A Level · AQA · Maths

2 hours · 19 questions

Practice Paper 1 (Pure)

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Total Marks /100

1 Given that $a > 0$, determine which of these expressions is **not** equivalent to the others.

Circle your answer.

$\log_2\left(\frac{1}{\sqrt{a}}\right)$	$\log_2 a^{\frac{1}{2}}$	$\frac{1}{2}\log_2(a^{-1})$	$-\frac{1}{2}\log_2 a$

(1 mark)

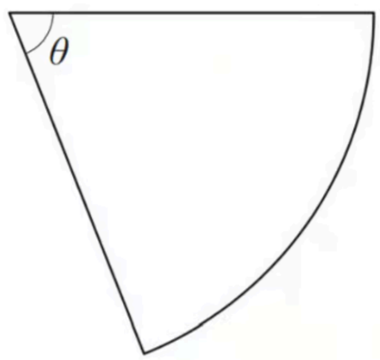
2 Given $y = e^{6x}$, find $\frac{dy}{dx}$.

Circle your answer.

$\frac{dy}{dx} = 6e^{6x}$	$\frac{dy}{dx} = 6xe^{6x-1}$	$\frac{dy}{dx} = e^{6x}$	$\frac{dy}{dx} = \frac{e^{6x}}{6}$

(1 mark)

3 The diagram below shows a sector of a circle.



The radius of the circle is 7 cm and $\theta = 0.9$ radians.

Find the area of the sector.

Circle your answer.

6.3 cm^2	44.1 cm^2	220.5 cm^2	22.05 cm^2

(1 mark)

4 (a) The line l passes through the points $(p, 2p)$ and $(3p, 9p)$.

Find an equation for the line l.

(3 marks)

(b) l intercepts the y-axis at $(0, 3)$.

Find the value of p.

(2 marks)

5 (a) The sum of the first n terms of an arithmetic series is

$$S_n = (k+7) + (2k+18) + (3k+29) + \ldots + 36$$

Show that $n = \dfrac{40}{k+11}$.

(2 marks)

(b) Hence show that the sum of the first n terms is $\dfrac{20k+860}{k+11}$.

(3 marks)

(c) Given that $S_n = 180$, find the value of k.

(1 mark)

6 (a) The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

(2 marks)

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

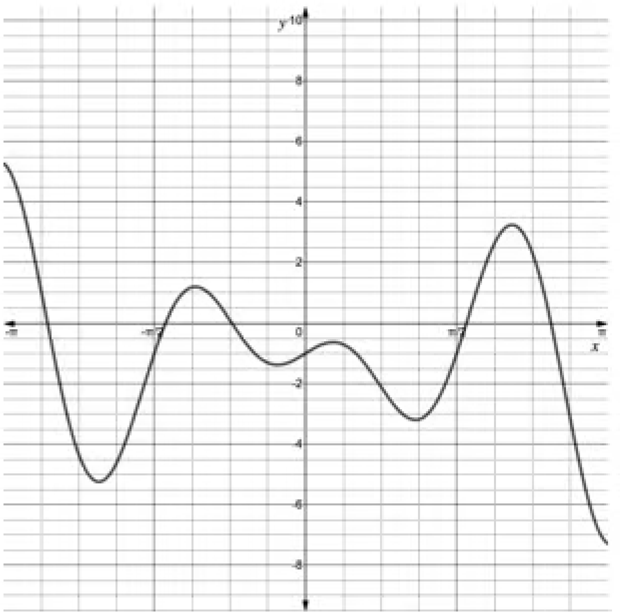
(2 marks)

(c) The domain of $f(x)$ is changed to $x > 0$.

Find an expression for $f^{-1}(x)$ and state its domain and range.

(4 marks)

7 (a) The diagram below shows part of the graph $y = f(x)$ where $f(x) = 2x\cos(3x) - 1$.



(i) Find $f(1.6)$ and $f(1.7)$, giving your answers to three significant figures.

(ii) Briefly explain the significance of your results from part (i).

(3 marks)

(b) One of the solutions to the equation $f(x) = 0$ is $x = 2.55$, correct to three significant figures.

(i) Write down the upper and lower bound of 2.55.

(ii) Hence, use the sign change rule to confirm that this is a solution (to three significant figures) to the equation $f(x) = 0$.

(3 marks)

8 (a) Show that the equation $x^3 + 3 = 5x$ can be rewritten as

$$x = \sqrt[3]{5x - 3}$$

(2 marks)

(b) Starting with $x_0 = 1.8$, use the iterative formula

$$x_{n+1} = \sqrt[3]{5x_n - 3}$$

to find a root of the equation $x^3 + 3 = 5x$, correct to two decimal places.

