A Level · AQA · Maths

2 hours   25 questions

Practice Paper 2 (Pure & Mechanics)

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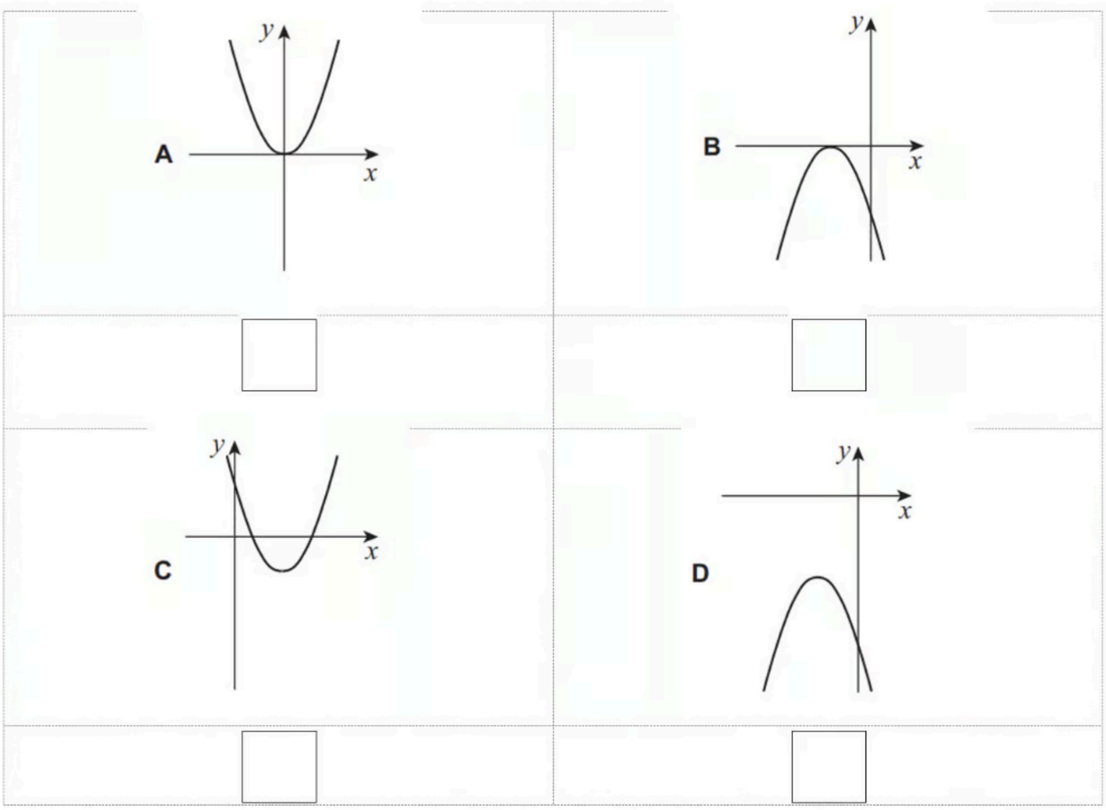
Total Marks	**/100**

1 Four possible sketches of $y = ax^2 + bx + c$ are shown below.

Given $b^2 - 4ac < 0$, which sketch is the only one that could possibly be correct?

Tick (✓) **one** box.



(1 mark)

2 A curve has equation $y = g(x)$.

The curve has a maximum point at $x = 2$ which is concave at that point.

It is given that $g'(2) = a$ and $g''(2) = b$ where a and b are real numbers.

Identify which one of the statements below must be true.

Circle your answer.

$g'(2) \neq 0$	$g''(2) \leq 0$	$g'(2) > 0$	$g''(2) > 0$

(1 mark)

3 A sequence is defined by

$$u_1 = k \text{ and } u_{n+1} = k - u_n$$

Find

$$\sum_{n=1}^{60} u_n$$

Circle your answer.

