

1

$$\begin{aligned}\int y \, dx &= \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{7x^2}{2} + 10x + C \\ &= \frac{1}{2}x^4 + \frac{5}{3}x^3 - \frac{7}{2}x^2 + 10x + C\end{aligned}$$

④

(Total for question 1 is 4 marks)

2

$$\begin{aligned}\frac{3x^3}{3} + \frac{7x^2}{2} - 2x + C \\ x^3 + \frac{7}{2}x^2 - 2x + C\end{aligned}$$

④

(Total for question 2 is 4 marks)

3

$$\int x^2 - 3x + 4x - 12 \, dx$$

$$\int x^2 + x - 12 \, dx$$

$$\begin{aligned}\frac{x^3}{3} + \frac{x^2}{2} - 12x + C \\ \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + C\end{aligned}$$

④

(Total for question 3 is 4 marks)

4

$$\begin{aligned}
 f(x) &= \frac{6x^3}{3} - \frac{3x^2}{2} + 8x + C \\
 &= 2x^3 - \frac{3}{2}x^2 + 8x + C \\
 y &= 2x^3 - \frac{3}{2}x^2 + 8x + C \\
 8 &= 2(1)^3 - \frac{3}{2}(1)^2 + 8(1) + C \\
 8 &= \frac{11}{2} + C \\
 C &= -\frac{1}{2} \\
 \underline{\underline{y &= 2x^3 - \frac{3}{2}x^2 + 8x - \frac{1}{2}}}
 \end{aligned}$$

⑤

(Total for question 4 is 5 marks)

5

$$\begin{aligned}
 y &= 4x^{\frac{1}{2}} + x^{-2} + 10 \\
 \int y \, dx &= \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + 10x + C \\
 &= \frac{8}{3}x^{\frac{3}{2}} - x^{-1} + 10x + C \\
 \underline{\underline{}}}
 \end{aligned}$$

④

(Total for question 5 is 4 marks)

6

$$\begin{aligned}
 &\int_1^3 x^2 + x - 12 \, dx \\
 &\left[\frac{x^3}{3} + \frac{x^2}{2} - 12x \right]_1^3 \\
 &\left(\frac{(3)^3}{3} + \frac{(3)^2}{2} - 12(3) \right) - \left(\frac{(1)^3}{3} + \frac{(1)^2}{2} - 12(1) \right) \\
 &= -\frac{34}{3} \\
 \underline{\underline{}}}
 \end{aligned}$$

⑤

(Total for question 6 is 5 marks)

7

$$y = \frac{10x^5}{5} - 5x + C$$

$$y = 2x^5 - 5x + C$$

$$30 = 2(2)^5 - 5(2) + C$$

$$30 = 54 + C$$

$$C = -24$$

$$\underline{\underline{y = 2x^5 - 5x - 24}}$$

(5)

(Total for question 7 is 5 marks)

8

$$\int_1^4 5 + x^{-\frac{1}{2}} dx$$

$$\left[5x + 2x^{\frac{1}{2}} \right]_1^4$$

$$\left(5(4) + 2(4)^{\frac{1}{2}} \right) - \left(5(1) + 2(1)^{\frac{1}{2}} \right)$$

$$\underline{\underline{17}}$$

(5)

(Total for question 8 is 5 marks)

9

$$f'(x) = 5 + 3x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$$f(x) = 5x + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$$

$$2 = 5(1) + \frac{6}{5}(1)^{\frac{5}{2}} + 4(1)^{\frac{1}{2}} + C$$

$$C = -\frac{41}{5}$$

$$\underline{\underline{f(x) = 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - \frac{41}{5}}}$$

(7)

(Total for question 9 is 7 marks)

10

(a)

$$4 = x(5 - x)$$

$$4 = 5x - x^2$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$\underline{x = 1} \quad \underline{x = 4}$$

$$\underline{\underline{(1, 4)}} \quad \text{and} \quad \underline{\underline{(4, 4)}}$$

②

(b)

$$\text{Area} = \int_1^4 5x - x^2 dx - 12$$

$$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 \right]_1^4 - 12$$

$$= \left(\frac{5}{2}(4)^2 - \frac{1}{3}(4)^3 \right) - \left(\frac{5}{2}(1)^2 - \frac{1}{3}(1)^3 \right) - 12$$

$$= \frac{33}{2} - 12$$

$$= \underline{\underline{\frac{9}{2} \text{ units}^2}}$$

⑥

(Total for question 10 is 8 marks)

