

Solutions

1

$$2x + 15 = \cos^{-1}(0.3)$$

$$2x + 15 = 72.54, 287.46$$

(M1)

$$x = 28.8^\circ, 136.2^\circ$$

(A1)

(Total for question 1 is 5 marks)

2

$$3\theta - 15 = \sin^{-1}(0.7)$$

(M1)

$$= 44.4^\circ, 135.6^\circ, 404.4^\circ, 495.6^\circ$$

(M1)

$$\theta = 19.81^\circ, 50.19^\circ, 139.81^\circ, 170.19^\circ$$

(A1)

(Total for question 2 is 5 marks)

3

$$\theta + 30 = \tan^{-1}(-2.5)$$

(M1)

$$\theta + 30 = -68.2^\circ, 111.8^\circ$$

(M1)

$$\theta = -98.2^\circ, 81.8^\circ$$

(A1)

(Total for question 3 is 4 marks)

4

$$\cos(x - 40) = \frac{2}{5}$$

(M1)

$$x - 40 = 66.42^\circ, 293.58^\circ$$

(M1)

$$x = 106.42^\circ, 333.58^\circ$$

(A1)

(Total for question 4 is 4 marks)

5

$$\tan x = \pm\sqrt{3}$$

(M1)

$$\tan x = \sqrt{3} \implies x = 60^\circ, 240^\circ$$

(A1)

$$\tan x = -\sqrt{3} \implies x = 120^\circ, 300^\circ$$

(A1)

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

(A1)

(Total for question 5 is 5 marks)

6

(a) Uses $\sin^2 x = 1 - \cos^2 x$ (M1)

$$2(1 - \cos^2 x) = 7 \cos x + 5$$

(M1)

$$2 - 2 \cos^2 x = 7 \cos x + 5$$

$$0 = 2 \cos^2 x + 7 \cos x + 3$$

(A1)

(b)

$$(2 \cos x + 1)(\cos x + 3) = 0$$

(M1)

$$\cos x = -\frac{1}{2} \text{ or } \cos x = -3$$

(A1)

$\cos x = -3$ has no solutions

(B1)

$$x = 120^\circ, 240^\circ$$

(A1)

(Total for question 6 is 8 marks)

7

(a)

$$6(1 - \sin^2 x) = 4 - \sin x$$

(M1)

$$6 - 6 \sin^2 x = 4 - \sin x$$

(M1)

$$0 = 6 \sin^2 x - \sin x - 2$$

(A1)

(b)

$$(3 \sin x - 2)(2 \sin x + 1) = 0$$

(M1)

$$\sin x = 2/3, \sin x = -1/2$$

(A1)

$$\sin x = 2/3 \implies x = 41.8^\circ, 138.2^\circ$$

(A1)

$$\sin x = -1/2 \implies x = 210^\circ, 330^\circ$$

(A1)**(Total for question 7 is 9 marks)**

8

$$2 \cos^2 x - 3(1 - \cos^2 x) = 14 \cos x$$

(M1)

$$5 \cos^2 x - 14 \cos x - 3 = 0$$

(M1)

$$(5 \cos x + 1)(\cos x - 3) = 0$$

(M1)

$$\cos x = -1/5, \cos x = 3$$

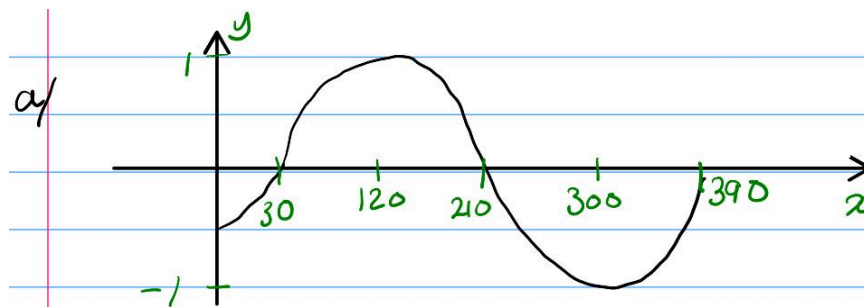
(A1) $\cos x = 3$ has no solutions**(B1)**

$$x = 101.5^\circ, 258.5^\circ$$

(A1)**(Total for question 8 is 8 marks)**

9

(a)



