A Level · AQA · Maths

2 hours 21 questions

Practice Paper 1 (Pure)

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Total Marks /100

1 (a) The first three terms, in ascending powers of x, of the binomial expansion of $(25 - 4x)^{\frac{3}{2}}$ are given by

$$(25 - 4x)^{\frac{3}{2}} \approx k - 30x + \frac{6x^2}{5}$$

where k is a constant.

State the range of values of x for which this expansion is valid.

Circle your answer.

$\lvert x \rvert < \frac{4}{5}$	$\lvert x \rvert < \frac{25}{4}$	$\lvert x \rvert < 1$	$\lvert x \rvert < \frac{4}{25}$

(1 mark)

(b) Find the value of k.

Circle your answer.

4	25	1	125

(1 mark)

2 A student is searching for a solution to the equation $g(x) = 0$.

They correctly evaluate

$$g(-1) = -2 \quad \text{and} \quad g(1) = 2$$

and conclude that there must be a root between -1 and 1 due to the change of sign.

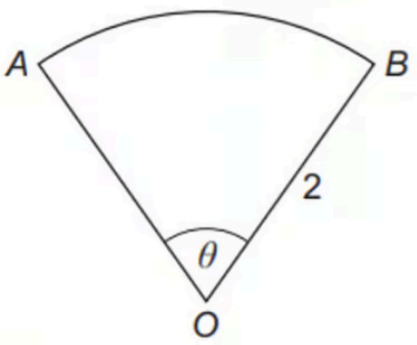
Select the function $g(x)$ for which the conclusion is **incorrect.**

Circle your answer.

$g(x) = \frac{2+4x}{x+2}$	$g(x) = 2x^3$	$g(x) = \frac{2}{x}$	$g(x) = 2x$

(1 mark)

3 The diagram shows a sector OAB of a circle with centre O and radius 2.



The angle AOB is θ radians and the perimeter of the sector is 7.

Find the value of θ.

Circle your answer.

3.5	1.5	$\frac{2}{7}$	$\sqrt{7}$

(1 mark)

4 (a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |g(x)|$ where

$$\mathrm{f}(x) = (x+2)^2 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 2x + 4 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) Solve the equation $\mathrm{f}(x) = |g(x)|$.

(2 marks)

5 Use a counter-example to prove that the difference between any two square numbers is not always odd.

(2 marks)

6 (i) Given that $\mathrm{f}(x) = 2x^2 + 5$, find $\mathrm{f}'(x)$.

(ii) Hence, or otherwise, find

$$\int \frac{4x}{2x^2+5}\,\mathrm{d}x.$$

(4 marks)

7 (a) A Fibonacci sequence can be expressed as the following recurrence relation

$$u_{n+2} = u_{n+1} + u_n, \qquad n \geq 1$$

Write down the first six terms of the Fibonacci sequence with $u_1 = u_2 = 1$.

(2 marks)

(b) Find

$$\sum_{r=1}^{5} u_r$$

with $u_1 = 2, u_2 = 4$

(2 marks)

8 (a) A lifejacket falls over the side of a boat from a height of 3 m.
The height, h m, of the lifejacket above or below sea level $(h = 0)$, at time t seconds after falling, is modelled by the equation $h = 3e^{-0.7t}\cos 4t$.

The lifejacket reaches its furthest point below sea level after 0.742 seconds.
Find the total distance it has fallen, giving your answer to three significant figures.

(2 marks)

(b) Write down the value of t for the first three times the lifejacket is at sea level.

(2 marks)

(c) (i) Find the value of $3e^{-0.7t}$ when $t = 6.2$.

(ii) Hence justify why, from 6.2 seconds on, the lifejacket will always be within 4 centimetres of sea level.

(3 marks)

9 (a) Use the factor theorem to show that $(x+1)$ is a factor of x^3-3x^2+4.

(2 marks)

(b) Hence, or otherwise, fully factorise x^3-3x^2+4.

(2 marks)

(c) Show that $\frac{19-8x}{x^3-3x^2+4}$ can be written in the form $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$, where A, B and C are integers to be found.

(4 marks)

10 (a) The 4^{th} and 8^{th} terms of an arithmetic sequence are 20 and 64 respectively.
Find the first term and the common difference.

