A Level · CIE · Maths

1 hour 12 questions

Practice Paper 3 (Pure 3)

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Total Marks /75

1 (a) Solve the equation

$$4^{3x+2} = 16^{x+6}$$

(2 marks)

(b) Solve the equation

$$4^{2x+3} - 8 = 92$$

giving your answer to 3 significant figures.

(3 marks)

2 Find, in ascending powers of x, the binomial expansion of

$$\frac{1}{(4+8x)^2}$$

up to and including the term in x^3.

(3 marks)

3 (a) Show that the equation $3\tan^2 x = 18 - 2\sec x$ can be written as

$$3\sec^2 x + 2\sec x - 21 = 0$$

(2 marks)

(b) Hence, or otherwise, solve the equation

$$3\tan^2 x = 18 - 2\sec x, \qquad -\pi \leq x \leq \pi$$

Give your answers to three significant figures.

(4 marks)

4 (a) Show that if $y = \text{cosec}2x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\text{cosec}\,2x\cot 2x$$

(5 marks)

(b) Hence find the gradient of the tangent to the curve $y = \text{cos}ec\,2x$ at the point with coordinates $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

(1 mark)

5 (a) Express

$$\frac{x^2 - 4x + 7}{(x-1)(x-3)^2}$$

as partial fractions.

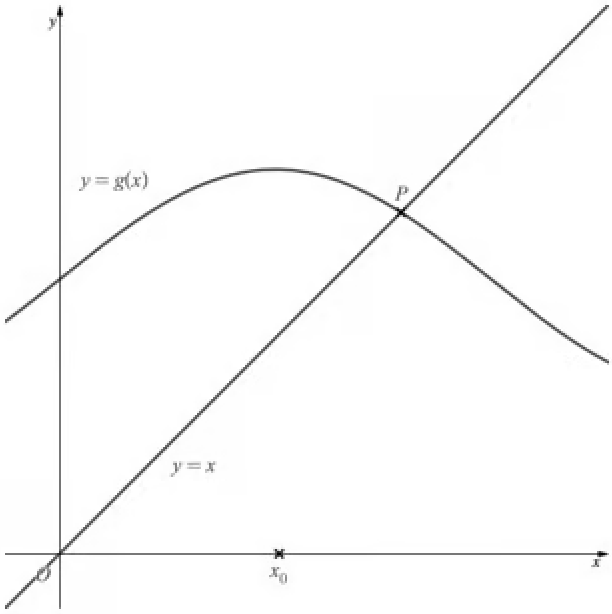
(4 marks)

(b) Hence, or otherwise, find

$$\int \frac{x^2 - 4x + 7}{(x-1)(x-3)^2}\, \mathrm{d}x$$

(3 marks)

6 (a) The diagram below shows the graphs of $y = x$ and $y = g(x)$.



Show on the diagram, using the value of x_0 indicated, how an iterative process will lead to a sequence of estimates that converge to the x-coordinate of the point P.
Mark the estimates x_1 and x_2 on your diagram.

(2 marks)

(b) By finding a suitable iterative formula, use $x_0 = 2$ to estimate a root to the equation $x - \sin 0.8x = 2.5$ correct to two significant figures.

(3 marks)

(c) Confirm that your answer to part (b) is correct to two significant figures.

(2 marks)

7 (a) Show that

$$\int \tan kx \, \mathrm{d}x = \frac{1}{k} \ln |\sec kx| + c$$

where k is a constant, and c is the constant of integration.

(3 marks)

(b) Use calculus to find the exact value of

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{\operatorname{cosec} 3\theta}{3 \cot 3\theta} \, \mathrm{d}\theta$$

writing your answer in the form $a \ln b$, where a and b are rational numbers to be found.

(5 marks)

8 A large container of water is leaking at a rate directly proportional to the square of the volume of water in the container.

(i) Given that the initial volume of water in the container is 4000 litres and that after 10 minutes the volume of water in the container has dropped by 30%, write down and solve a differential equation connecting the volume, V, of water in the container to the time, t.

(ii) What does your solution predict will happen to the volume of water in the container after a very long time?

(8 marks)

9 (a) The coordinates of three points are $A(2,-1,5)$, $B(4,1,1)$ and $C(-2,-5,9)$.

Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

(2 marks)

(b) By considering the scalar product $\overrightarrow{BA} \cdot \overrightarrow{BC}$, or otherwise, calculate the angle between $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Give your answer in degrees, accurate to 1 decimal place.

(3 marks)

10 (a) Given that $z_1 = 5 + 7i$ and $z_2 = 2 - i$:

Work out $z_1 \times z_2^*$ and $z_2 \times z_2^*$

(3 marks)

(b) Hence or otherwise work out $\frac{z_1}{z_2}$, giving your answer in the form $a + bi$ where a and b are real numbers.

(2 marks)

11 (a) On an Argand diagram, sketch the loci (i.e., sets of points) for which each of the following equations is true:

(i) $\arg(z+2-2i)=\dfrac{\pi}{4}$

(ii) $|z-3-2i|=5$

(4 marks)

(b) Shade the region of your diagram that satisfies both of the following inequalities:

$$0\leq \arg(z+2-2i)\leq \frac{\pi}{4} \quad \text{and} \quad |z-3-2i|\leq 5$$

(2 marks)

12 (a) The rare Leaping Unicorn jumps in such a way that the length of a jump is always the same distance. However, the maximum height a Leaping Unicorn reaches during a jump reduces gradually over time as the unicorn tires.

The way in which Leaping Unicorns jump can be modelled by the function

$$h(x) = |A(e^{-kx})\sin x| \qquad x \geq 0$$

where x is the horizontal distance covered and h is the height, both measured in metres.
A and k are both positive constants.

(i) Write down the length of a Leaping Unicorn jump.

(ii) Briefly describe how changing the value of the constant k would affect the model.

(2 marks)

(b) During its first jump, a Leaping Unicorn reaches a maximum height of 1.288 metres after covering 1.471 metres over the ground.
Find the values of A and k.

(5 marks)

(c) What is the total distance of ground covered by a Leaping Unicorn when it is at the maximum height of its third jump?

(2 marks)