(B2)

(b)

$$x - 30 = 17.5^\circ, 162.5^\circ$$

(M1)

$$x = 47.5^\circ, 192.5^\circ$$

(A1)

(Total for question 9 is 6 marks)

10

$$3 \frac{\sin x}{\cos x} = 4 \sin x$$

(M1)

$$3 \sin x = 4 \sin x \cos x$$

$$0 = \sin x (4 \cos x - 3)$$

(M1)

$$\sin x = 0 \implies x = 0^\circ, 180^\circ$$

(A1)

$$\cos x = 3/4 \implies x = 41.4^\circ, 318.6^\circ$$

(A1)

(Total for question 10 is 7 marks)

11

- (a)
- Use $\tan 2x = \frac{\sin 2x}{\cos 2x}$ to get $3 \sin 2x \cdot \frac{\sin 2x}{\cos 2x} = \cos 2x + 2$ (M1)
 - Multiply by $\cos 2x$ to get $3 \sin^2 2x = \cos^2 2x + 2 \cos 2x$ (M1)
 - Substitute $\sin^2 2x = 1 - \cos^2 2x$ (M1)
 - Rearrange to $4 \cos^2 2x + 2 \cos 2x - 3 = 0$ (A1)
- (b)
- Solve quadratic $4y^2 + 2y - 3 = 0$ using formula: $\cos 2x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-3)}}{2(4)} = \frac{-1 \pm \sqrt{13}}{4}$ (M1)
 - Identify valid solution $\cos 2x = \frac{-1 + \sqrt{13}}{4} \approx 0.651...$ (A1)
 - Find $2x = 49.35...$ (M1)
 - Find second value $2x = 360 - 49.35 = 310.65...$ (M1)
 - Divide by 2 to get $x = 24.68^\circ$ and $x = 155.32^\circ$ (A1) (A1)

(Total for question 11 is 10 marks)

12

- (a)
- Use $\tan x = \frac{\sin x}{\cos x}$ to get $1 + \cos x = 3 \frac{\sin^2 x}{\cos x}$ (M1)
 - Multiply by $\cos x$ to get $\cos x + \cos^2 x = 3 \sin^2 x$ (M1)
 - Substitute $\sin^2 x = 1 - \cos^2 x$ (M1)
 - Rearrange to $4 \cos^2 x + \cos x - 3 = 0$ (A1)
- (b)
- Factorise quadratic: $(4 \cos x - 3)(\cos x + 1) = 0$ (M1)
 - Identify $\cos x = \frac{3}{4}$ and $\cos x = -1$ (A1)
 - Solve $\cos x = \frac{3}{4}$ to get $x = 41.4^\circ$ and $x = 318.6^\circ$ (M1) (A1)
 - Solve $\cos x = -1$ to get $x = 180^\circ$ (A1)

(Total for question 12 is 9 marks)

13

- (a)
- Substitute $\cos^2 \theta = 1 - \sin^2 \theta$ into numerator: $6(1 - \sin^2 \theta) + 7 \sin \theta - 8$ (M1)
 - Simplify numerator to $-6 \sin^2 \theta + 7 \sin \theta - 2$ (M1)
 - Factorise numerator: $-(2 \sin \theta - 1)(3 \sin \theta - 2)$ or $(1 - 2 \sin \theta)(3 \sin \theta - 2)$ (M1)
 - Cancel $(1 - 2 \sin \theta)$ to get $3 \sin \theta - 2$ (A1)
- (b)
- Set $3 \sin \theta - 2 = 2 \cos \theta - 2$ (M1)
 - Simplify to $3 \sin \theta = 2 \cos \theta$ and use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to get $\tan \theta = \frac{2}{3}$ (M1)
 - Solve to get $\theta = 33.7^\circ$ and $\theta = 213.7^\circ$ (A1)

(Total for question 13 is 7 marks)

