 2) \implies 9y = -4x - 8 \implies 4x + 9y + 8 = 0$ (M1A1M1A1M1A1)

(Total for question 10 is 6 marks)

- 11
- (a)
- Differentiate C with respect to v : $\frac{dC}{dv} = -4500v^{-2} + 1$ (M1)
 - Set derivative to zero: $-4500v^{-2} + 1 = 0$ (M1)
 - Rearrange to find v^2 : $1 = \frac{4500}{v^2} \implies v^2 = 4500$ (M1)
 - Solve for v : $v = \sqrt{4500}$ (M1)
 - Final answer $v = 67.1$ mph (A1)
- (b)
- Find second derivative: $\frac{d^2C}{dv^2} = 9000v^{-3}$ (M1)
 - Substitute $v = 67.1$: $\frac{d^2C}{dv^2} = 0.0298$ which is positive, therefore a minimum (A1)
- (c)
- Substitute $v = 67.1$ into the original equation: $C = \frac{4500}{67.1} + 67.1 + 10$ (M1)
 - Final answer $C = £144$ (3sf) (A1)

(Total for question 11 is 9 marks)

12

- (a)
- State surface area formula: $S.A = 2\pi r^2 + 2\pi r h = 500$ (B1)
 - Rearrange for h : $h = \frac{250 - \pi r^2}{\pi r}$ (M1)
 - Substitute h into $V = \pi r^2 h$: $V = \pi r^2 \left(\frac{250 - \pi r^2}{\pi r} \right)$ (M1)
 - Simplify to show $V = 250r - \pi r^3$ (A1)
- (b)
- Differentiate V : $\frac{dV}{dr} = 250 - 3\pi r^2$ (M1)
 - Set derivative to zero: $250 - 3\pi r^2 = 0$ (M1)
 - Solve for r : $r = \sqrt{\frac{250}{3\pi}} \approx 5.15$ (M1)
 - Substitute r back into V : $V = 250(5.15) - \pi(5.15)^3$ (M1)
 - Calculate $V = 858.3...$ (M1)
 - Final answer 858 (A1)
- (c)
- Find second derivative: $\frac{d^2V}{dr^2} = -6\pi r$ (M1)
 - Evaluate at $r = 5.15$: $\frac{d^2V}{dr^2} = -97.1$, which is negative, therefore a maximum (A1)

(Total for question 12 is 12 marks)

13

- Differentiate: $\frac{dy}{dx} = 12x^2 + 30x - 18$ (M1)
- Set to zero and solve: $12x^2 + 30x - 18 = 0 \implies 2x^2 + 5x - 3 = 0 \implies (2x - 1)(x + 3) = 0$ (M1)
- Find x values: $x = \frac{1}{2}, x = -3$ (A1)
- Find y values: $(\frac{1}{2}, \frac{1}{4})$ and $(-3, 86)$ (A1)
- Use second derivative: $\frac{d^2y}{dx^2} = 24x + 30$ (M1)
- Test $x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = 42$ (+ve) $\therefore$ Minimum
- Test $x = -3$: $\frac{d^2y}{dx^2} = -42$ (-ve) $\therefore$ Maximum (A1)

(Total for question 13 is 6 marks)