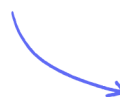


Practice Paper 2 (Pure 2)

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Total Marks /50

Solve the equation

1 $\log_3(x + 4) = 4 + 2 \log_3 x$

giving your answers correct to 3 significant figures.

(3 marks)

2 Show that $(5x - 2)$ is a factor of $25x^3 + 55x^2 - 56x + 12$.

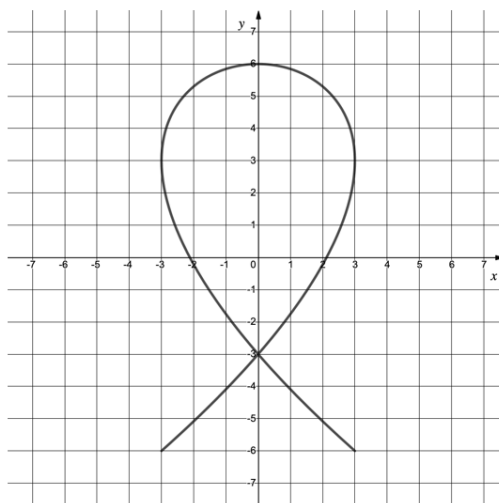
Hence find all the real solutions to the equation $25x^3 + 55x^2 - 56x + 12 = 0$.

(5 marks)

The graph of the curve C shown below is defined by the parametric equations

$$x = 3 \sin 3\theta \quad y = 6 \cos 2\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

3 (a)



(a) (i) Write down the value of $\frac{dy}{d\theta}$ at the point $(0, 6)$.

(ii) Write down the value of $\frac{dx}{d\theta}$ at the points $(-3, 3)$ and $(3, 3)$.

(2 marks)

(b) (b) Find an expression for $\frac{dy}{dx}$ in terms of θ .

(3 marks)

(a) On the same axes, sketch the graphs of $y = f(x)$ and $y = |g(x)|$ where

4 (a)

$$\begin{array}{ll} f(x) = \sqrt{x} & x \geq 0 \\ g(x) = 2x - 3 & x \in \mathbb{R} \end{array}$$

Label the points at which the graphs intersect the coordinate axes.

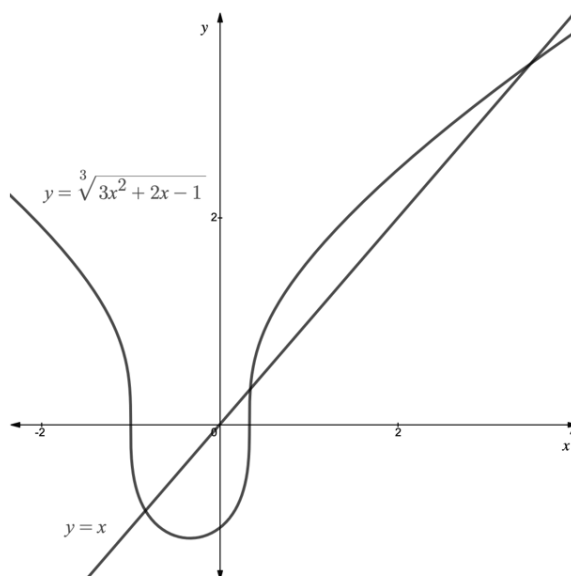
(3 marks)

(b) (b) Solve the equation $f(x) = |g(x)|$.

(3 marks)

The diagram below shows a sketch of the graphs $y = x$, and $y = \sqrt[3]{3x^2 + 2x - 1}$.

5 (a)



- (a) An iterative formula is used to find roots to the equation $x^3 - 3x^2 - 2x + 1 = 0$.
On the diagram above show that the iterative formula

$$x_{n+1} = \sqrt[3]{3x_n^2 + 2x_n - 1}$$

would converge to the root close to $x = 3.5$ when using a starting value of $x_0 = 0.5$.

(2 marks)

- (b) (i) Use $x_0 = 0.5$ in the iterative formula from part (a) to find three further approximations to the root close to $x = 3.5$.
Give each approximation correct to three significant figures.
(ii) Comment on your approximations and what they suggest about convergence to the root close to $x = 3.5$.

(2 marks)

- (c) (c) Confirm that the root close to $x = 3.5$ is 3.49 correct to three significant figures.

(3 marks)

Show that

6

$$\cos 4\theta + \cos \frac{\pi}{3} \equiv 8 \sin^4 \theta - 8 \sin^2 \theta + \frac{3}{2}$$

(5 marks)

- 7 Solve the equation $\sec^2 2x = 1 + \tan 2x$ for $0^\circ \leq x \leq 180^\circ$.

(4 marks)

(a) Show that

8 (a)

$$\left(\cos\left(\theta + \frac{\pi}{8}\right) + \sin\left(\theta + \frac{\pi}{8}\right)\right)\left(\sin\left(\theta + \frac{\pi}{8}\right) - \cos\left(\theta + \frac{\pi}{8}\right)\right) \equiv -\cos\left(2\theta + \frac{\pi}{4}\right)$$

(3 marks)

(b) Hence, or otherwise, find the exact value of

(b)

$$\int_0^{\frac{\pi}{8}} \left(\sin^2\left(\theta + \frac{\pi}{8}\right) - \cos^2\left(\theta + \frac{\pi}{8}\right)\right) d\theta$$

(3 marks)

It is given that

9 (a)
$$\frac{f(x)}{g(x)} = 2x + 3 - \frac{4}{x + 1}$$

(a) Why would assuming that $g(x) = x + 1$ be a logical first step in attempting to determine the precise forms of $f(x)$ and $g(x)$?

(1 mark)

(b) (b) By first making the assumption from part (a), find $f(x)$.

(2 marks)

(c) (c) Explain, with an example, why the forms of $f(x)$ and $g(x)$ determined in parts (a) and (b) are not the only possible forms for those functions.

(2 marks)

(a) Find the integral

10 (a)

$$\int 5(e^{5x} - e^{-5x}) \, dx$$

(2 marks)

(b) Find an expression for y given that

(b)

$$y = \int (\sin x + \cos x) \, dx$$

(2 marks)