(3 marks)

9 A geometric series is given by $1 + 2x + 4x^2 + \dots$

(i) Write down the common ratio, r, of the series.

(ii) Given that the series is convergent, and that $\sum_{n=1}^{\infty} 2x^{n-1} = 19$, calculate the value of x.

(4 marks)

10 Prove by contradiction that $\sqrt{11}$ is an irrational number. You may use without proof the fact that if n^2 is a multiple of 11, then n is a multiple of 11.

(6 marks)

11 (a) In a computer animation, the side length, s mm, of a square is increasing at a constant rate of 2 millimetres per second.

(i) Write down the value of $\frac{\mathrm{d}s}{\mathrm{d}t}$, where t is time and measured in seconds.

(ii) Write down a formula for the area, A mm^2, of the square and hence find $\frac{\mathrm{d}A}{\mathrm{d}s}$.

(3 marks)

(b) Use the chain rule to find an expression for $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of s and hence find the rate at which the area is increasing when $s=10$.

(3 marks)

12 (a) Given that $f(x) = \sin x$

Show that

$$f'(x) = \lim_{h \to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right)$$

(4 marks)

(b) Hence prove that $f'(x) = \cos x$.

(3 marks)

13 (a) Show that the equation $3\tan^2 x = 18 - 2\sec x$ can be written as

$$3\sec^2 x + 2\sec x - 21 = 0$$

(2 marks)

(b) Hence, or otherwise, solve the equation

$$3\tan^2 x = 18 - 2\sec x, \qquad -\pi \leq x \leq \pi$$

Give your answers to three significant figures.

(4 marks)

14 (a) Show that if $y = \operatorname{cosec} 2x$, then

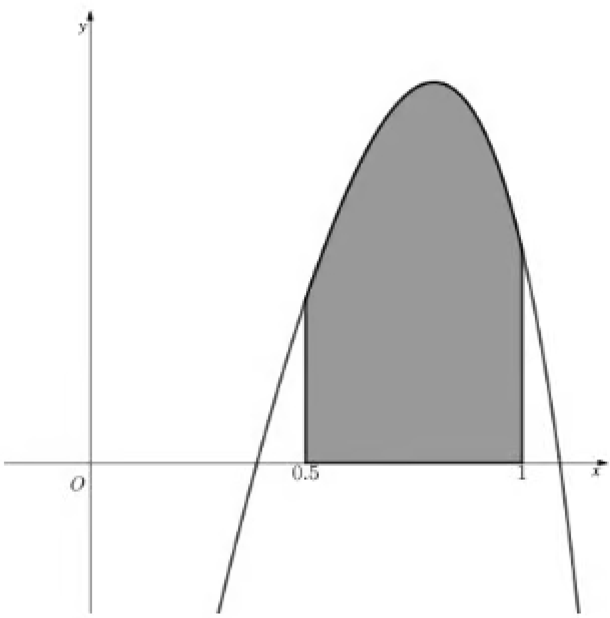
$$\frac{dy}{dx} = -2\operatorname{cosec} 2x \cot 2x$$

(5 marks)

(b) Hence find the gradient of the tangent to the curve $y = \operatorname{cosec} 2x$ at the point with coordinates $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

(1 mark)

15 The diagram below shows part of the graph with equation $y = 3x - e^{x^2}$.



Use the trapezium rule with 5 strips to find an estimate for the shaded area, giving your answer to three significant figures.

(5 marks)

16 Use a suitable substitution to find the following

$$\int 8x \sin\left(3x^2 + 1\right) \, dx$$

(5 marks)

17 (a) A soft ball is thrown upwards from the top of a building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = 15 + 8.4t - 4.9t^2 \qquad t > 0$$

What is the significance of the constant 15 in the function?

(1 mark)

(b) At what time is the ball at the same height as when it was thrown?

(2 marks)

(c) Find the time at which the ball is at its maximum height and what this maximum height is.

(3 marks)

(d) How long does it take for the ball to first hit the ground?

(2 marks)

(e) Given that the ball first hits the ground at a distance 20 m from the base of the building find the shortest distance between this point and where the ball was thrown from.

(2 marks)

18 (a) Differentiate with respect to x, simplifying your answers as far as possible:

$$(4\cos x - 3\sin x)\, e^{3x-5}$$

(3 marks)

(b) $(x^3 - 4x^2 + 7)\ln x$

(3 marks)

19 (a) Integrate

$$\int 2\sin x \cos x \ \mathrm{d}x$$

(2 marks)

(b) Use calculus to find the value of

$$\int_1^3 (4x+1)^5 \ \mathrm{d}x$$

(4 marks)