A Level · AQA · Maths

2 hours 20 questions

Practice Paper 1 (Pure)

Scan here to return to the course
or visit savemyexams.com

Total Marks **/100**

1 State the set of values of x which satisfies the inequality

$$(x+2)(3x-1)<0$$

Tick (✓) **one** box.

$\left\{x : x < -\frac{1}{3} \text{ or } x > 2\right\}$	☐
$\left\{x : -2 < x < \frac{1}{3}\right\}$	☐
$\left\{x : x < -2 \text{ or } x > \frac{1}{3}\right\}$	☐
$\left\{x : -\frac{1}{3} < x < 2\right\}$	☐

(1 mark)

2 Given that $y = e^{5x}$ find $\frac{dy}{dx}$.

Circle your answer.

$\frac{dy}{dx} = e^{5x}$	$\frac{dy}{dx} = 5xe^{5x}$	$\frac{dy}{dx} = 5e^{5x}$	$\frac{dy}{dx} = \frac{1}{5}e^{5x}$

(1 mark)

3 A geometric sequence has a sum to infinity of $-\frac{1}{2}$.

A second sequence is formed by multiplying each term of the original sequence by -10.

What is the sum to infinity of the new sequence?

Circle your answer.

5	–5	the sum to infinity does not exist	$-\frac{1}{2}$

(1 mark)

4 Milo is attempting to use proof by contradiction to show that the result of multiplying a prime number by a multiple of 3 is never a prime number.

Select the assumption he should make to start his proof.

Tick (✓) **one** box.

Every prime multiplied by a multiple of 3 is never prime.	☐
Every prime multiplied by a multiple of 3 is prime.	☐
There exists a prime and a multiple of 3 whose product is prime.	☐
There exists a prime and a multiple of 3 whose product is not prime.	☐

(1 mark)

5 (a) The line l passes through the points (3, 4) and (9, 2).

Find the equation of the line l, giving your answer in the form $y = mx + c$.

(2 marks)

(b) Write down the gradient of a line perpendicular to l.

(1 mark)

6 The line segment AB is the diameter of a circle.

A has coordinates (-7,-9) and B has coordinates (9, 3).

Find the coordinates of the centre of the circle and the length of the diameter.

(3 marks)

7 (a) An arithmetic series is defined by

$$S_n = (k + 13) + (2k + 9) + (3k + 5) + ... + u_n + ...$$

Find an expression for n in terms of u_n and k.

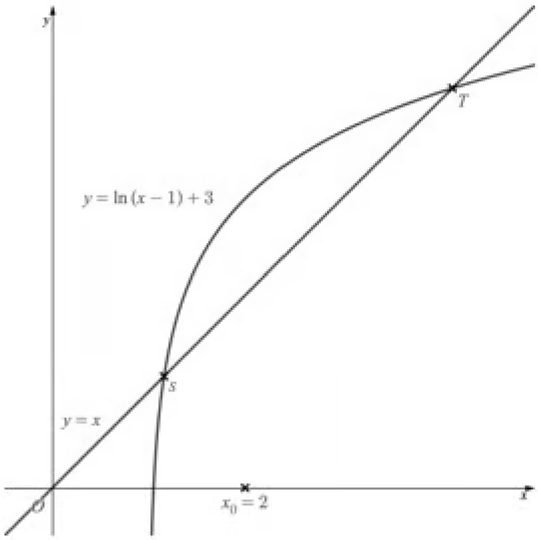
(2 marks)

(b) For a particular value of n, $u_n = -16$ and $S_n = -11$.

Find the value of k.

(5 marks)

8 (a) The diagram below shows the graphs of $y = x$ and $y = \ln(x-1) + 3$.



The iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3$$

is to be used to find an estimate for a root, α, of the function $f(x)$.

Write down an expression for $f(x)$.

(1 mark)

(b) Using an initial estimate, $x_0 = 2$, show, by adding to the diagram above, which of the two points (S or T) the sequence of estimates $x_1, x_2, x_3, \ldots$ will converge to.
Hence deduce whether α is the x-coordinate of point S or point T.

(2 marks)

(c) Find the estimates x_1, x_2, x_3 and x_4, giving each to three decimal places.

(2 marks)

(d) Confirm that α = 4.146 correct to three decimal places.

