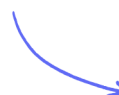


Practice Paper 2 (Pure & Mechanics)

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Total Marks

/100

1 Which one of these functions is decreasing for all real values of x ?

Circle your answer.

$h(x) = -2^{x-1}$	$h(x) = 2^x$	$h(x) = -2^{-x}$	$h(x) = -2^{1-x}$
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(1 mark)

2 Which one of the following equations has no real solutions?

Tick (✓) **one** box.

$\ln(x-2) = 0$	☐
$\operatorname{cosec} x = 0$	☐
$\cot x = 0$	☐
$ 2-x = 0$	☐

(1 mark)

3 In the expansion of $(4 - px)^6$, the coefficient of the x^4 term is 19 440.
Given that p is a positive integer find the value of p .

(3 marks)

4 (a) Given that x is small, show that

$$16 - 7 \sin x + 2 \tan 2x - 18 \cos x \approx 9x^2 - 3x - 2$$

(3 marks)

(b) Hence, or otherwise, find approximate solution(s) to the equation

$$16 - 7 \sin x + 2 \tan 2x - 18 \cos x = 0$$

(3 marks)

5 Use a suitable substitution to find

$$\int -15 \sin(5x - 2) \, dx$$

(5 marks)

6 The line with equation $y = x - 1$ intersects the circle with equation $(x - 5)^2 + (y - 4)^2 = 18$ at two distinct points.
Find the coordinates of the two points of intersection.

(5 marks)

7 Determine if the circles with equations

$$(x + 4)^2 + y^2 = 9 \text{ and } (x - 2)^2 + y^2 = 9$$

intersect once, twice or not at all. Fully explain your answer.

(3 marks)

8 Prove that the square of an odd number is always odd.

(3 marks)

9 Prove by contradiction that there are an infinite number of even numbers.

(4 marks)

- 10** (i) Find the Cartesian equation for the curve defined by the parametric equations

$$x = 4\cos \theta \quad y = 4\sin \theta$$

- (ii) Describe the shape of the curve.

(4 marks)

11 (a) The curve C has parametric equations

$$x = t^2 - 4 \quad y = 3t$$

Show that at the point $(0, 6)$, $t = 2$ and find the value of $\frac{dy}{dx}$ at this point.

(4 marks)

(b) The tangent at the point $(0, 6)$ is parallel to the normal at the point P .
Find the exact coordinates of point P

(3 marks)

- 12 (a)** A large block of ice used by sculptors is in the shape of a cuboid with dimensions x m by $2x$ m by $5x$ m. The block melts uniformly with its surface area decreasing at a constant rate of $k \text{ m}^2 \text{ s}^{-1}$. You may assume that as the block melts, the shape remains mathematically similar to the original cuboid.

Show that the rate of melting, by volume, is given by

$$\frac{15kx}{34} \text{ m}^3 \text{ s}^{-1}.$$

(5 marks)

- (b)** In the case when $k = 0.2$, the block of ice remains solid enough to be sculpted as long as the rate of melting, by volume, does not exceed $0.05 \text{ m}^3 \text{ s}^{-1}$.

Find the value of x for the largest block of ice that can be used for ice sculpting under such conditions, giving your answer as a fraction in its lowest terms.

(3 marks)

- 13** A tractor is driven at a constant speed of 8 ms^{-1} along a straight horizontal path.

Only one of the statements below is correct.

Identify the correct statement.

Tick (✓) **one** box.

The driving force of the tractor exceeds the total force resisting its motion.	☐
The resultant force acting on the tractor depends on its mass.	☐
The tractor is accelerating.	☐
The resultant force acting on the tractor is zero.	☐

(1 mark)

- 14 A number of forces act on a particle such that the resultant force is $\begin{pmatrix} -4 \\ 10 \end{pmatrix}$ N.

One of the forces acting on the particle is $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ N.

Calculate the total of the other forces acting on the particle.

Circle your answer.