14

- (a)
- Use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cos^2 \theta = 1 - \sin^2 \theta$ to get $3(1 - \sin^2 \theta) = 8 \sin \theta$ (M1)
 - Form quadratic $3 \sin^2 \theta + 8 \sin \theta - 3 = 0$ (M1)
 - Factorise to $(3 \sin \theta - 1)(\sin \theta + 3) = 0$ (M1)
 - Identify $\sin \theta = \frac{1}{3}$ (as $\sin \theta = -3$ is impossible) (A1)
 - Find solutions in range: $\theta = 360 + 19.5 = 379.5^\circ$ and $\theta = 540 - 19.5 = 520.5^\circ$ (A1)
- (b)
- Find principal values for $2a + 50$: $\cos^{-1}(0.7) = 45.57^\circ, 314.43^\circ, 405.57^\circ, 674.43^\circ, 765.57^\circ$ (M1)
 - Identify the 5th value of the bracket to get the 4th positive value of a (since the first value 45.57 gives a negative a): $2a + 50 = 765.57$ (M1)
 - Solve $2a = 715.57$ to get $a_4 = 358^\circ$ (A1)

(Total for question 14 is 8 marks)

15

- $\tan^2 2x = 3 \implies \tan 2x = \pm\sqrt{3}$ (M1)
- Find values for $2x$ in range $0 \leq 2x < 720$: $2x = 60, 120, 240, 300, 420, 480, 600, 660$ (M1)
- Divide by 2 to get $x = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$ (A2)

(Total for question 15 is 4 marks)

16

- $\tan 75 = \frac{\sin 75}{\cos 75} = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ (M1)
- Rationalise or simplify $\tan 75 = 2 + \sqrt{3}$ (M1)
- $\tan^2 75 = (2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$ (A1)

(Total for question 16 is 3 marks)

17

- (a)
- $a = -270$ and $b = 4$ (Note: The solution provided in the markscheme image for Q17 shows $y = -4 \sin x$ which gives $a = -90$, but the question text says $y = 4 \sin x$. Based on $y = 4 \sin x$, $a = -270$, $b = 4$. Based on the solution provided: $a = -90$, $b = 4$) (B1)
- (b)
- (i) Shift left 28: $(-90 - 28, 4) = (-118, 4)$ (B1)
 - (ii) Horizontal stretch scale factor $1/3$: $(-90/3, 4) = (-30, 4)$ (B1)
- (c)
- $4 \sin \theta = \frac{\sin \theta}{\cos \theta} \implies 4 \sin \theta \cos \theta - \sin \theta = 0$ (M1)
 - $\sin \theta(4 \cos \theta - 1) = 0$ (M1)
 - $\sin \theta = 0 \implies \theta = 360^\circ, 540^\circ$ (A1)
 - $\cos \theta = 0.25 \implies \theta = 435.5^\circ, 644.5^\circ$ (A2)

(Total for question 17 is 8 marks)

18

- $\tan 2\theta = 1$ (M1)
- $2\theta = 45, 225, 405, 585$ (M1)
- $\theta = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$ (A1)

(Total for question 18 is 3 marks)

19

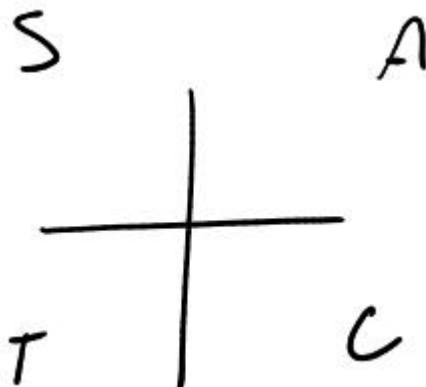
- (a)
- $6(1 - \cos^2 \theta) = \cos \theta + 4 \implies 6 \cos^2 \theta + \cos \theta - 2 = 0$ (M1)
 - $(2 \cos \theta - 1)(3 \cos \theta + 2) = 0$ (M1)
 - $\cos \theta = 0.5 \implies \theta = 60^\circ, 300^\circ$ (A1)
 - $\cos \theta = -2/3 \implies \theta = 131.8^\circ, 228.2^\circ$ (A1)
- (b)
- $2\theta = 60, 300, 420, 660, 131.8, 228.2, 491.8, 588.2$ (M1)
 - $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ, 65.9^\circ, 114.1^\circ, 245.9^\circ, 294.1^\circ$ (A1)