(2 marks)

9 Show that

$$\sin\theta\left(\operatorname{cosec}^2\theta - 2\right) \equiv \frac{\cos 2\theta}{\sin\theta}$$

(4 marks)

10 (a) Show that the equation $\text{cosec}^2 x = 2 \text{ cosec } x - 1$ can be written as

$$(\text{cosec } x - 1)^2 = 0$$

(2 marks)

(b) Hence, or otherwise, solve the equation

$$\text{cosec}^2 x = 2 \text{ cosec } x - 1, \qquad -2\pi \leq x \leq 2\pi$$

(3 marks)

11 (a) Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_0 and k are constants.

For an object initially containing 100g of carbon-14, write down the value of m_0.

(1 mark)

(b) Briefly explain why, if $m_0 = 100, m$ will equal 50g when $t = 5700$ years.

(2 marks)

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

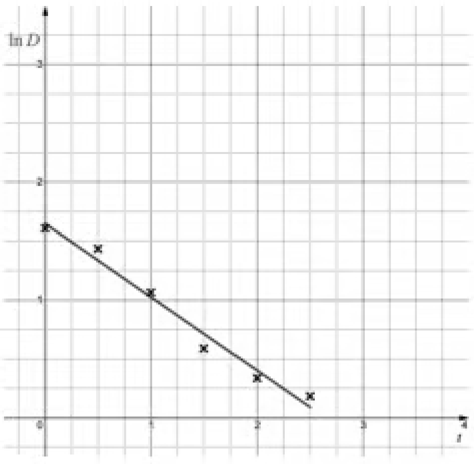
(2 marks)

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

(2 marks)

12 (a) An exponential model of the form $D = Ae^{-kt}$ is used to model the amount of a pain-relieving drug (D mg/ml) there is in a patient's bloodstream, t hours after the drug was administered by injection. A and k are constants.

The graph below shows values of $\ln D$ plotted against t with a line of best fit drawn.



(i) Use the graph and line of best fit to estimate $\ln D$ at time $t = 0$.

(ii) Work out the gradient of the line of best fit.

(2 marks)

(b) Use your answers to part (a) to write down an equation for the line of best fit in the form $\ln D = mt + \ln c$, where m and c are constants.

(1 mark)

(c) Show that $D = Ae^{-kt}$ can be rearranged to give $\ln D = -kt + \ln A$

(1 mark)

(d) Hence find estimates for the constants A and k.

(2 marks)

(e) Find the time when the amount of the pain-relieving drug in the patient's bloodstream is 1.5 mg/ml.

(2 marks)

13 Differentiate $\dfrac{5x^7}{\sin 2x}$ with respect to x.

(4 marks)

14 Use the substitution $u = 4x + 1$ to find

$$\int_2^6 4(4x+1)^{\frac{1}{2}}\,dx$$

(4 marks)

15 (a) Show that the general solution to the differential equation

$$\frac{dy}{dx} = 3x^2y, \qquad y \neq 0$$

is

$$y = Ae^{x^3}$$

where A is a constant.

(5 marks)

(b) On the same set of axes sketch the graphs of the solution for the instances where

(i) the constant A is greater than 0

(ii) the constant A is less than 0

In each case be sure to state where the graph intercepts the y-axis.

(3 marks)

16 (a) Show that the derivative function of the curve given by

$$\ln y - 2xy^3 = 8$$

is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^4}{1 - 6xy^3}.$$

(5 marks)

(b) Find the equation of the normal to the curve given in part (a) at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3 marks)

17 (a) $f(x) = 6x^3 - 19x^2 + 11x + 6$

Show that $f(x) = (2x - 3)(ax^2 + bx + c)$ where a, b and c are constants to be found.

(2 marks)

(b) Hence factorise $f(x)$ completely.