$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ N	$\begin{pmatrix} -6 \\ 17 \end{pmatrix}$ N	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ N	$\begin{pmatrix} 6 \\ -17 \end{pmatrix}$ N
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(1 mark)

- 15 A particle, P , is moving with constant velocity $-3\mathbf{i} + 15\mathbf{j}$.

A second particle, Q , is moving with constant velocity $b\mathbf{i} - 20\mathbf{j}$.

Q travels in a direction which is parallel to the motion of P .

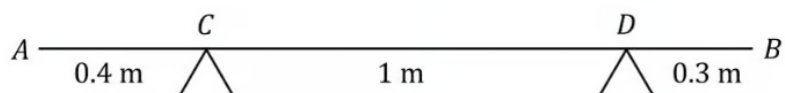
Find b .

Circle your answer.

4	8	-4	-8
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(1 mark)

- 16** AB is a uniform rod of length 1.7 m and weight 50 N. AB rests horizontally on supports placed at points C and D , with $AC = 0.4$ m, $CD = 1$ m and $DB = 0.3$ m as shown in the diagram below:



Calculate the magnitude of the reaction force at each of the support points.

(4 marks)

17 (a) The position vector of a particle at time t seconds is given by

$$\mathbf{r} = (12e^{-0.1t})\mathbf{i} + (24e^{-0.2t})\mathbf{j} \text{ m}$$

- (i) Write down the initial position of the particle.
- (ii) Briefly explain why the particle gets closer to the origin but never actually meets it.

(2 marks)

(b) Find an expression for the velocity of the particle at time t seconds.

(2 marks)

- (c) (i)** Find an expression for the acceleration of the particle at time t seconds.
- (ii) Find the magnitude of acceleration of the particle at time $t = 4$ seconds, giving your answer to three significant figures.

(3 marks)

18 (a) A TV camera runs along a horizontal track next to the pitch at a football match.

The pitch is **100 m** long and the camera is initially positioned at the halfway line.

The camera moves from rest at a constant velocity and after 5 seconds has displacement **35 m**.

It remains in this location for 3 seconds before moving back to the halfway line at a constant speed of **10 m s^{-1}** .

For the next 4 seconds the camera moves with constant velocity **-2.5 m s^{-1}** .

Sketch a displacement-time graph to illustrate the motion of the camera.

(3 marks)

- (b)** (i) Write down the final displacement of the camera and the time when it reaches this position.
- (ii) How far has the camera travelled by the time it reaches this position?

(2 marks)

- 19** A ball is projected upwards from ground level. 1.6 seconds after reaching its maximum height the ball hits the ground. Find the maximum height the ball reaches and the velocity with which it was projected. Give your answers to an appropriate degree of accuracy.

(5 marks)

- 20 (a)** The flight of a particle projected with an initial velocity of $U \text{ m s}^{-1}$ at an angle α above the horizontal is modelled as a projectile moving under gravity only. The particle is projected from the point $(0, h)$ with the upward direction being taken as positive, and with the coordinates being expressed in metres. $g \text{ m s}^{-2}$ is the constant of acceleration due to gravity.

Find, in terms of U, α, h, g and time t as appropriate, expressions for

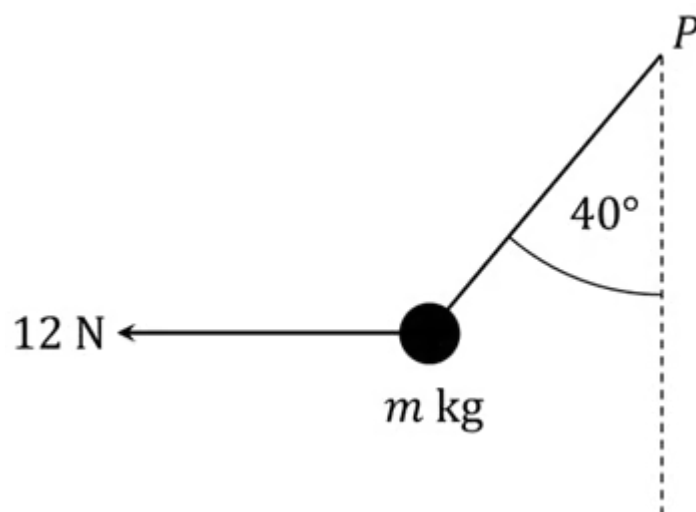
- (i) the x - coordinate of the projectile at time t seconds,
- (ii) the y - coordinate of the projectile at time t .