$30k$	0	$60k$	k

(1 mark)

4 (a) On the same axes, sketch the graphs of $y = |f(x)|$ and $y = |g(x)|$ where

$$f(x) = 3x - 1 \qquad x \in \mathbb{R}$$
$$g(x) = 2x + 2 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

(3 marks)

(b) Solve the equation = $|f(x)| = |g(x)|$.

(3 marks)

5 Write

$$\frac{2(x-11)}{x^2+2x-15}$$

in the form

$$\frac{A}{x+5} + \frac{B}{x-3}$$

where A and B are integers to be found.

(3 marks)

6 (a) Simplify

$$2 \ln 3^4 + \ln 3^3 - \ln 9$$

giving your answer in the form $a \ln b$, where a and b are integers to be found.

(2 marks)

(b) Write

$$2 \log_a x + 3 \log_a (x+1) - \log_a 4(x+2)$$

as a single logarithm.

(2 marks)

7 (a) Show that $x^2 + y^2 + 5x - 2y - 5 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$, where a, b and r are constants to be found.

(2 marks)

(b) Hence write down the centre and radius of the circle with equation $x^2 + y^2 + 5x - 2y - 5 = 0$.

(2 marks)

8 The line $y + 2x = 11$ meets the circle with equation $x^2 + y^2 + 6x - 14y = -38$.

(i) Show that the line and circle meet at one point only.

(ii) Find the coordinates of the point of intersection.

(4 marks)

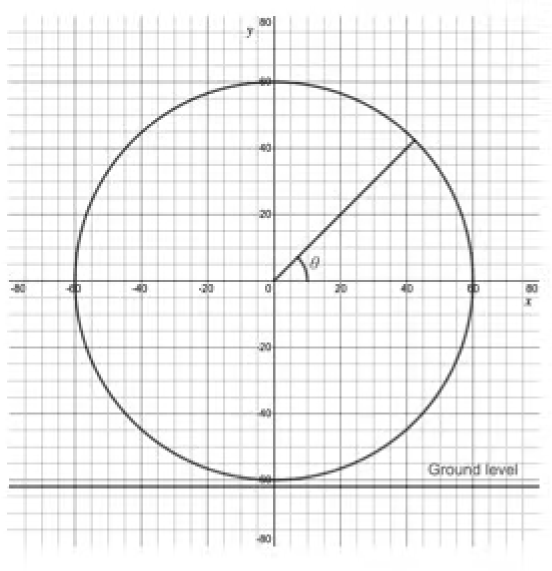
9 Prove that the sum of any three consecutive even numbers is always a multiple of 2, but not always a multiple of 4.

(3 marks)

10 Prove that the (positive) difference between an integer and its cube is the product of three consecutive integers.

(3 marks)

11 (a) A Ferris wheel with 30 passenger "pods" is modelled as a circle with centre $(0,0)$ and radius 60 m. A pod's position can be determined by the angle θ radians, which is measured anticlockwise from the positive x-direction, as shown in the diagram below.



The coordinates of a pod, (x, y) are given by $(A\cos(\theta), B\sin(\theta))$ where A and B are positive constants. Ground level is represented by the line with equation y=-62.

(i) Write down the values of A and B.

(ii) The pods are evenly distributed around the wheel.
Find the angle between each pod.

(3 marks)

(b) Find the height above the ground of a passenger pod when $\theta = \frac{7\pi}{6}$ radians.

(3 marks)

(c) Find the angle θ, to three significant figures, for a passenger pod located at the point $(48, -36)$.

(2 marks)

(d) What would you be able to say about the Ferris wheel in the case where $A \neq B$?

