1

(a)

- Integrate acceleration to find velocity: $v = \int (4t - 8)\,dt = 2t^2 - 8t + C$ **M1**
- Use initial conditions $v = 6$ when $t = 0$ to find $C = 6$ **M1**
- State final expression: $v = 2t^2 - 8t + 6$ **A1**

(b)

- Set $v = 0$ and solve for t: $2t^2 - 8t + 6 = 0 \implies t^2 - 4t + 3 = 0 \implies (t - 3)(t - 1) = 0$ **M1**
- Identify $t = 1$ and $t = 3$ **A1**
- Integrate velocity to find displacement: $s = \int (2t^2 - 8t + 6)\,dt = \frac{2}{3}t^3 - 4t^2 + 6t + C$ **M1**
- Evaluate s at $t = 3$: $s(3) = 18 - 36 + 18 + C = C$ **M1**
- Evaluate s at $t = 1$: $s(1) = \frac{2}{3} - 4 + 6 + C = \frac{8}{3} + C$ **M1**
- Calculate difference: $(\frac{8}{3} + C) - C = \frac{8}{3}$ **M1**
- Final answer $\frac{8}{3}$ m **A1**

(Total for question 1 is 10 marks)

2

(a)

- Integrate velocity: $s = \int (6t - 2)\,dt = 3t^2 - 2t + C$ **M1**
- Use $t = 5, s = 75$: $75 = 3(5)^2 - 2(5) + C \implies 75 = 65 + C$ **M1**
- Find $C = 10$ and state $s = 3t^2 - 2t + 10$ **A1**

(b)

- Substitute $t = 10$ into displacement formula: $s = 3(10)^2 - 2(10) + 10$ **M1**
- $s = 300 - 20 + 10 = 290$ m **A1**

(Total for question 2 is 5 marks)

3

(a)

- Integrate a for $0 \le t \le 3$: $v = \int (4t - t^2)\,dt = 2t^2 - \frac{1}{3}t^3 + C$ **M1**
- Use $t = 0, v = 0 \implies C = 0$ **B1**
- Substitute $t = 3$: $v = 2(3)^2 - \frac{1}{3}(3)^3 = 18 - 9 = 9\ \text{m s}^{-1}$ **A1**

(b)

- Integrate a for $t > 3$: $v = \int 27t^{-2}\,dt = -27t^{-1} + C$ **M1**
- Use continuity at $t = 3$: $v = 9$ when $t = 3$ **M1**
- $9 = -\frac{27}{3} + C \implies 9 = -9 + C \implies C = 18$ **A1**
- Substitute $t = 6$: $v = -\frac{27}{6} + 18 = -4.5 + 18 = 13.5\ \text{m s}^{-1}$ **M1A1**

(Total for question 3 is 8 marks)

4

(a)

- Differentiate v for $0 \leq t \leq 5$: $a = \frac{dv}{dt} = 7 - 2t$ **M1**
- Substitute $t = 4$: $a = 7 - 2(4)$ **M1**
- $a = -1 \text{ ms}^{-2}$ **A1**

(b)

- Integrate v for $0 \leq t \leq 5$: $s_1 = [\frac{7}{2}t^2 - \frac{1}{3}t^3]_0^5$ **M1**
- $s_1 = \frac{7}{2}(25) - \frac{1}{3}(125) = \frac{175}{2} - \frac{125}{3} = \frac{525-250}{6} = \frac{275}{6}$ m **A1**
- For $t > 5$, $v = 10 - 2t$. $v = 0$ when $t = 5$. For $5 < t \leq 10$, v is negative. **M1**
- Distance for $5 \leq t \leq 10$: $|\int_5^{10}(10 - 2t)\,dt| = |[10t - t^2]_5^{10}|$ **M1**
- $|(100 - 100) - (50 - 25)| = |-25| = 25$ m **A1**
- Total distance = $\frac{275}{6} + 25 = \frac{275+150}{6} = \frac{425}{6}$ m **M1A1**

(Total for question 4 is 10 marks)

5

- Differentiate velocity to find acceleration: $a = \frac{dv}{dt} = 6 - 4t$ **M1**
- Set $a = 0$: $6 - 4t = 0$ **M1**
- Solve for t: $t = 1.5$ s **A1**

(Total for question 5 is 3 marks)

6

(a)

- Integrate acceleration: $v = \int (6t - 24)\,dt = 3t^2 - 24t + C$ **M1**
- Use $t = 0, v = 60 \implies C = 60$, so $v = 3t^2 - 24t + 60$ **A1**
- Complete the square or find minimum: $v = 3(t^2 - 8t + 20) = 3[(t-4)^2 + 4]$ **M1**
- Minimum $v = 12$ at $t = 4$. Since $v_{min} > 0$, v is always positive. **A1**

(b)