(2 marks)

- (b)** For a particular projectile, $\sin \alpha = \frac{8}{17}$, $U = 51 \text{ m s}^{-1}$ and the particle is projected from the point $(0, 6)$. Find an expression for the trajectory of the particle, giving your answer in the form $y = ax + bgx^2 + c$ where a, b and c are rational constants.

(4 marks)

- 21** A particle of mass $m \text{ kg}$ is suspended by a light inextensible string, with the other end of the string attached to a fixed point P . With the string at an angle of 40° to the vertical, equilibrium is maintained by a horizontal force of 12 N which acts on the particle as shown in the diagram below:

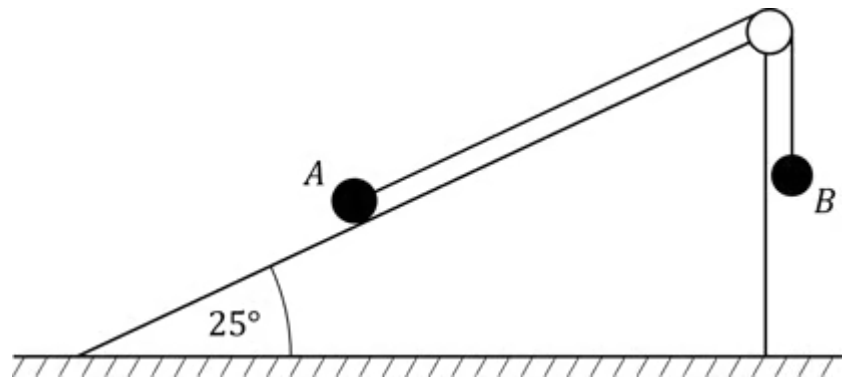


Find

- (i) the tension in the string
- (ii) the value of m .

(5 marks)

- 22** Two particles A and B , of masses 2.7 kg and 2.2 kg respectively, are connected by means of a light inextensible string. Particle A is held motionless on a rough fixed plane inclined at 25° to the horizontal. The string passes over a smooth light pulley fixed at the top of the plane so that B is hanging vertically downwards as shown in the diagram below:



The string between A and the pulley lies along a line of greatest slope of the plane, and B hangs freely from the pulley. The coefficient of friction between particle A and the plane is μ .

The system is released from rest with the string taut. Given that particle B descends 1.82 m in the first 3 seconds after it is released, find the value of μ .

(8 marks)

- 23 (a)** A remote-controlled car is driven around a large playground with velocity, $\mathbf{v}_c$ m s⁻¹, at time t seconds, given by

$$\mathbf{v}_c = (0.45t^2 + 2t - 16)\mathbf{i} + (0.75t^2 - 1)\mathbf{j}$$

- (i) The remote-controlled car is initially set in motion from position $(-6, 15)$. Find the position vector $\mathbf{r}_c$ of the car at time t seconds.
- (ii) Find the distance of the remote-controlled car from the origin after 15 seconds.

(4 marks)

- (b)** At the same time as the remote-controlled car is started, a remote-controlled truck is also set into motion. The truck has position vector, $\mathbf{r}_T$ m, at time t seconds given by

$$\mathbf{r}_T = (0.15t^3 - 6)\mathbf{i} + (0.25t^3 - 1)\mathbf{j}$$

Determine the time(s) at which the car and the truck will collide, if at all.

(3 marks)