(3 marks)

(b) The 12^{th} and 16^{th} terms of an arithmetic sequence differ by 20.
Find the possible values of the common difference.

(2 marks)

11 The third term of an arithmetic series is 32. The eleventh term is 0. The sum of the first n terms is -44.

Find the value of n.

(5 marks)

12 (a) Traffic is monitored by three average speed cameras, along a stretch of the road where the speed limit is 30 mph.

A car passes the first camera at time zero.
The car's speed and the time it passes each camera, are recorded.

The results are shown in the table below.

Camera	1	2	3
Time (hours)	0	0.25	0.5
Speed (mph)	32	38	27

Use the trapezium rule with **all** the data in the table to estimate the distance between the first and last camera.
Give your estimate to three significant figures.

(3 marks)

(b) Is the car driving within the speed limit? You must show clearly how you achieve your answer.

(2 marks)

13 (a) The drilling rig AlphaBeta began leaking oil into the North Sea following a technical fault. It took 14 hours for engineers to trace and repair the fault.

During that time the rate of oil leaking, measured in tonnes per hour, was recorded every 2 hours. The results are shown in the table below.

Time	0	2	4	6	8	10	12	14
Rate of leak	0	8	12	18	26	38	18	0

For safety reasons oil rigs are required to shut down and stop all operations until an inspection is carried out should the total amount of oil leaked during any incident exceed 250 tonnes.

Use all the data in the table to decide if the AlphaBeta rig should be shut down or not.

(3 marks)

(b) Explain why using the trapezium rule to estimate the total amount of oil spilled from AlphaBeta in the first 10 hours would be an over-estimate.

(1 mark)

14 Find the gradient of the curve with equation

$$3x^2y + 4x - y = 39$$

at the point with coordinates (2 , 3).

(4 marks)

15 (a) Verify that the point $A(1\ ,\ 1)$ lies on the curve with equation

$$\ln(xy) + xy^2 = 1.$$

(1 mark)

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C.
Find the area of the triangle OBC.

(8 marks)

16 (a) The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = 3x + 5 \qquad x \in \mathbb{R}$$

$$g(x) = -2x \qquad x \in \mathbb{R}$$

Find

(i) $fg(x)$

(ii) $gf(x)$

(4 marks)

(b) Solve the equation $f(x) = g(x)$.

(2 marks)

17 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $f: x \mapsto 2 - x^3$

(ii) $f: x \mapsto \sin x$

(iii) $f: x \mapsto \frac{1}{x^2}$

(iv) $f: x \mapsto \ln x$

(4 marks)

18 (a) The function $f(x)$ is defined as

$$f: x \mapsto \sqrt{25 - x^2} \qquad x \in \mathbb{R},\ -5 \leq x \leq 5$$

Explain why the inverse of $f(x)$ does not exist.

(1 mark)

(b) Suggest an adaption to the domain of $f(x)$ so the following conditions are met:

- the inverse of $f(x)$ exists,
- the graph of $y = f(x)$ lies in the first quadrant only, and,
- the domain of $f(x)$ is as large as possible.

State the range for your adapted $f(x)$.

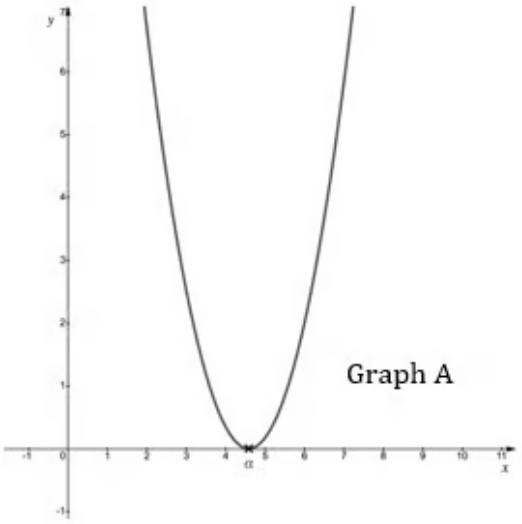
(2 marks)

