

- 1
- Take logs of both sides: $x = \log_3 13$ (M1)
 - Evaluate: $x = 2.33$ (A1)

(Total for question 1 is 2 marks)

- 2
- Recognize $32 = 2^5$ or use logs (M1)
 - $x = 5$ (A1)

(Total for question 2 is 2 marks)

- 3
- Use power law: $\log_2 x^2 - \log_2 5 = 1$ (M1)
 - Use subtraction law: $\log_2 \left(\frac{x^2}{5}\right) = 1$ (M1)
 - Remove logs: $2^1 = \frac{x^2}{5} \Rightarrow 2 = \frac{x^2}{5}$ (M1)
 - Solve for x : $x^2 = 10 \Rightarrow x = \sqrt{10}$ (A1)

(Total for question 3 is 4 marks)

- 4
- Use addition law: $\log_3 4x = 2$ (M1)
 - Remove logs: $3^2 = 4x$ (M1)
 - $9 = 4x$ (M1)
 - $x = \frac{9}{4}$ (A1)

(Total for question 4 is 4 marks)

- 5
- Use power law: $\log_a (x+1)^2 - \log_a 4$ (M1)
 - Use subtraction law: $\log_a \left(\frac{(x+1)^2}{4}\right)$ (A1)

(Total for question 5 is 2 marks)

- 6
- Rearrange second equation: $x - 2 = e^{2y-1} = \frac{e^{2y}}{e}$ (M1)
 - Substitute $e^{2y} = x + 1$: $x - 2 = \frac{x+1}{e}$ (M1)
 - Solve for x : $ex - 2e = x + 1 \Rightarrow x(e - 1) = 2e + 1 \Rightarrow x = \frac{2e+1}{e-1}$ (M1)
 - $x \approx 3.75$ (A1)
 - Find y : $e^{2y} = 3.75 + 1 = 4.75$ (M1)
 - $2y = \ln(4.75) \Rightarrow y = \frac{\ln(4.75)}{2}$ (M1)
 - $y \approx 0.78$ (A1)

(Total for question 6 is 7 marks)

- 7
- Remove $\ln$: $2x + 5 = e^1$ (M1)
 - Rearrange: $2x = e - 5$ (M1)
 - $x = \frac{e-5}{2}$ (A1)

(Total for question 7 is 3 marks)

- 8
- (a)
- Use power law: $2 \log_2 x$ (M1)
 - Substitute y : $2y$ (A1)
- (b)
- Use addition law: $\log_2 x + \log_2 2$ (M1)
 - Simplify: $y + 1$ (A1)
- (c)
- Use change of base: $\frac{\log_2 x}{\log_2 8}$ (M1)
 - Simplify: $\frac{y}{3}$ (A1)

(Total for question 8 is 6 marks)

- 9
- Multiply by e^y : $2e^{2y} + 15 = 11e^y$ (M1)
 - Form quadratic: $2e^{2y} - 11e^y + 15 = 0$ (M1)
 - Factorise: $(2e^y - 5)(e^y - 3) = 0$ (M1)
 - Solve for e^y : $e^y = \frac{5}{2}$ or $e^y = 3$ (A1)
 - Solve for y : $y = \ln\left(\frac{5}{2}\right)$ or $y = \ln 3$ (A1)

(Total for question 9 is 5 marks)

10

- (a) • $t = 0 \Rightarrow P = 50e^0 = 50$ (B1)
- (b) • Differentiate: $\frac{dP}{dt} = 5e^{0.1t}$ (M1)
 • Substitute $t = 10$: $5e^{0.1(10)} = 5e$ (A1)
- (c) • Set $P = 300$: $300 = 50e^{0.1t}$ (M1)
 • $6 = e^{0.1t}$ (M1)
 • $\ln 6 = 0.1t \Rightarrow t = 10 \ln 6$ (M1)
 • $t \approx 17.9$, so 18 weeks (A1)

(Total for question 10 is 7 marks)

