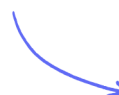


Practice Paper 3 (Pure 3)

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Total Marks

/75

1 Solve the equation $|x^2 - 4| = 3$, giving your answers in exact form.

(3 marks)

- 2 (a)** Show that $2 \sin \theta + 4 \cos \theta$ can be written as $2\sqrt{5} \cos(\theta - \alpha)$, where $\alpha = 0.464$ to three significant figures.

(4 marks)

- (b)** Hence solve the equation

$$2 \sin \theta + 4 \cos \theta = 3 \quad -\pi \leq \theta \leq \pi$$

giving your answers correct to 3 significant figures.

(3 marks)

- 3** Differentiate $\frac{5x^7}{\sin 2x}$ with respect to x .

(4 marks)

- 4** Use calculus and the substitution $x = \cos \theta$ to find the exact value of

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} dx$$

(5 marks)

5 Solve the equation

$$8 \cos^4 \theta - 5 \cos 2\theta - 2 = 0 \quad 0 \leq \theta \leq \pi$$

State your answers as multiples of π .

(5 marks)

6 (a) The functions $f(x)$ and $g(x)$ are given as follows

$$f(x) = (4 + 3x)^{\frac{1}{2}} \qquad g(x) = (9 - 2x)^{-\frac{1}{2}}$$

Expand $f(x)$, in ascending powers of x up to and including the term in x^2 .

(2 marks)

(b) Expand $g(x)$, in ascending powers of x up to and including the term in x^2 .

(2 marks)

(c) Find the expansion of $\sqrt{\frac{4+3x}{9-2x}}$ in ascending powers of x , up to and including the term in x^2 .

(2 marks)

(d) Find the values of x for which your expansion in part (c) is valid.

(2 marks)

7 (a) A tree disease is spreading throughout a large forested area.

When the disease was first discovered, three trees were infected.

Ten days later ten trees were infected.

The differential equation

$$\frac{1}{t} \frac{dN}{dt} = kN$$

where k is a positive constant, is used to model the number of infected trees, N , at a time t days after the disease was first discovered.

Find the particular solution to the differential equation.

(7 marks)

- (b)** Scientists believe the majority of the forest can be saved from infection if action is taken before 30 trees are infected.

Measured from the time when the disease was first discovered, how many days does the model predict the scientists have to take action in order to save the majority of the forest from infection?

(4 marks)

8 Use integration by parts to find

$$\int (2x^2 - 1)e^x \, dx$$

(5 marks)

9 (a) Show that the equation $x^3 + 3 = 5x$ can be rewritten as

$$x = \sqrt[3]{5x - 3}$$

(2 marks)

(b) Starting with $x_0 = 1.8$, use the iterative formula

$$x_{n+1} = \sqrt[3]{5x_n - 3}$$

to find a root of the equation $x^3 + 3 = 5x$, correct to two decimal places.

(3 marks)

10 Determine whether the following pairs of lines intersect, are parallel, or are skew.

If the lines intersect, find the coordinates of the point of intersection.

$$\mathbf{r} = -7\mathbf{i} - \mathbf{j} + 6\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \text{ and } \mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + 11\mathbf{k} + t(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}).$$

(5 marks)

11 Find the perpendicular distance of the point $P(6, 0, 3)$ to the line

$$\mathbf{r} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k} + s(3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k})$$

(5 marks)

12 (a) $f(z) = z^4 - 2z^3 + 6z^2 + pz + 125$, where p is a real constant.

Given that $3 - 4i$ is a root of the equation $f(z) = 0$, show that $z^2 + 4z + 5$ is a factor of $f(z)$.

(4 marks)

(b) Hence find the value of p and solve completely the equation $f(z) = 0$.

(3 marks)

13 On an Argand diagram, shade the region which satisfies both of the following inequalities:

$$|z - 4i| > |z| \quad \text{and} \quad |z - 4i| < |z + 2|$$

(5 marks)