A Level · CIE · Maths

1 hour 14 questions

Practice Paper 3 (Pure 3)

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Total Marks /75

1 The functions $f(x), g(x)$ are defined as follows

$$f(x) = |x-2| - 5 \qquad x \in \mathbb{R}$$
$$g(x) = |x| \qquad x \in \mathbb{R}$$

Sketch the graph of $y = gf(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

(4 marks)

2 Given that $|z+1-2i| = |z-7+4i|$:

(i) On an Argand diagram, sketch the locus (i.e., set of points) for which the equation is true.

(ii) Shade the region of your diagram that satisfies the inequality $|z+1-2i| > |z-7+4i|$.

(4 marks)

3 (a) Find an expression for $\frac{dy}{dx}$ in terms of t for the parametric equations

$$x = \sin 2t \quad y = e^t$$

(3 marks)

(b) Verify that the graph of x against y passes through the point (0 , 1) and find the gradient at that point.

(2 marks)

4 Solve the equation

$$2\log_x(x + 2) = 3$$

giving your answer correct to 3 significant figures.

(3 marks)

5 Solve the equation

$$5^{2x} - 8 \times 5^x + 12 = 0,$$

giving your answers in the form $\log_a b$.

(3 marks)

6 (a) The function, $f(x)$ is defined by $f(x) = \frac{1}{e^x} - x + 1 \qquad x \in \mathbb{R}$

Show that the equation $f(x) = 0$ can be written in the form

$$x = e^{-x} + 1$$

(2 marks)

(b) On the same diagram sketch the graphs of $y = x$ and $y = e^{-x} + 1$.

(2 marks)

(c) The equation $f(x) = 0$ has a root, α, close to $x = 1$.
The iterative formula $x_{n+1}=e^{-x_n}+1$ with $x_0=2$ is to be used to find correct to three significant figures.

Show, using a diagram and your answer to part (b), that this formula and initial x value will converge to the root α.

(2 marks)

(d) (i) Find the values of x_1, x_2 and x_3, giving each correct to three significant figures.

(ii) How many iterations are required before x_n and x_{n-1} agree to two decimal places?

(3 marks)

7 (a) Use the identity

$$R\sin(A+B) \equiv R\cos B \sin A + R\sin B \cos A$$

to show that

$$3\sin\theta + 4\cos\theta$$

can be written as

$$5\sin(\theta+\alpha), \qquad \text{where } \alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

(4 marks)

(b) Hence, or otherwise, solve the equation $3\sin\theta + 4\cos\theta = 1$ for $0 \leq \theta \leq \pi$.
Give your answers to three significant figures.

(3 marks)

8 (a) Show that $\alpha = -1 + 4i$ is a root of the cubic equation

$$z^3 + 5z^2 + 23z + 51 = 0$$

(3 marks)

(b) Find the other two roots of the cubic equation in part (a), being sure to show clear algebraic working.

(3 marks)

9 Find the particular solution to the following differential equation, using the given boundary conditions.

$$\sin^2 x \frac{\mathrm{d}y}{\mathrm{d}x} = \cos^2 y \qquad x = \frac{\pi}{4}, y = 0$$

(6 marks)

10 (a) Express $\frac{2(2-5x+x^2)}{(x+2)(2-x)^2}$ in partial fractions.

(3 marks)

(b) Express $\frac{2(2-5x+x^2)}{(x+2)(2-x)^2}$ as the first three terms of a binomial expansion in ascending powers of x.

(5 marks)

11 (a) Write $e^{2x}(1 + e^{2x} + e^2)$ in the form $e^{f(x)} + e^{g(x)} + e^{h(x)}$, where $f(x)$, $g(x)$ and $h(x)$ are all linear functions of x.

(2 marks)

(b) Hence show that

$$\int_0^1 e^{2x}(1 + e^{2x} + e^2)\, dx = \frac{3}{4}(e^4 - 1)$$

(4 marks)

12 The coordinates of three points are A(—3, 4, 0), B(2, —2, —3) and C(—l, —1, —4).

Calculate the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$. Give your answer in degrees, accurate to 1 decimal place.

(5 marks)

13 The coordinates of three points are A(—2, 3, —5), B(2, —5, —9) and C(—10, —1, —1).

The point M is the midpoint of AB, and the point N lies on BC.

Given that $\left|\overrightarrow{BN}\right| = 3\left|\overrightarrow{NC}\right|$, find the equation of the line through points M and N in vector form.

(6 marks)

14 Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[a^{kx}\right] = ka^{kx} \ln a$$

where a and k are constants.

(3 marks)