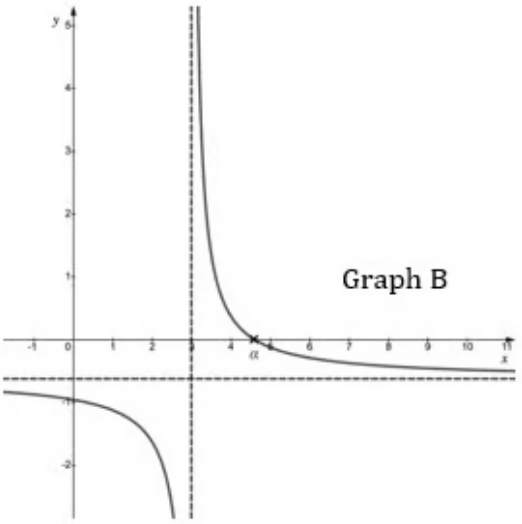
(c) The domain of $f(x)$ is changed to $-5 \leq x \leq 0$.

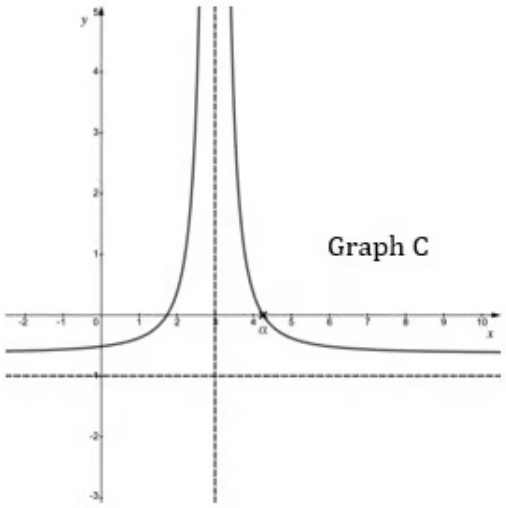
Find an expression for $f^{-1}(x)$ and state its domain and range.

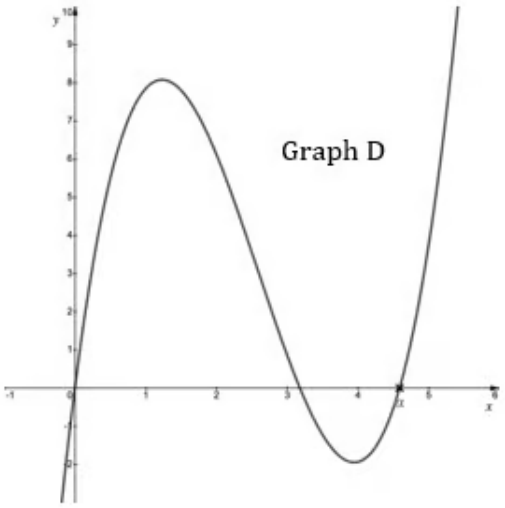
(3 marks)

19 The diagrams below show the graphs of four different functions.









Match each graph above with the correct statement below.

1. The sign change rule with values of $x = 2$ and $x = 4$ would indicate a root but has failed due to the discontinuity (asymptote) at $x = 3$.
2. The sign change rule with values of $x = 1$ and $x = 5$ would indicate no root but has failed because there are two roots in the interval (1 , 5).
3. The sign change rule with values of $x = 3$ and $x = 5$ would indicate no root but fail as there are two roots in the interval (3 , 5).
4. The sign change rule with values of $x = 3$ and $x = 5$ would indicate no root but has failed to find the root as the graph has a turning point at $x = \alpha$.

(3 marks)

20 (a) The function $f(x)$ is defined as

$$f(x) = \sin 3x - \ln 2x \quad x > 0, \text{ where } x \text{ is in radians.}$$

Find $f'(x)$.

(2 marks)

(b) Use the Newton-Raphson method with $x_0 = 0.8$ to find a root, α, of the equation $f(x) = 0$, correct to four decimal places.

(4 marks)

(c) The graph of $y = f(x)$ has a local maximum point at $x = \beta$. Briefly explain why the Newton-Raphson method would fail if the exact value of β was used for x_0.

(1 mark)

21 (a) Show that

$$\int \tan kx \, \mathrm{d}x = \frac{1}{k} \ln |\sec kx| + c$$

where k is a constant, and c is the constant of integration.

(3 marks)

(b) Use calculus to find the exact value of

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{\operatorname{cosec}^2 3\theta}{3 \cot 3\theta} \, \mathrm{d}\theta$$

writing your answer in the form $a \ln b$, where a and b are rational numbers to be found.

(5 marks)