(Total for question 19 is 6 marks)

20

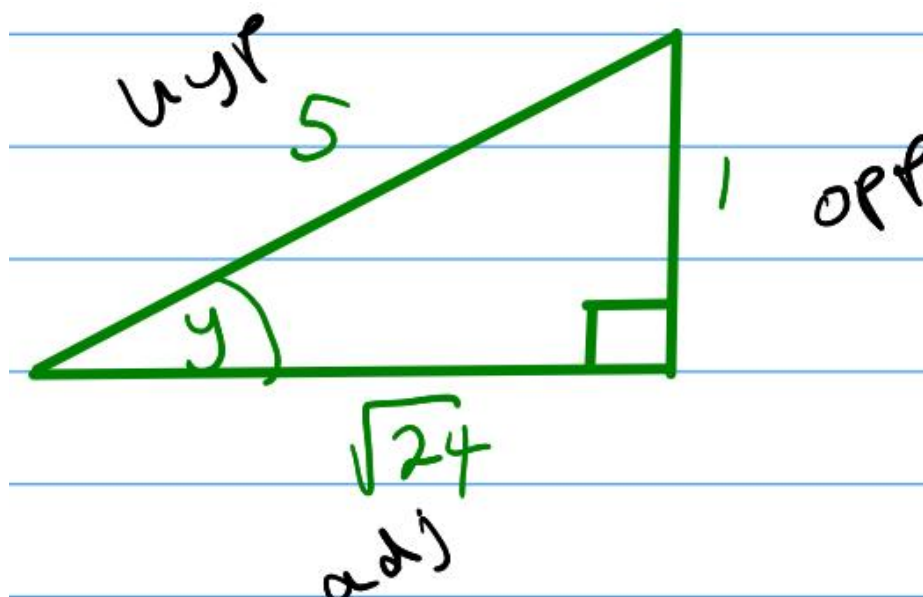
- (a)
- At $h = 0, T = 22 \implies 22 = A + B \sin(0) \implies A = 22$ (B1)
- (b)
- At $h = 6, T = 25 \implies 25 = 22 + B \sin(15 \times 6) \implies 3 = B \sin(90) \implies B = 3$ (B1)
- (c)
- At 9 pm, $h = 9: T = 22 + 3 \sin(15 \times 9) = 22 + 3 \sin(135) = 24.1^\circ\text{C}$ (B1)
- (d)
- $23.5 = 22 + 3 \sin(15h) \implies \sin(15h) = 0.5$ (M1)
 - $15h = 30, 150$ (M1)
 - $h = 2, 10$ (A1)
 - Duration = $10 - 2 = 8$ hours (A1)

(Total for question 20 is 7 marks)



$$\sin y = -\frac{1}{5}$$

cos is negative between 180 and 270



$$\sqrt{5^2 - 1^2} = \sqrt{24}$$

$$\cos y = -\frac{\sqrt{24}}{5} \quad (\text{B2})$$

(Total for question 21 is 2 marks)

22

- (a)
- Jacob should not have divided through by $\sin x$. This results in not getting the answer $\sin x = 0$ (B1)
 - He did not find all solutions for $\sin x = 0.5$ in the range (B1)
- (b) $2 \sin^2 x - \sin x = 0 \implies \sin x(2 \sin x - 1) = 0 \implies \sin x = 0 \implies x = 0^\circ, -180^\circ$ (B1) $\sin x = 0.5 \implies x = 30^\circ, 150^\circ$ (B1)

(Total for question 22 is 4 marks)

23 $6(1 - \sin^2 x) + 5 \sin x - 7 = 0$ $6 - 6 \sin^2 x + 5 \sin x - 7 = 0$ $-6 \sin^2 x + 5 \sin x - 1 = 0$
 $6 \sin^2 x - 5 \sin x + 1 = 0$ (M1) $(3 \sin x - 1)(2 \sin x - 1) = 0$ (M1) $\sin x = \frac{1}{3} \Rightarrow x =$
 $19^\circ, 161^\circ$ (A1) $\sin x = \frac{1}{2} \Rightarrow x = 30^\circ, 150^\circ$ (A1)

