AS · CIE · Maths

50 mins   8 questions

Practice Paper 2 (Pure 2)

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Total Marks /50

1 (a)

(a) Write $e^{2x}(1 + e^{2x} + e^2)$ in the form $e^{f(x)} + e^{g(x)} + e^{h(x)}$, where $f(x), g(x)$ and $h(x)$ are all linear functions of x.

(2 marks)

(b)

(b) Hence show that

$$\int_0^1 e^{2x}(1 + e^{2x} + e^2)\,dx = \frac{3}{4}(e^4 - 1)$$

(4 marks)

(a) On the same axes, sketch the graphs of $y = |\mathrm{f}(x)|$ and $y = |\mathrm{g}(x)|$ where

2 (a)

$$\mathrm{f}(x) = 3x - 1 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 2x + 2 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) (b) Solve the equation $|\mathrm{f}(x)| = |\mathrm{g}(x)|$.

(3 marks)

An exponential growth model for the number of bacteria in an experiment is of the form

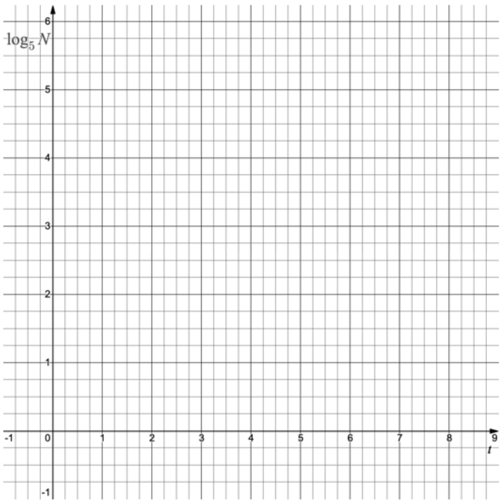
$$N = N_0 a^{kt}.$$

N is the number of bacteria and t is the time in hours since the experiment began.
N_0, a and k are constants.

A scientist records the number of bacteria at various points over a six-hour period.
The results are in the table below.

t, hours	0	2	4	6
N, no. of bacteria	200	350	600	1100

3 (a) (a) Plot the observations on the graph below - plotting $\log_5 N$ against t.
Draw a line of best fit.



(2 marks)

(b) (b) Find an equation for your line of best fit in the form $\log_5 N = mt + \log_5 c$.

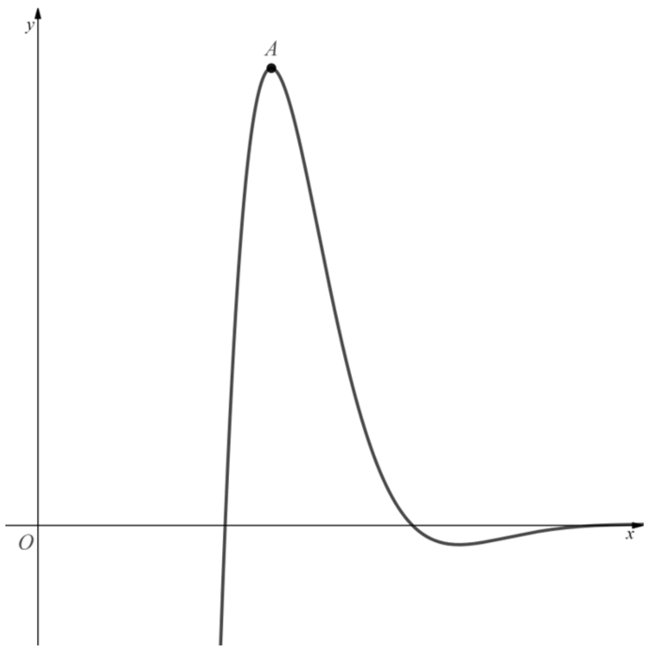
(2 marks)

(c) (c) Estimate the values of N_0, a, and k.

(2 marks)

4 The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = \frac{\sin x}{1 - e^x}, \quad x > 0$$



Point A is a maximum point on the graph.

Show that the x-coordinate of A is a solution to the equation

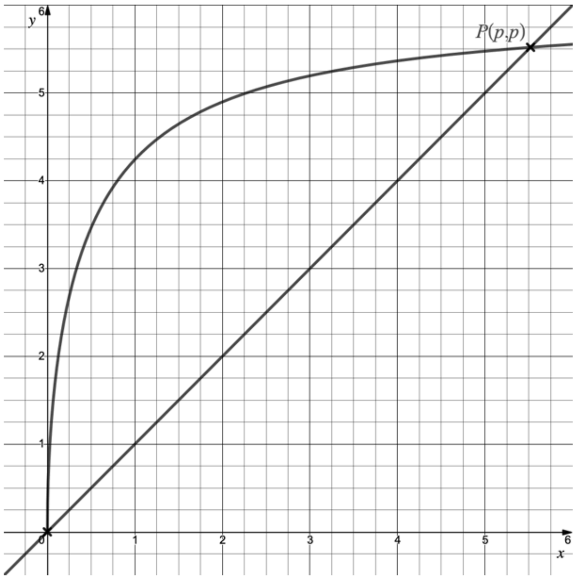
$$\frac{\cos x + e^x(\sin x - \cos x)}{e^{2x} - 2e^x + 1} = 0$$

(5 marks)

The village of Greendale lies on a straight road, as modelled by the line $y = x$ on the graph below. To ease rush hour congestion, a bypass is to be built around Greendale.

The path of the bypass is modelled by the equation $y = 6\sqrt{1 - \frac{1}{x+1}}$.

5 (a)



The bypass runs from the origin to the point $P(p, p)$.

(a) On the diagram show how using the iterative formula $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n+1}}$ with $0 < x_0 < p$ will lead to convergence at the point P.

(1 mark)

(b) (b) Use the iterative method $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n+1}}$ with $x_0 = 5$ to find the value of p, correct to two decimal places.

(3 marks)

The curve C has parametric equations

6 (a)

$$x = t^2 - 4 \qquad\qquad y = 3t$$

(a) Show that at the point $(0\,,6)$, $t = 2$ and find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at this point.

(4 marks)

(b) (b) The tangent at the point $(0\,,6)$ is parallel to the normal at the point P.
Find the exact coordinates of point P

(3 marks)

7 (a) $f(x) = x^3 + rx^2 + sx - 30$. Given that $f(2) = 0$ and $f(-3) = -240$:

(a) find the values of r and s.

(6 marks)

(b) (b) Factorise $f(x)$ completely.

(3 marks)

(a) Show that, for $\theta \neq k\pi$ (where k is an integer),

8 (a)

$$\frac{2 - 2\cos^2\theta}{\sin 2\theta} = \tan\theta$$

(3 marks)

(b) Use calculus and your result from part (a) to show that

(b)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2 - 2\cos^2\theta}{\sin 2\theta}\,\mathrm{d}\theta = \frac{1}{2}\ln 3$$

(4 marks)