(1 mark)

12 State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto \tan x$

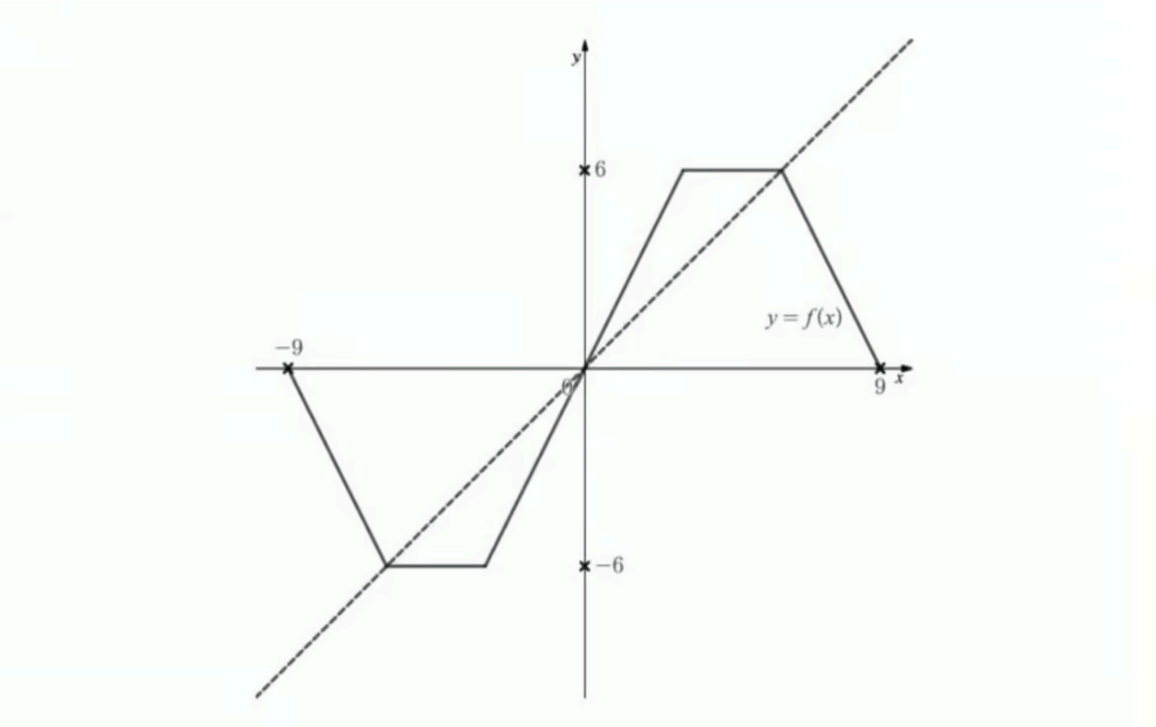
(ii) $\mathrm{f}: x \mapsto \left|\frac{1}{x}\right|$

(iii) $\mathrm{f}: x \mapsto \sqrt{x^2}$

(iv) $\mathrm{f}: x \mapsto \pm\sqrt{25 - x^2}$

(4 marks)

13 (a) The graphs of $y = f(x)$ and $y = x$ (dotted line) are shown in the diagram below. $f(x)$ has rotational symmetry about the origin and for $x > 0$, there is a vertical line of symmetry at $x = 4.5$.



(i) Use the graph to write down the domain and range of $f(x)$.

(ii) On the diagram above sketch the reflection of $f(x)$ in the line $y = x$ and explain why this cannot be the graph of $f^{-1}(x)$.

(4 marks)

(b) (i) Given that the maximum solution to $f(x) = 6$ is $x = 6$, state the restriction on the domain of $f(x)$ such that $f^{-1}(x)$ exists.

(ii) Hence, or otherwise, write down the domain and range of $f^{-1}(x)$.

(3 marks)

14 A particle's displacement, r metres, with respect to time, t seconds, is defined by the equation

$$r = 10 \sin 4t$$

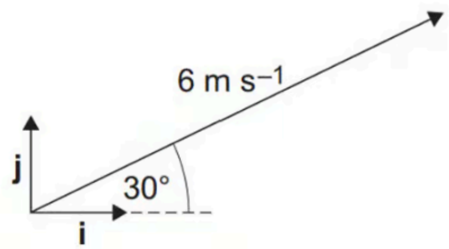
Find an expression for the velocity, v ms^{-1}, of the particle at time t seconds.