(Total for question 23 is 4 marks)

24 (a) $2(1 - \cos^2 x) = 4 \cos^2 x - \cos x$ (M1) $2 - 2 \cos^2 x = 4 \cos^2 x - \cos x$ (M1) $6 \cos^2 x - \cos x - 2 = 0$ (A1)

(b) $6 \cos^2 2\theta - \cos 2\theta - 2 = 0$ $(3 \cos 2\theta - 2)(2 \cos 2\theta + 1) = 0$ (M1) $\cos 2\theta = \frac{2}{3} \Rightarrow$
 $2\theta = 48.2^\circ, 311.8^\circ$ (M1) $\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = 120^\circ, 240^\circ$ (M1) $\theta =$
 $24.1^\circ, 155.9^\circ, 60^\circ, 120^\circ$ (A2)

(Total for question 24 is 8 marks)

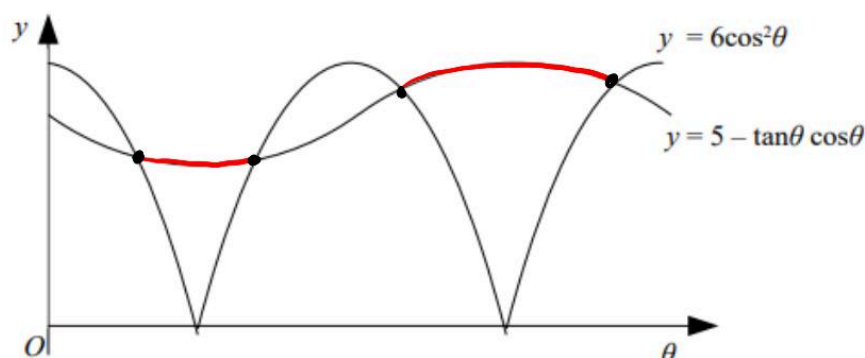
25 (a) $\sin x = \pm \sqrt{0.25} = \pm 0.5$ (M1) $\sin x = 0.5 \Rightarrow x = 30^\circ, 150^\circ$ (A1) $\sin x =$
 $-0.5 \Rightarrow x = 210^\circ, 330^\circ$ (A1)

(b) $3x = 45^\circ, 225^\circ, 405^\circ$ (M1) $x = 15^\circ, 75^\circ, 135^\circ$ (A2)

(Total for question 25 is 6 marks)

26 (a) $5 - \frac{\sin \theta}{\cos \theta} \cos \theta = 6(1 - \sin^2 \theta)$ (M1) $5 - \sin \theta = 6 - 6 \sin^2 \theta$ $6 \sin^2 \theta - \sin \theta - 1 = 0$
(A1)

(b)



$6 \sin^2 \theta - \sin \theta - 1 = 0$ $(3 \sin \theta + 1)(2 \sin \theta - 1) = 0$ $\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ, 150^\circ$
(M1) $\sin \theta = -\frac{1}{3} \Rightarrow \theta = 199.5^\circ, 340.5^\circ$ (M1) From graph, $30 < \theta < 150$ or $199.5 <$
 $\theta < 340.5^\circ$ (A3)

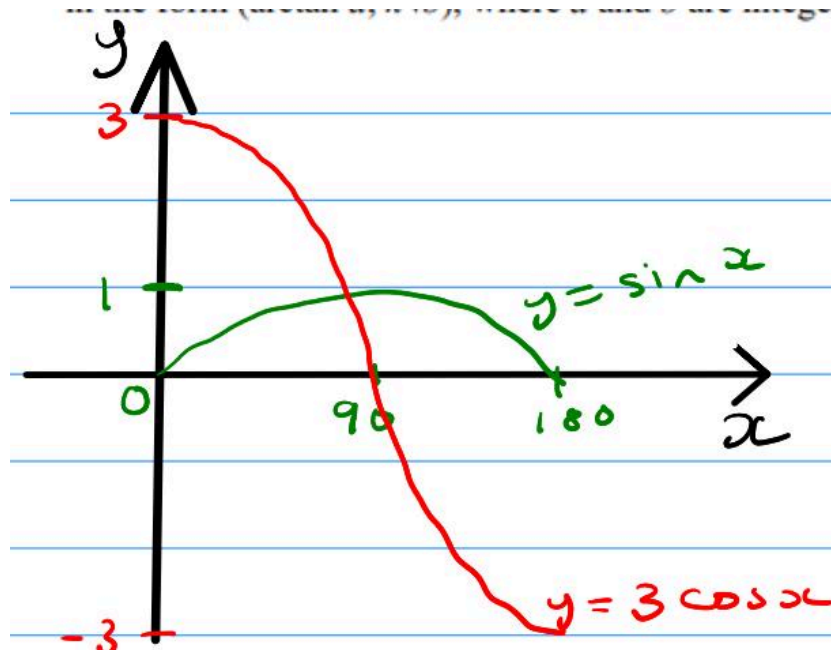
(Total for question 26 is 7 marks)