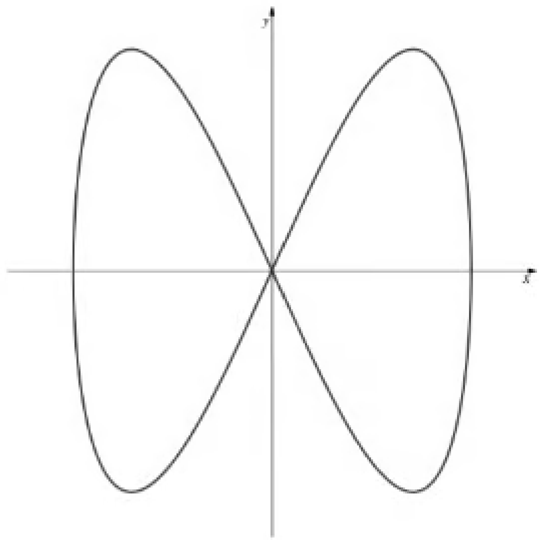
(4 marks)

(c) Write down all the real roots of the equation $f(x) = 0$.

(2 marks)

18 (a) The diagram below shows a sketch of the curve defined by the parametric equations

$$x = 3\cos t \quad y = 5\sin 2t \quad 0 \leq t \leq 2\pi$$

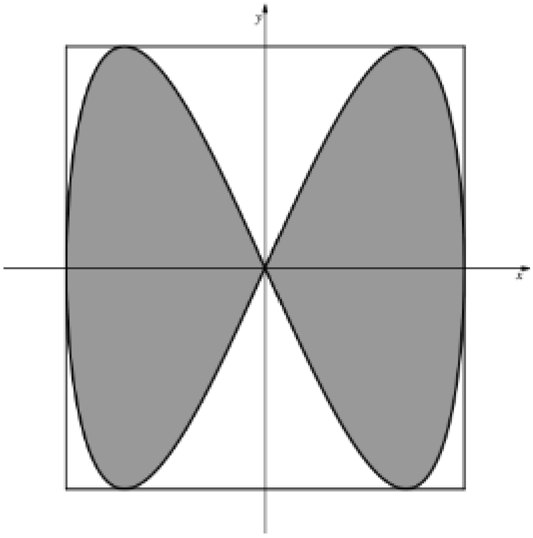


(i) Write down the equations of the two horizontal tangents to the curve.

(ii) Write down the equations of the two vertical tangents to the curve.

(2 marks)

(b) The four tangents from part (a) create a rectangle around the curve as shown below.



Find the percentage of the area of the rectangle enclosed by the curve

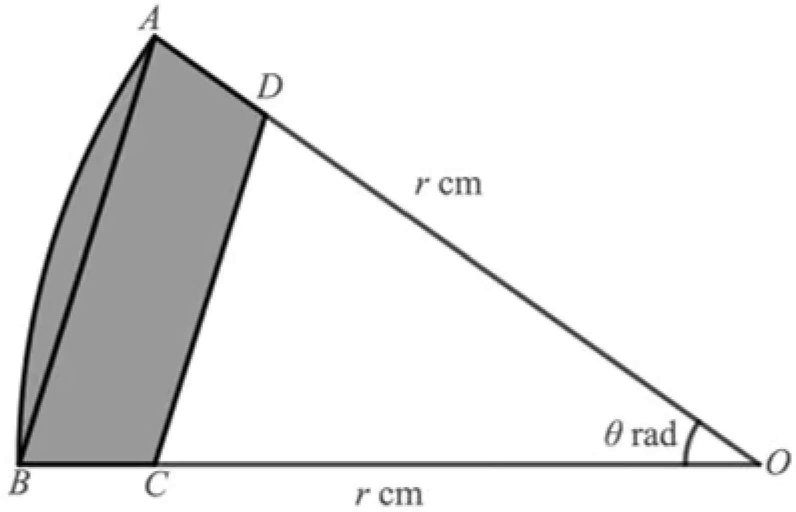
(the shaded area on the diagram).

(8 marks)

19 (a) The circle sector OAB is shown in the diagram below.

The angle at the centre is θ radians , and the radii OA and OB are each equal to r cm

Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



In the case when $AD = BC = 1$ cm, show that the area of the shaded shape $ABCD$ is given by $\frac{1}{2}\theta r^2 - \frac{1}{2}(r-1)^2 \sin\theta$.

.

(4 marks)

(b) Show that for small values of θ, the area of $ABCD$ is approximately $\frac{1}{2}\theta(2r-1)$.

(3 marks)

20 Given that θ is small, write an approximation in terms of θ for

$$\frac{\sin^2 \theta + \cos \theta}{\tan \theta + \sin \theta}$$

(3 marks)