11

- (a) • $t = 0 \Rightarrow N = 1000$ (B1)
- (b) • Set $N = 500, t = 14.4$: $500 = 1000e^{-14.4k}$ (M1)
 • $0.5 = e^{-14.4k}$ (M1)
 • $\ln(0.5) = -14.4k$ (M1)
 • $k = 0.0481$ (A1)
- (c) • $N = 1000e^{-0.0481(30)} \approx 236$ (B1)

(Total for question 11 is 6 marks)

12

- (a) • Differentiate: $\frac{dT}{dt} = -75ke^{-kt}$ (M1 A1)
- (b) • $70 = 75e^{-5k} + 22$ (M1)
 • $48 = 75e^{-5k} \Rightarrow \frac{16}{25} = e^{-5k}$ (M1)
 • $\ln(0.64) = -5k \Rightarrow k = 0.0893$ (A1)
- (c) • $55 = 75e^{-0.0893t} + 22$ (M1)
 • $33 = 75e^{-0.0893t} \Rightarrow \frac{11}{25} = e^{-0.0893t}$ (M1)
 • $\ln(0.44) = -0.0893t$ (M1)
 • $t = 9.20$, so 9 minutes (A1)

(Total for question 12 is 9 marks)

13

- (a)
- $f(x) > -3$ (B1)
- (b)
- A (where $x = 0$): $y = e^{2(0)+1} - 3 = e - 3$. $A : (0, e - 3)$ (M1 A1)
 - B (where $y = 0$): $0 = e^{2x+1} - 3 \Rightarrow e^{2x+1} = 3$ (M1)
 - $2x + 1 = \ln 3 \Rightarrow x = \frac{\ln 3 - 1}{2}$. $B : (\frac{\ln 3 - 1}{2}, 0)$ (A1)
- (c)
- $f'(x) = 2e^{2x+1}$ (M1)
 - At $A(x = 0)$, $m = 2e^1 = 2e$ (M1)
 - $y - (e - 3) = 2e(x - 0)$ (M1)
 - $y = 2ex + e - 3$ (A1)

(Total for question 13 is 9 marks)

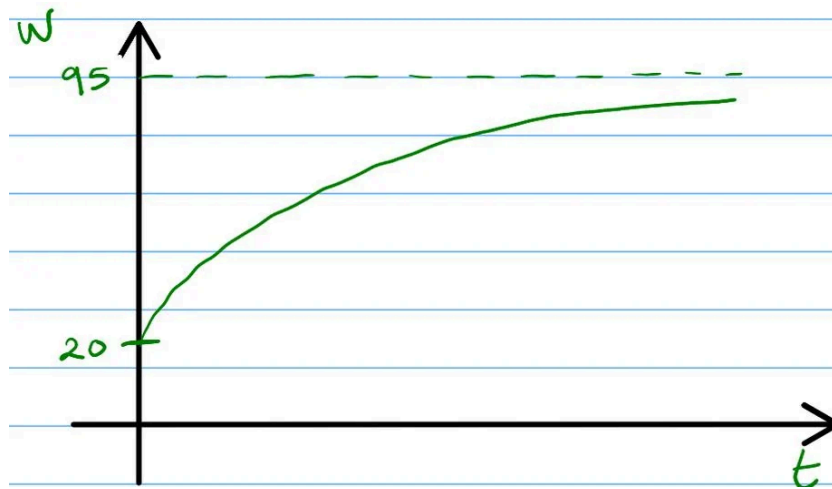
14

- (a)
- $P = 10^{0.5t+1.398}$ (M1)
 - $P = 10^{1.398} \times (10^{0.5})^t$ (M1)
 - $a = 10^{1.398} = 25$ (A1)
 - $b = 10^{0.5} = 3.16$ (A1)
- (b)
- a is the initial population of bacteria (at $t = 0$) (B1)
- (c)
- $P = 25 \times 3.16^{10}$ (M1)
 - $P = 2,500,000$ (A1)

(Total for question 14 is 7 marks)