Circle your answer.

$v = -2.5 \cos 4t$	$v = 40 \sin 4t$	$v = -160 \sin 4t$	$v = 40 \cos 4t$

(1 mark)

15 A particle has a speed of 6 ms^{-1} in a direction relative to unit vectors **i** and **j** as shown in the diagram below.



The velocity of this particle can be expressed as a vector $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ms^{-1}.

Find the correct expression for v_1.

Circle your answer.

$v_1 = 6 \sin 30°$	$v_1 = -6 \cos 30°$	$v_1 = 6 \cos 30°$	$v_1 = -6 \sin 30°$

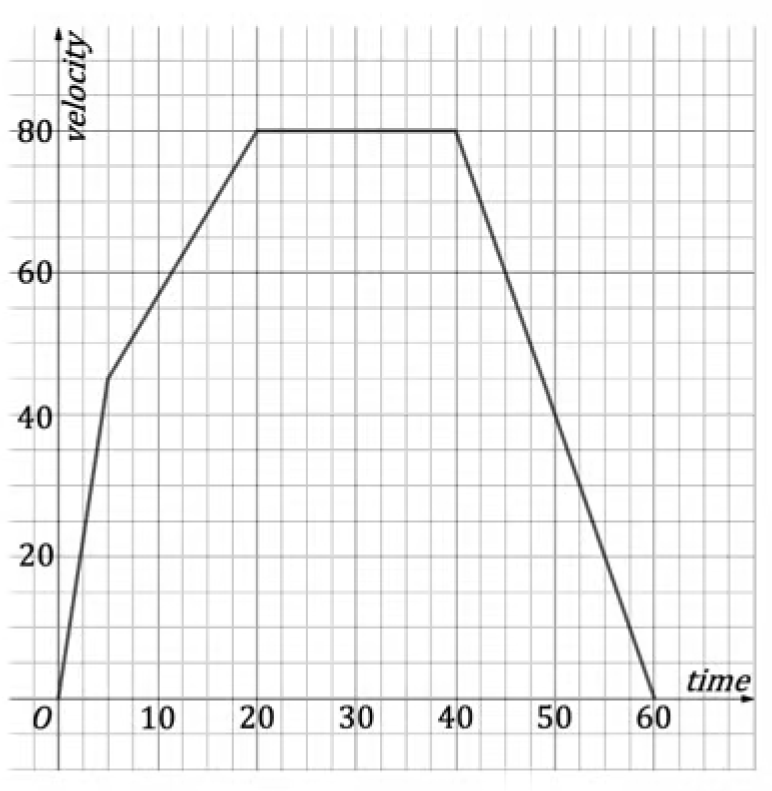
(1 mark)

16 Two particles A and B are connected by a light inextensible string. Particle A has a mass of 7 kg, particle B has a mass of 3 kg, and particle B hangs directly below particle A. A force of 120 N is applied vertically upwards on particle A, causing the particles to accelerate.

By considering particles A and B as a single object, use Newton's Second Law of Motion to find the magnitude of the acceleration.

(3 marks)

17 The diagram below shows the velocity-time graph for a train travelling between two stations, starting at station P and finishing at station Q. The graph indicates velocity in kilometres per hour and time in minutes.



(i) Find the distance between station P and station Q.

(ii) Find the deceleration of the train in the last 20 minutes.

(4 marks)

18 (a) A train locomotive of mass 7000 kg and a carriage of mass 2000 kg are at rest on a section of horizontal track. The connection between the locomotive and carriage may be modelled as a light rod parallel to the direction of their motion forward or backward along the track. The resistances to motion of the locomotive and the carriage are modelled as constant forces of 2300 N and 1000 N respectively.