11

(a) $x = 0 \quad x = 2 \quad x = 5$ ①

(b)

$$\int_0^2 x(x - 2)(x - 5) dx = \frac{16}{3}$$

$$\int_2^5 x(x - 2)(x - 5) dx = -\frac{63}{4}$$

$$\text{Total Area} = \frac{16}{3} + \frac{63}{4} = \frac{253}{12} \text{ units}^2$$

⑧

(Total for question 11 is 9 marks)

12

(a)

$$10x - x^2 = 2x$$

$$0 = x^2 - 8x$$

$$0 = x(x - 8)$$

$$x = 0 \quad x = 8$$

$$y = 0 \quad y = 16$$

(0, 0) and (8, 16) (2)

(b) Area of triangle = $\frac{1}{2} \cdot 8 \cdot 16 = 64 \text{ units}^2$

$$\int_0^8 10x - x^2 dx = \left[5x^2 - \frac{1}{3}x^3 \right]_0^8 = \frac{448}{3} \text{ units}^2$$

$$\text{Shaded Area} = \frac{448}{3} - 64 = \frac{256}{3} \text{ units}^2$$

(6)

(Total for question 12 is 8 marks)

13 Max point where $x = 3.5$ (middle of -1 and 8)

$$y = (3.5 + 1)(8 - 3.5) = \frac{81}{4}$$

$$P : \left(\frac{7}{2}, \frac{81}{4} \right)$$

$$\text{Area of rectangle} = \frac{7}{2} \cdot \frac{81}{4} = \frac{567}{8} \text{ units}^2$$

$$\int_0^{\frac{7}{2}} (x + 1)(8 - x) dx = \frac{679}{12} \text{ units}^2$$

$$\text{Shaded Area} = \frac{567}{8} - \frac{679}{12} = \frac{343}{24} \text{ units}^2$$

(10)

(Total for question 13 is 10 marks)

14

$$\frac{dy}{dx} = \frac{1}{3}x^{-2}$$

$$y = -\frac{1}{3}x^{-1} + C$$

(3)

(Total for question 14 is 3 marks)

15

$$f'(x) = 4x^2 - 4x + 1$$

$$f(x) = \frac{4}{3}x^3 - 2x^2 + x + C$$

$$5 = \frac{4}{3}(2)^3 - 2(2)^2 + 2 + C$$

$$5 = \frac{14}{3} + C$$

$$C = \frac{1}{3}$$

$$\underline{f(x) = \frac{4}{3}x^3 - 2x^2 + x + \frac{1}{3}}$$

④

(Total for question 15 is 4 marks)

16

$$\frac{dy}{dx} = 8x^{-2}$$

$$y = -8x^{-1} + C$$

$$0 = -8(2)^{-1} + C$$

$$0 = -4 + C$$

$$C = 4$$

$$\underline{\underline{y = -8x^{-1} + 4}}$$

④

(Total for question 16 is 4 marks)

17

$$y = \frac{3}{2}x^{-\frac{1}{2}}$$

$$\int_1^k \frac{3}{2}x^{-\frac{1}{2}}dx = 12$$

$$\left[3x^{\frac{1}{2}}\right]_1^k = 12$$

$$3k^{\frac{1}{2}} - 3(1)^{\frac{1}{2}} = 12$$

$$3\sqrt{k} - 3 = 12$$

$$\sqrt{k} = 5$$

$$\underline{\underline{k = 25}}$$

⑤

(Total for question 17 is 5 marks)

18

$$\frac{dy}{dx} = 5x^{-2}$$

$$\underline{\underline{y = -5x^{-1} + C}}$$

(3)

(Total for question 18 is 3 marks)

19

(a) A: (0, 5) (when $x = 0, y = 5$)

$$0 = x^2 - 6x + 5$$

$$0 = (x - 5)(x - 1)$$

$$x = 5 \quad x = 1$$

B: (1, 0) C: (5, 0) (3)

(b)

$$\int_0^1 x^2 - 6x + 5 \, dx = \frac{7}{3}$$

$$\int_1^5 x^2 - 6x + 5 \, dx = -\frac{32}{3}$$

$$\frac{7}{3} + \frac{32}{3} = \underline{\underline{13 \text{ units}^2}}$$

(5)

(Total for question 19 is 8 marks)

20

$$\int 4x^{\frac{1}{2}} - 2 \, dx$$

$$\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + C$$

$$\frac{8}{3}x^{\frac{3}{2}} - 2x + C$$

(5)

(Total for question 20 is 5 marks)