15

- (a) • $t = 0 \Rightarrow W = 95 - 75e^0 = 20$ (B1)
- (b) • $35 = 95 - 75e^{50k}$ (M1)
 • $75e^{50k} = 60 \Rightarrow e^{50k} = 0.8$ (M1)
 • $50k = \ln 0.8 \Rightarrow k = -0.00446$ (M1)
 • $W = 95 - 75e^{-0.00446t}$ (A1)
- (c) • $W = 95 - 75e^{-0.00446(365)} \approx 80$ (B1)
- (d) • Curve starting at $(0, 20)$ (B1)
 • Increasing concave down shape (B1)
 • Asymptote at $W = 95$ (B1)



(Total for question 15 is 9 marks)

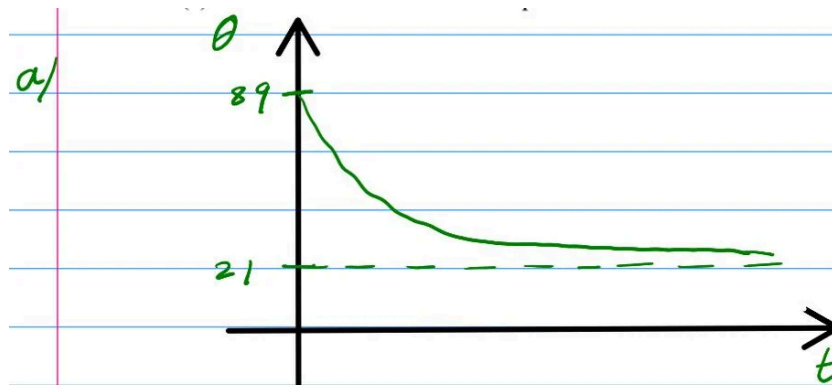
16

- (a) • $a + b = 120$ (1) (B1)
 • $ae^{1.6} + be^{1.2} = 480$ (2) (M1)
 • Substitute $b = 120 - a$ into (2): $ae^{1.6} + (120 - a)e^{1.2} = 480$
 • $a(e^{1.6} - e^{1.2}) = 480 - 120e^{1.2}$
 • $a = 49.96 \approx 50, b = 70$ (A1)
- (b) • $P_{total} = 50e^{0.2(10)} + 70e^{0.15(10)} = 50e^2 + 70e^{1.5} \approx 683$ (B1)
- (c) • $50e^{0.2t} > 70e^{0.15t}$ (M1)
 • $\frac{e^{0.2t}}{e^{0.15t}} > \frac{70}{50} \Rightarrow e^{0.05t} > 1.4$ (M1)
 • $0.05t > \ln 1.4 \Rightarrow t > 6.73$, so day 7 (A1)

(Total for question 16 is 7 marks)

17

- (a)
- Curve starting at $(0, 89)$ (B1)
 - Decreasing concave up shape with asymptote at $\theta = 21$ (B1)



- (b)
- $t = 0 \Rightarrow \theta = 68 + 21 = 89^\circ$ (B1)
- (c)
- $40 = 68e^{-0.15t} + 21$ (M1)
 - $19 = 68e^{-0.15t} \Rightarrow e^{-0.15t} = \frac{19}{68}$ (M1)
 - $-0.15t = \ln\left(\frac{19}{68}\right) \Rightarrow t = 8.50 \text{ mins}$ (A1)

(Total for question 17 is 6 marks)

18

- At $(0, 18)$: $18 = A + B$ (1) (B1)
- At $(6, 7)$: $7 = A + Be^{-1}$ (2) (M1)
- Subtract (2) from (1): $11 = B(1 - e^{-1})$ (M1)
- $B = \frac{11}{1 - e^{-1}} \approx 17.4$, $A = 18 - 17.4 = 0.598$ (A1)
- Asymptote $k = A = 0.598$ (B1)

(Total for question 18 is 5 marks)

19

- LHS: $\log_2 x - \log_2 8 + \log_2 3^2$ (M1)
- $\log_2 x - 3 + \log_2 9$ (M1)
- $\log_2(9x) - 3$ (A1)

(Total for question 19 is 3 marks)