- Integrate velocity: $s = \int (3t^2 - 24t + 60)\,dt = t^3 - 12t^2 + 60t + C$ **M1**
- Use $t = 0, s = 0 \implies C = 0$ **B1**
- Substitute $t = 10$: $s = (10)^3 - 12(10)^2 + 60(10) = 1000 - 1200 + 600 = 400$ m **A1**

(Total for question 6 is 7 marks)

7

(a)
- Integrate velocity: $s = \int (12 - 2t^2)\,dt = 12t - \frac{2}{3}t^3 + C$ **M1**
- Use $t = 0, s = 0 \implies C = 0$ **B1**
- Substitute $t = 1$: $s = 12(1) - \frac{2}{3}(1)^3 = 12 - \frac{2}{3} = \frac{34}{3}$ m **A1**

(b)
- Set $v = 0$: $12 - 2t^2 = 0$ **M1**
- $t^2 = 6 \implies t = \sqrt{6}$ s **A1**

(c)
- Set displacement $s = 0$: $12t - \frac{2}{3}t^3 = 0$ **M1**
- $t(12 - \frac{2}{3}t^2) = 0 \implies \frac{2}{3}t^2 = 12$ **M1**
- $t^2 = 18 \implies t = \sqrt{18} = 3\sqrt{2}$ s **A1**

(Total for question 7 is 8 marks)

8

(a)
- Differentiate displacement: $v = \frac{dx}{dt} = 3t^2 - 30t + 62$ **M1A1**
- Substitute $t = 0$: $v = 62$ ms^{-1} **A1**

(b)
- Differentiate velocity: $a = \frac{dv}{dt} = 6t - 30$ **M1**
- Set $a = 0$: $6t - 30 = 0 \implies t = 5$ s **A1**

(Total for question 8 is 5 marks)

9

(a)
- Differentiate displacement: $v = 6t^2 - 36t + 48$ **M1**
- Set $v = 0$: $6(t^2 - 6t + 8) = 0$ **M1**
- Solve: $(t - 2)(t - 4) = 0 \implies t = 2, t = 4$ **A1**

(b)
- Calculate displacement at key times:
  - $t = 0, x = 0$
  - $t = 2, x = 2(8) - 18(4) + 48(2) = 16 - 72 + 96 = 40$ **M1**
  - $t = 4, x = 2(64) - 18(16) + 48(4) = 128 - 288 + 192 = 32$ **M1**
  - $t = 5, x = 2(125) - 18(25) + 48(5) = 250 - 450 + 240 = 40$ **M1**
- Calculate distances between turns:
  - 0 to 2: $|40 - 0| = 40$
  - 2 to 4: $|32 - 40| = 8$
  - 4 to 5: $|40 - 32| = 8$ **M1**
- Total distance = $40 + 8 + 8 = 56$ m **A1**

(Total for question 9 is 8 marks)

10

- Integrate acceleration: $v = \int (3t + 5)\, dt = \frac{3}{2}t^2 + 5t + C$ **M1**
- Use $t = 0, v = 2 \implies C = 2$ **B1**
- Set $v = 6$ at $t = T$: $\frac{3}{2}T^2 + 5T + 2 = 6$ **M1**
- Rearrange to quadratic: $3T^2 + 10T - 8 = 0$ **M1**
- Factorise: $(3T - 2)(T + 4) = 0$ **M1**
- Since $T \geq 0$, $T = \frac{2}{3}$ **A1**

(Total for question 10 is 6 marks)

11

- Expand $s = t^3 + kt^2$ and differentiate: $v = 3t^2 + 2kt$ **M1A1**
- Use $v = 0$ at $t = 4$: $0 = 3(16) + 2k(4) \implies 48 + 8k = 0$ **M1**
- Find $k = -6$ **A1**
- Differentiate v to find a: $a = 6t + 2k = 6t - 12$ **M1A1**
- Substitute $t = 10$: $a = 6(10) - 12 = 48 \text{ ms}^{-2}$ **M1A1**

(Total for question 11 is 8 marks)

12

- Set $v = 0$: $6t - 2t^2 = 0 \implies 2t(3 - t) = 0$ **M1**
- Identify $t = 3$ (as $t = 0$ is start) **A1**
- Integrate velocity: $s = \int (6t - 2t^2)\, dt = 3t^2 - \frac{2}{3}t^3 + C$ **M1**
- Use $t = 0, s = 0 \implies C = 0$ **B1**
- Substitute $t = 3$: $s = 3(3)^2 - \frac{2}{3}(3)^3 = 27 - 18 = 9 \text{ m}$ **A1**

(Total for question 12 is 5 marks)

13

(a)

- Expand $x = \frac{3}{4}t^4 - \frac{4}{3}t^3 + \frac{1}{2}t^2$ **M1**
- Differentiate: $v = 3t^3 - 4t^2 + t$ **M1A1**
- Set $v = 0$: $t(3t^2 - 4t + 1) = 0$ **M1**
- Factorise: $t(3t - 1)(t - 1) = 0 \implies t = 0, t = \frac{1}{3}, t = 1$ **A1**