21

$$y = 8x^{\frac{1}{3}} - 4x^{-\frac{1}{3}}$$

$$\int_1^a 8x^{\frac{1}{3}} - 4x^{-\frac{1}{3}} dx = 36$$

$$\left[6x^{\frac{4}{3}} - 6x^{\frac{2}{3}} \right]_1^a = 36$$

$$\left(6a^{\frac{4}{3}} - 6a^{\frac{2}{3}} \right) - \left(6(1)^{\frac{4}{3}} - 6(1)^{\frac{2}{3}} \right) = 36$$

$$6a^{\frac{4}{3}} - 6a^{\frac{2}{3}} = 36$$

$$a^{\frac{4}{3}} - a^{\frac{2}{3}} - 6 = 0$$

$$\left(a^{\frac{2}{3}} - 3 \right) \left(a^{\frac{2}{3}} + 2 \right) = 0$$

$$a^{\frac{2}{3}} = 3 \implies a^2 = 27 \implies a = \sqrt{27}$$

⑨

(Total for question 21 is 9 marks)

22

$$\int 2x^3 + x^{\frac{1}{2}} + 2 dx$$

$$\underline{\underline{\underline{\frac{1}{2}x^4 + \frac{2}{3}x^{\frac{3}{2}} + 2x + C}}}}$$

④

(Total for question 22 is 4 marks)

23

$$y = 8x^{-2} + 4x$$

$$\frac{dy}{dx} = -16x^{-3} + 4$$

when $x = 2$, $\frac{dy}{dx} = 2$, so normal gradient $m = -\frac{1}{2}$

$$l : y - 10 = -\frac{1}{2}(x - 2) \implies 2y = -x + 22$$

Crosses x when $y = 0 \implies x = 22$ Area of triangle $= \frac{1}{2} \cdot 20 \cdot 10 = 100 \text{ units}^2$

$$\int_1^2 8x^{-2} + 4x dx = 10 \text{ units}^2$$

$$100 + 10 = 110 \text{ units}^2$$

⑩

(Total for question 23 is 10 marks)

24

(a)

$$\int kx^{-3} + 2x \, dx$$

$$-\frac{1}{2}kx^{-2} + x^2 + c \quad (3)$$

(b)

$$\left[-\frac{1}{2}kx^{-2} + x^2 \right]_1^2 = 12$$

$$\left(-\frac{1}{8}k + 4 \right) - \left(-\frac{1}{2}k + 1 \right) = 12$$

$$\frac{3}{8}k + 3 = 12 \implies \frac{3}{8}k = 9 \implies k = 24$$

(3)

(Total for question 24 is 6 marks)

25 turning point where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 14x + 15 = 0$$

$$(3x - 5)(x - 3) = 0 \implies x = 3 \text{ (minimum)}$$

$$\int_0^3 x^3 - 7x^2 + 15x \, dx = \left[\frac{1}{4}x^4 - \frac{7}{3}x^3 + \frac{15}{2}x^2 \right]_0^3$$

$$= \left(\frac{81}{4} - 63 + \frac{135}{2} \right) - 0 = \frac{99}{4}$$

(7)

(Total for question 25 is 7 marks)

26

(a)

$$\int_1^k \frac{3}{2}x^{-\frac{1}{2}} + 9 dx = 8$$

$$\left[3x^{\frac{1}{2}} + 9x\right]_1^k = 8$$

$$(3k^{\frac{1}{2}} + 9k) - (3(1)^{\frac{1}{2}} + 9(1)) = 8$$

$$3k^{\frac{1}{2}} + 9k - 12 = 8 \implies 9k + 3\sqrt{k} - 20 = 0$$

(4)

(b)

$$(3\sqrt{k} - 4)(3\sqrt{k} + 5) = 0$$

$$\sqrt{k} = \frac{4}{3} \implies k = \frac{16}{9}$$

$$\sqrt{k} = -\frac{5}{3} \implies \text{No real solution for } k$$

(4)

(Total for question 26 is 8 marks)

27

(a)

$$f(-2) = 2(-2)^3 - (-2)^2 - 8(-2) + 4 = -16 - 4 + 16 + 4 = 0$$

Since $f(-2) = 0$, $(x + 2)$ is a factor. (2)

(b)

$$(x + 2)(2x^2 - 5x + 2)$$

$$(x + 2)(2x - 1)(x - 2)$$

(4)

(c) Region is between $x = 1/2$ and $x = 2$.