20

- (a)
- $4^x = (2^2)^x = (2^x)^2 = y^2$ (B1)
 - $2^{x+2} = 2^x \cdot 2^2 = 4y$ (B1)
 - Result follows: $y^2 - 4y - 5 = 0$
- (b)
- Factorise: $(y - 5)(y + 1) = 0$ (M1)
 - $y = 5$ or $y = -1$ (A1)
 - $2^x = 5 \Rightarrow x = \log_2 5$ (M1)
 - $2^x = -1$ has no solution as $2^x > 0$ (A1)

(Total for question 20 is 6 marks)

21

- Rearrange line: $2y = 9x + 3 \Rightarrow y = 4.5x + 1.5$, so $m = 4.5$ (M1)
- Differentiate: $\frac{dy}{dx} = 2e^{2x}$ (M1)
- Set equal: $2e^{2x} = 4.5 \Rightarrow e^{2x} = 2.25 = \frac{9}{4}$ (M1)
- Solve for x : $2x = \ln(\frac{9}{4}) \Rightarrow x = \frac{1}{2} \ln(\frac{9}{4}) = \ln(\frac{3}{2})$ (M1 A1)
- Find y : $y = e^{2 \ln(1.5)} = (1.5)^2 = 2.25 = \frac{9}{4}$ (M1)
- $P : (\ln \frac{3}{2}, \frac{9}{4})$ (A1)

(Total for question 21 is 7 marks)

22

- (a)
- a is the population in the year 2001 (when $t = 0$) (B1)
- (b)
- $54000 = a(10^{7b})$ (1)
 - $59000 = a(10^{12b})$ (2) (M1)
 - Divide (2) by (1): $\frac{59}{54} = 10^{5b}$ (M1)
 - $5b = \log_{10}(\frac{59}{54}) \Rightarrow b = 0.00769$ (A1)
 - $a = \frac{54000}{10^{7(0.00769)}} = 47700$ (A1)

(Total for question 22 is 5 marks)

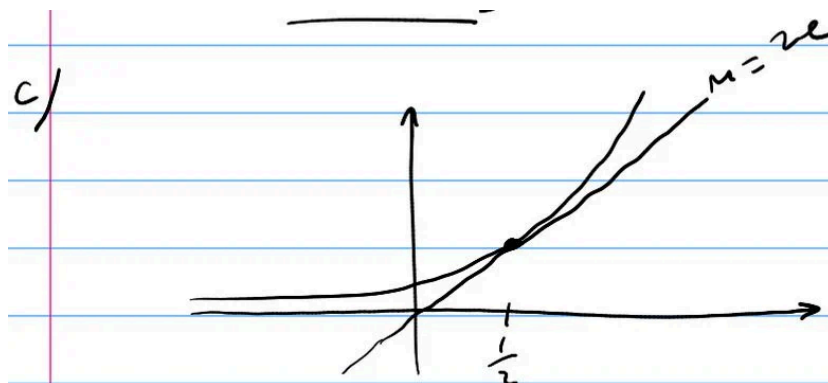
23

- $2x = \log_5 9$ (M1)
- $x = \frac{1}{2} \log_5 9 = \log_5 9^{1/2}$ (M1)
- $x = \log_5 3$ (A1)

(Total for question 23 is 3 marks)