27 (a) $\sin^2 x = \frac{\sin^2 x}{\cos^2 x} \sin^2 x \cos^2 x - \sin^2 x = 0$ (M1) $\sin^2 x (\cos^2 x - 1) = 0$ (M1) $\sin x =$
 $0 \Rightarrow x = 0^\circ, 180^\circ$ (A1) $\cos x = \pm 1 \Rightarrow x = 0^\circ, 180^\circ$ (A2)

(b) $\frac{2 \sin x - (1 - \sin^2 x) + 1}{2 + \sin x}$ (M1) $\frac{2 \sin x + \sin^2 x}{2 + \sin x}$ (M1) $\frac{\sin x (2 + \sin x)}{2 + \sin x} = \sin x$ (A1)

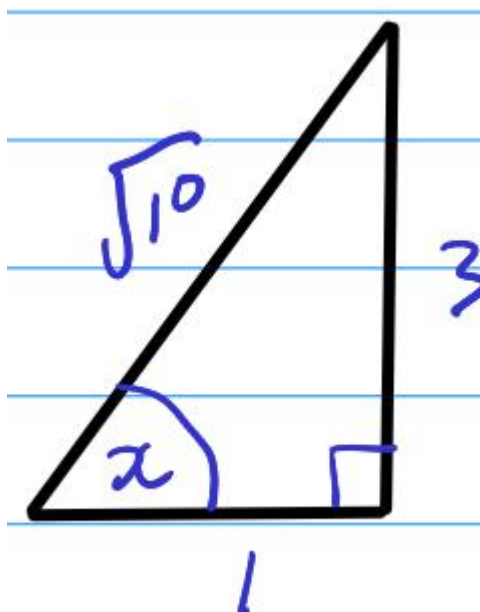
(Total for question 27 is 8 marks)

(a)



- Correct cosine curve starting at (0,3) and ending at (180,-3) (B1)
- Correct sine curve starting at (0,0), peaking at (90,1) and ending at (180,0) (B1)

(b) $\sin x = 3 \cos x \implies \tan x = 3 \implies x = \arctan 3$ (M1) Using triangle: opp = 3, adj = 1, hyp = $\sqrt{10}$ (M1)



$$y = \sin x = \frac{3}{\sqrt{10}} = \frac{3}{10}\sqrt{10} \quad (\text{A1}) \quad \text{Intersection at } (\arctan 3, \frac{3}{10}\sqrt{10}) \quad (\text{A1})$$

(Total for question 28 is 6 marks)

29 $5 \tan x = 3 \implies \tan x = 0.6$ (M1) $x = 31.0^\circ$ (A1) $x = 211.0^\circ$ (A1)

(Total for question 29 is 3 marks)

30 $\frac{24 \sin x}{\cos x} + 5 \cos x = 0$ $24 \sin x + 5 \cos^2 x = 0$ (M1) $24 \sin x + 5(1 - \sin^2 x) = 0$ (M1)
 $5 \sin^2 x - 24 \sin x - 5 = 0$ (M1) $(5 \sin x + 1)(\sin x - 5) = 0$ $\sin x = -\frac{1}{5}$ (as $\sin x = 5$ has no
sols) (A1) $x = 192^\circ, 348^\circ$ (A2)

(Total for question 30 is 6 marks)