The locomotive begins to accelerate in the backwards direction, with its engine providing a constant driving force of 15000 N.

Find:

the magnitude of the acceleration of the locomotive and carriage

(3 marks)

(b) the thrust in the connecting rod.

(2 marks)

19 AB is a non-uniform rod of mass 12 kg and length 4 m. AB is held horizontally in equilibrium by a support placed at point C and a vertical wire attached to point D such that $AC = 0.8$ m and $DB = 1$ m as shown in the diagram below:

The distance from point A to the centre of mass of the rod is 1.75 m.

Find the ratio of the reaction force at C to the tension in the wire at D. Give your answer in the form $p: q$ where p and q are integers with no common factors other than 1.

(4 marks)

20 (a) A high-speed train has a maximum acceleration of $0.6\ m\ s^{-2}$ which, from rest, takes 20 seconds to reach.

One such train leaves a station at $t = 0$ seconds and its displacement, s m, from the station is modelled using the equation

$$s = \frac{1}{m}t^3 \qquad 0 \le t \le 20$$

where m is a constant.

Find an expression for the velocity of the high-speed train for $0 \le t \le 20$.

(1 mark)

(b) (i) Find an expression for the acceleration of the high-speed train for $0 \le t \le 20$.

(ii) Thus find the value of the constant m, assuming that the train reaches its maximum acceleration in the quickest time possible.

(3 marks)

21 (a) A home-made rocket is launched from rest, at time t = 0 seconds, from ground level with an acceleration of 56 m s^{-2}. The rocket's acceleration is then modelled by the equation.

$$a = 56 + t - t^2 \qquad\qquad t \geq 0$$

(i) Find an expression for the velocity of the home-made rocket.

(ii) Other than at launch, find the time when the velocity of the rocket is 0 m s^{-1}.

(3 marks)

(b) Find the greatest height the rocket reaches, giving your answer in kilometres to three significant figures.

(4 marks)

22 For a particle modelled as a projectile with initial velocity $U \text{ m s}^{-1}$ at an angle of α° above the horizontal, show that the equation of the trajectory of the particle is given by

$$y = (tan\ \alpha)\ x - \frac{gx^2}{2U^2 cos^2\ \alpha}$$

(5 marks)

23 (a) The flight of a particle projected with an initial velocity of $U \text{ m s}^{-1}$ at an angle α above the horizontal is modelled as a projectile moving under gravity only. The particle is projected from the point (x_0, y_0) with the upward direction being taken as positive, and with the coordinates being expressed in metres. $g \text{ m s}^{-2}$ is the constant of acceleration due to gravity.

Write down expressions for

(i) the x-coordinate of the projectile at time t seconds

(ii) the y-coordinate of the projectile at time t seconds.

(2 marks)

(b) For a particular projectile, $\tan\alpha = \frac{3}{4}$, $U = 10 \text{ m s}^{-1}$ and the particle is projected from the point $(3, 8)$. Find an expression for the trajectory of the particle, giving your answer in the form

$$y = \frac{ax^2 + bx + c}{128}$$

where the constants a, b and c are expressed in terms of g.

(4 marks)

24 A particle moves with acceleration $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ m s^{-2} and after 7 seconds of motion has velocity $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ m s^{-1}. Find the displacement of the particle in this time.

(3 marks)

25 (a) Two stones are slid across a large icy pond. The first stone is released from rest at the origin with constant acceleration $(2\mathbf{i} + 3\mathbf{j})\ \mathrm{m\,s^{-2}}$. The second stone is released from rest from the position with coordinates $(50,\ -100)$ with constant acceleration $(\mathbf{i} + 5\mathbf{j})\ \mathrm{m\,s^{-2}}$.

(i) Find the position vectors of both stones after 5 seconds.

(ii) Find the distance between the two stones after 5 seconds.

(5 marks)

(b) Show that the two stones collide after 10 seconds.

(2 marks)