24

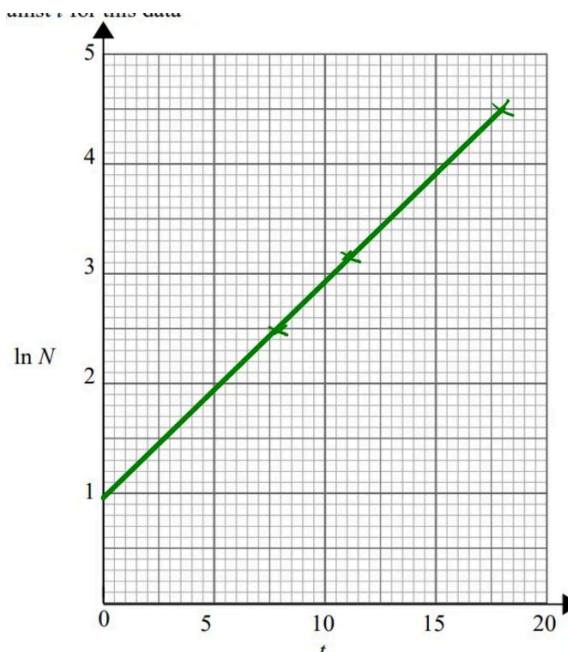
- (a)
- $\frac{dy}{dx} = 2e^{2x}$ (M1)
 - $m = 2e^{2a}$ (M1)
 - $y - e^{2a} = 2e^{2a}(x - a) \Rightarrow y = 2e^{2a}x - 2ae^{2a} + e^{2a}$ (A1)
- (b)
- Pass through $(0, 0) \Rightarrow 0 = 2e^{2a}(0) - 2ae^{2a} + e^{2a}$ (M1)
 - $e^{2a}(1 - 2a) = 0 \Rightarrow a = 0.5$ (A1)
- (c)
- Tangent gradient at $a = 0.5$ is $m = 2e^{2(0.5)} = 2e$ (M1)
 - No solutions when gradient is less than tangent but non-negative: $0 \leq m < 2e$ (M1 A1)



(Total for question 24 is 8 marks)

25

- (a)
- $\ln N = \ln(Ae^{kt}) = \ln A + \ln e^{kt}$ (M1)
 - $\ln N = kt + \ln A$ (A1)
- (b)
- Points: (8, 2.48), (11, 3.18), (18, 4.50) (M1)
 - Straight line of best fit drawn (A1)



- (c)
- Intercept $\ln A \approx 1 \Rightarrow A = e^1 \approx 2.7$ (M1 A1)
 - Gradient $k = \frac{3.4}{17.5} \approx 0.19$ (M1 A1)

(Total for question 25 is 8 marks)

26

- (a)
- The room temperature (the temperature the water approaches as $t \rightarrow \infty$) (B1)
- (b)
- $\frac{dT}{dt} = -75ke^{-kt}$ (M1)
 - At $t = 0$, rate is -20 : $-75k = -20$ (M1)
 - $k = \frac{20}{75} = \frac{4}{15}$ (A1)
- (c)
- $55 = 75e^{-\frac{4}{15}t} + 25$ (M1)
 - $30 = 75e^{-\frac{4}{15}t} \Rightarrow 0.4 = e^{-\frac{4}{15}t}$ (M1)
 - $\ln 0.4 = -\frac{4}{15}t \Rightarrow t = 3.44$ mins (A1)

(Total for question 26 is 7 marks)

27

- Point: $x = 3, y = e^1 = e$. Point is $(3, e)$ (B1)
- Gradient: $\frac{dy}{dx} = \frac{1}{3}e^{\frac{1}{3}x}$ (M1)
- At $x = 3, m = \frac{1}{3}e$ (M1)
- Equation: $y - e = \frac{1}{3}e(x - 3) \Rightarrow y - e = \frac{1}{3}ex - e \Rightarrow y = \frac{1}{3}ex$ (A1)
- Since $c = 0$, it passes through the origin.

(Total for question 27 is 4 marks)

28

- Set equal: $e^x = 3 - 2e^{\frac{1}{2}x}$ (M1)
- Form quadratic in $e^{\frac{1}{2}x}$: $(e^{\frac{1}{2}x})^2 + 2e^{\frac{1}{2}x} - 3 = 0$ (M1)
- Factorise: $(e^{\frac{1}{2}x} + 3)(e^{\frac{1}{2}x} - 1) = 0$ (M1)
- $e^{\frac{1}{2}x} = 1$ (since $e^{\frac{1}{2}x} \neq -3$) (A1)
- $\frac{1}{2}x = 0 \Rightarrow x = 0$ (M1)
- $y = e^0 = 1$. Point is $(0, 1)$ (A1)

(Total for question 28 is 6 marks)