$$\begin{aligned} \int_{\frac{1}{2}}^2 2x^3 - x^2 - 8x + 4 dx &= \left[\frac{1}{2}x^4 - \frac{1}{3}x^3 - 4x^2 + 4x \right]_{\frac{1}{2}}^2 \\ &= (8 - 8/3 - 16 + 8) - (1/32 - 1/24 - 1 + 2) = -\frac{117}{32} \end{aligned}$$

$$\text{Area} = \frac{117}{32} \quad (4)$$

(Total for question 27 is 10 marks)

28

$$\begin{aligned} \int \frac{4x^3}{3x^2} - \frac{5}{3x^2} dx &= \int \frac{4}{3}x - \frac{5}{3}x^{-2} dx \\ &= \underline{\underline{\frac{2}{3}x^2 + \frac{5}{3}x^{-1} + C}} \end{aligned}$$

(4)

(Total for question 28 is 4 marks)

29

$$\int_1^8 kx^{-\frac{1}{3}} = 22.5$$

$$\left[\frac{3}{2} kx^{\frac{2}{3}} \right]_1^8 = 22.5$$

$$\frac{3}{2}k(8)^{\frac{2}{3}} - \frac{3}{2}k(1)^{\frac{2}{3}} = 22.5$$

$$6k - 1.5k = 22.5 \implies 4.5k = 22.5 \implies k = 5$$

④

(Total for question 29 is 4 marks)

30

$$\int_k^9 10x^{-\frac{1}{2}} dx = 20$$

$$\left[20x^{\frac{1}{2}} \right]_k^9 = 20$$

$$20(3) - 20\sqrt{k} = 20 \implies 60 - 20\sqrt{k} = 20 \implies 20\sqrt{k} = 40 \implies k = 4$$

④

(Total for question 30 is 4 marks)

31

(a)

$$\frac{2x^2}{2} - \frac{x^3}{3} + C = x^2 - \frac{1}{3}x^3 + C$$

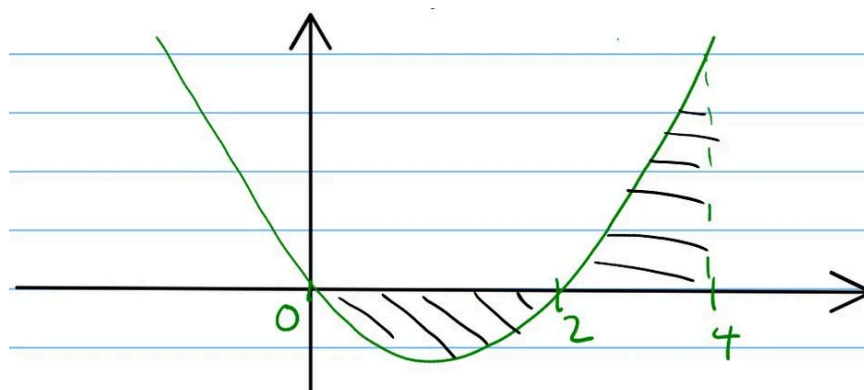
②

(b)

$$\left[x^2 - \frac{1}{3}x^3 \right]_0^4 = (16 - 64/3) - 0 = -\frac{16}{3}$$

②

(c)



- between 0 and 4 some of the area is above and some below the x axis.
- the answer will be the difference between the two areas. ①

(Total for question 31 is 5 marks)

32

$$0 = x^2 - 9x + 14 \implies (x - 2)(x - 7) = 0 \implies x = 2, 7$$

$$\int_2^7 x^2 - 9x + 14 \, dx = \left[\frac{1}{3}x^3 - 4.5x^2 + 14x \right]_2^7 = -\frac{125}{6}$$

Area 1 = $125/6$. Area 2 = $29 - 125/6 = 49/6$.

$$\int_7^a x^2 - 9x + 14 \, dx = \frac{49}{6}$$

$$\left[\frac{1}{3}x^3 - 4.5x^2 + 14x \right]_7^a = \frac{49}{6}$$

$$\frac{1}{3}a^3 - 4.5a^2 + 14a = 0 \implies a = \frac{27 + \sqrt{57}}{4}$$

(8)

(Total for question 32 is 8 marks)