(b)

- Calculate displacement at turning points and endpoints:
  - $t = 0, s = 0$
  - $t = 1/3, s = \frac{1}{12}(\frac{1}{9})(9(\frac{1}{9}) - 16(\frac{1}{3}) + 6) = \frac{1}{108}(1 - \frac{16}{3} + 6) = \frac{1}{108}(\frac{5}{3}) = \frac{5}{324}$
  - $t = 1, s = \frac{1}{12}(1)(9 - 16 + 6) = -\frac{1}{12}$
  - $t = 2, s = \frac{1}{12}(4)(9(4) - 16(2) + 6) = \frac{1}{3}(36 - 32 + 6) = \frac{10}{3}$ **M1**
- Calculate distances:
  - 0 to $1/3$: $5/324$
  - $1/3$ to 1: $|-1/12 - 5/324| = |-27/324 - 5/324| = 32/324$
  - 1 to 2: $|10/3 - (-1/12)| = |40/12 + 1/12| = 41/12$ **M1**
- Total distance = $\frac{5}{324} + \frac{32}{324} + \frac{41}{12} = \frac{37}{324} + \frac{1107}{324} = \frac{1144}{324} \approx 3.53$ m **A1**

(Total for question 13 is 8 marks)

14

(a)

- Factorise the expression: $x = t^2(2t - 1)^2$ **M1**
- Since both t^2 and $(2t - 1)^2$ are squares, they are always ≥ 0, so $x \geq 0$. **A1**

(b)

- Expand $x = 4t^4 - 4t^3 + t^2$ **M1**
- Differentiate: $v = 16t^3 - 12t^2 + 2t$ **M1A1**
- Set $v = 0$: $2t(8t^2 - 6t + 1) = 0$ **M1**
- Factorise: $2t(2t - 1)(4t - 1) = 0 \implies t = 0, t = \frac{1}{2}, t = \frac{1}{4}$ **A1**

(c)

- Calculate displacement at turning points:
  - $t = 0, s = 0$
  - $t = 1/4, s = (1/16)(4(1/16) - 4(1/4) + 1) = (1/16)(1/4 - 1 + 1) = 1/64$
  - $t = 1/2, s = (1/4)(4(1/4) - 4(1/2) + 1) = (1/4)(1 - 2 + 1) = 0$ **M1**
- Calculate displacement at endpoint:
  - $t = 5, s = 25(4(25) - 4(5) + 1) = 25(100 - 20 + 1) = 25(81) = 2025$ **M1**
- Total distance = $(1/64 - 0) + (1/64 - 0) + (2025 - 0) = 2025 + 2/64 = 2025 + 1/32 = \frac{64801}{32}$ m **A1**

(Total for question 14 is 10 marks)

15

(a)

- Set $v = 0$: $16 + 6t - t^2 = 0 \implies t^2 - 6t - 16 = 0$ **M1**
- Factorise: $(t - 8)(t + 2) = 0 \implies t = 8$ (since $t \geq 0$) **A1**
- Differentiate v to find a: $a = 6 - 2t$ **M1**
- Substitute $t = 8$: $a = 6 - 2(8) = -10$ **M1**
- Magnitude is $10\ \text{ms}^{-2}$ **A1**

(b)

- Check for turning points in $0 \leq t \leq 5$: $v = 0$ at $t = 8$, so no turns. **M1**
- Integrate velocity: $s = \int (16 + 6t - t^2)\,dt = 16t + 3t^2 - \frac{1}{3}t^3 + C$ **M1**
- Distance = $s(5) - s(0) = 16(5) + 3(25) - \frac{125}{3} = 80 + 75 - \frac{125}{3} = 155 - \frac{125}{3} = \frac{465 - 125}{3} = \frac{340}{3}$ m **A1**

(Total for question 15 is 8 marks)

16

(a)

- Differentiate v to find a: $a = 3 - 4t$ **M1A1**
- Set $a = 0$: $3 - 4t = 0$ **M1**
- $t = 0.75$ s **A1**

(b)

- Set $v = 0$: $3t + 5 - 2t^2 = 0 \implies 2t^2 - 3t - 5 = 0$ **M1**
- Factorise: $(2t - 5)(t + 1) = 0 \implies t = 2.5$ s **A1**
- Integrate velocity: $s = \int (3t + 5 - 2t^2)\,dt = \frac{3}{2}t^2 + 5t - \frac{2}{3}t^3 + C$ **M1**
- Use $t = 0, s = 0 \implies C = 0$ **B1**
- Substitute $t = 2.5$: $s = \frac{3}{2}(6.25) + 5(2.5) - \frac{2}{3}(15.625) = 9.375 + 12.5 - 10.4166... = 11.4583...$ m **A1**
- Final answer $\frac{275}{24}$ m or 11.5 m (3sf) **A1**

(Total for question 16 is 9 marks)