29

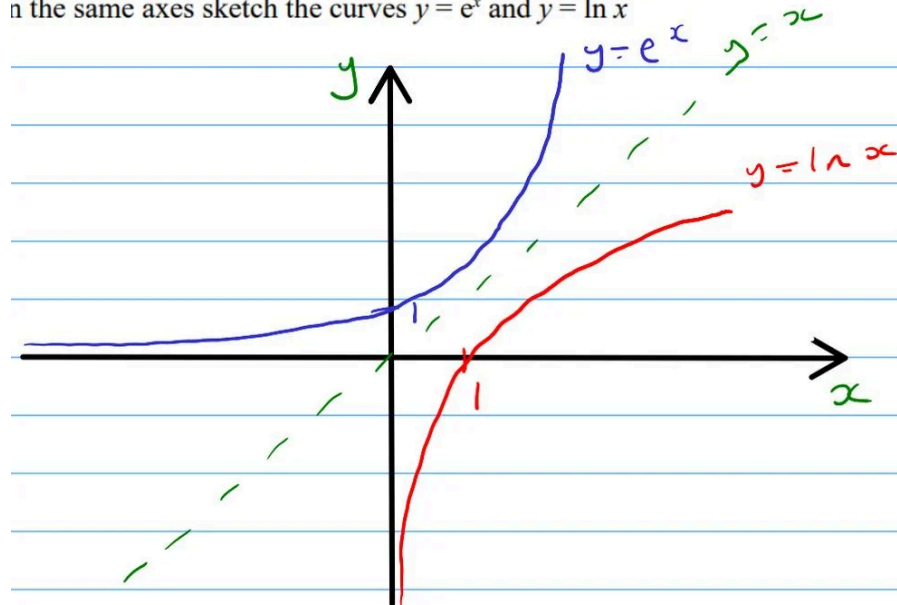
- Take logs: $\ln(2^{2x}) = \ln(2 \cdot 3^x)$ (M1)
- $2x \ln 2 = \ln 2 + x \ln 3$ (M1)
- $x(2 \ln 2 - \ln 3) = \ln 2$ (M1)
- $x = \frac{\ln 2}{2 \ln 2 - \ln 3}$ (A1)
- $x = \frac{\ln 2}{\ln 4 - \ln 3} = \frac{\ln 2}{\ln(4/3)}$ (A1)

(Total for question 29 is 5 marks)

30

- $y = e^x$ curve through $(0, 1)$ (B1)
- $y = \ln x$ curve through $(1, 0)$ (B1)
- Curves are reflections in $y = x$ (B1)
- Correct asymptotes (x -axis for e^x , y -axis for $\ln x$) (B1)

On the same axes sketch the curves $y = e^x$ and $y = \ln x$



(Total for question 30 is 4 marks)

31

- (a) • 1 (B1)
- (b) • 2 (B1)

(Total for question 31 is 2 marks)

32

- (a)
- $\log_2 x^2 - \log_2(x + a) = 3$ (M1)
 - $\log_2\left(\frac{x^2}{x+a}\right) = 3$ (M1)
 - $\frac{x^2}{x+a} = 2^3 = 8 \Rightarrow x^2 = 8x + 8a$ (A1)
- (b)
- Discriminant $b^2 - 4ac = 0$ (M1)
 - $(-8)^2 - 4(1)(-8a) = 0$ (M1)
 - $64 + 32a = 0 \Rightarrow a = -2$ (A1)

(Total for question 32 is 6 marks)

- 33
- $\log_2\left(\frac{(x+6)^2}{x+4}\right) = 3$ (M1)
 - $\frac{x^2+12x+36}{x+4} = 8$ (M1)
 - $x^2 + 12x + 36 = 8x + 32$ (M1)
 - $x^2 + 4x + 4 = 0 \Rightarrow (x + 2)^2 = 0$ (M1)
 - $x = -2$ (A1)

(Total for question 33 is 5 marks)