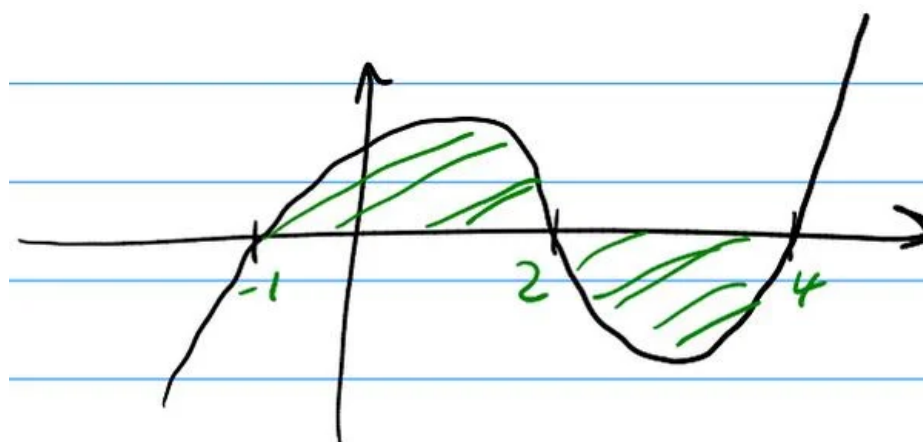
33

$$\int 10 - 6x^{-2} \, dx = 10x + 6x^{-1} + C$$

(4)

(Total for question 33 is 4 marks)

34



$$\int_{-1}^2 (x - 4)(x - 2)(x + 1) \, dx = -\frac{16}{3}$$

$$\int_{-1}^2 (x - 4)(x - 2)(x + 1) \, dx = \frac{63}{4}$$

$$\frac{16}{3} + \frac{63}{4} = \underline{\underline{\frac{253}{12} \text{ units}^2}}$$

(5)

(Total for question 34 is 5 marks)

35

$$y = k(x - 1)(x - 5) \implies 10 = k(-1)(-5) \implies k = 2$$

$$y = 2(x^2 - 6x + 5) = 2x^2 - 12x + 10$$

$$\int_1^5 2x^2 - 12x + 10 \, dx = \left[\frac{2}{3}x^3 - 6x^2 + 10x \right]_1^5 = -\frac{64}{3}$$

$$\text{Area} = \frac{64}{3} \quad \textcircled{8}$$

(Total for question 35 is 8 marks)

36

$$\int_1^8 16x^{-2} \, dx = \left[-16x^{-1} \right]_1^8 = -16(8)^{-1} - (-16(1)^{-1}) = -2 + 16 = 14$$

 $\textcircled{4}$

(Total for question 36 is 4 marks)

37

$$\int_1^4 1 + 3x^{\frac{1}{2}} \, dx = \left[x + 2x^{\frac{3}{2}} \right]_1^4 = (4 + 2(8)) - (1 + 2) = 20 - 3 = 17$$

 $\textcircled{4}$

(Total for question 37 is 4 marks)

38

(a)

$$\int 2x^3 + 25x^{\frac{3}{2}} \, dx = \frac{1}{2}x^4 + 10x^{\frac{5}{2}} + C$$

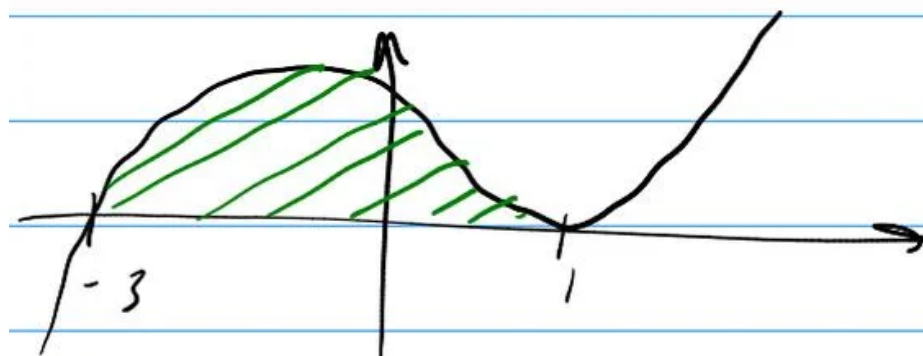
 $\textcircled{5}$

(b)

$$\left[\frac{1}{2}x^4 + 10x^{\frac{5}{2}} \right]_1^9 = (3280.5 + 2430) - (0.5 + 10) = 5710.5 - 10.5 = 5700$$

 $\textcircled{3}$

(Total for question 38 is 8 marks)



$$(x + 3)(x^2 - 2x + 1) = x^3 + x^2 - 5x + 3$$

$$\int_{-3}^1 x^3 + x^2 - 5x + 3 dx = \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{5}{2}x^2 + 3x \right]_{-3}^1 = \frac{64}{3}$$

⑥

(Total for question 39 is 6 marks)