- 34
- $2\log_2 x + \frac{1}{2}\log_2 x = 10$ (M1)
 - $\frac{5}{2}\log_2 x = 10$ (M1)
 - $\log_2 x = 4$ (M1)
 - $x = 2^4 = 16$ (A1)

(Total for question 34 is 4 marks)

- 35
- (a)
- $P = 10^{0.008t+0.6} = 10^{0.6} \times (10^{0.008})^t$ (M1)
 - $a = 10^{0.6} = 3.981$ (A1)
 - $b = 10^{0.008} = 1.019$ (A1)
- (b)
- (i) a is the population in 1974 (3.981 billion) (B1)
 - (ii) b represents a growth rate of 1.9% per year (B1)
- (c)
- $t = 2020 - 1974 = 46$ (B1)
 - $P = 3.981 \times 1.019^{46} = 9.46$ billion (B1)
- (d)
- The model overestimates the population (9.46 vs 7.8), so it is not suitable for long-term prediction. (B1)

(Total for question 35 is 8 marks)

- 36
- (a)
- $t = 0 \Rightarrow V = 17100 + 2000 = £19100$ (B1)
- (b)
- (i) $\frac{dV}{dt} = -0.2(17100)e^{-0.2t} = -3420e^{-0.2t}$ (M1)
 - Rate of decrease is 1000: $3420e^{-0.2T} = 1000$ (A1)
 - (ii) $e^{-0.2T} = \frac{1000}{3420} = \frac{50}{171}$ (M1)
 - $-0.2T = \ln\left(\frac{50}{171}\right) \Rightarrow T = 6.15$ years (M1)
 - $0.15 \times 12 = 1.8$ months ≈ 2 months (M1)
 - 6 years 2 months (A1)

(Total for question 36 is 7 marks)

37

- (a)
- $\log_{10} N = B \log_{10} P + \log_{10} A$ (M1)
 - $B = -1.32$ (B1)
 - $\log_{10} A = 7.6 \Rightarrow A = 10^{7.6} = 39,800,000$ (A1)
- (b)
- $N = 39800000 \times (74.95)^{-1.32}$ (M1)
 - $N \approx 133,000$ (A1)
- (c)
- Profit per unit = $74.95 - 20 = 54.95$ (M1)
 - Total profit = $133000 \times 54.95 = \text{£}7,310,000$ (A1)

(Total for question 37 is 7 marks)

38

- $\log_a 4^3 - \log_a 2$ (M1)
- $\log_a 64 - \log_a 2 = \log_a 32$ (A1)

(Total for question 38 is 2 marks)

39

- $V = 3.2e^{kt}$. At $t = 20$, $V = 6.1$: $6.1 = 3.2e^{20k}$ (M1)
- $e^{20k} = \frac{6.1}{3.2} \Rightarrow k = 0.0323$ (M1)
- At $t = 30$, $V = 3.2e^{0.0323(30)} = 8.4$ million (A1)

(Total for question 39 is 3 marks)

40

- Intercept $c = 2.29$ (B1)
- Gradient $m = \frac{2.13 - 2.29}{0.4 - 0} = -0.4$ (M1)
- $\log_{10} P = -0.4 \log_{10} Q + 2.29$ (M1)
- $P = 10^{2.29} \times Q^{-0.4}$ (M1)
- $P = 195Q^{-0.4}$ (A1)

(Total for question 40 is 5 marks)

41

- (a) • The initial value of the car (at $t = 0$) (B1)
- (b) • $\ln V = \ln(Ae^{-kt}) = \ln A + \ln e^{-kt} = \ln A - kt$ (B1)
- (c) • $\ln 47200 = \ln A - 2k$ (1)
• $\ln 29200 = \ln A - 5k$ (2) (M1)
• Subtract: $\ln\left(\frac{47200}{29200}\right) = 3k \Rightarrow k = 0.16$ (M1 A1)
• $\ln A = \ln 47200 + 2(0.16) \Rightarrow A = 65000$ (A1)
- (d) • $V = 65000e^{-0.16(10)} = £13100$ (M1 A1)

(Total for question 41 is 8 marks)
