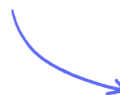


Practice Paper 3 (Pure & Statistics)

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Total Marks

/100

1 Given that

$$\int_0^8 g(x) \, dx = 5$$

deduce the value of

$$\int_0^8 (g(x) + 1) \, dx$$

Circle your answer.

5	13	6	-3
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(1 mark)

2 Given that

$$5 \cos \alpha - 12 \sin \alpha \equiv R \cos(\theta + \alpha)$$

find the value of R .

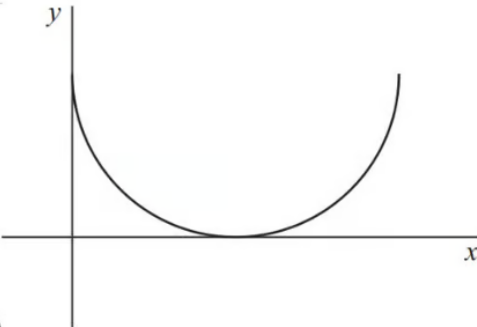
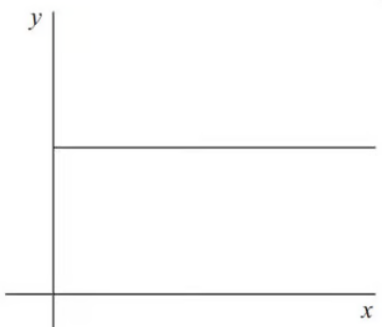
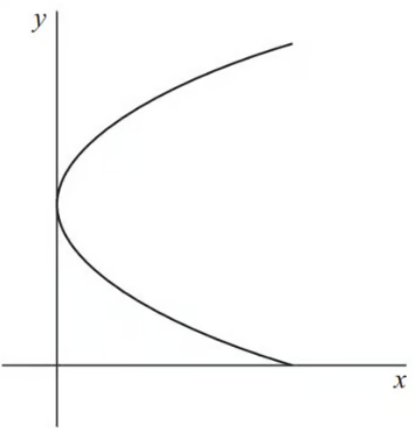
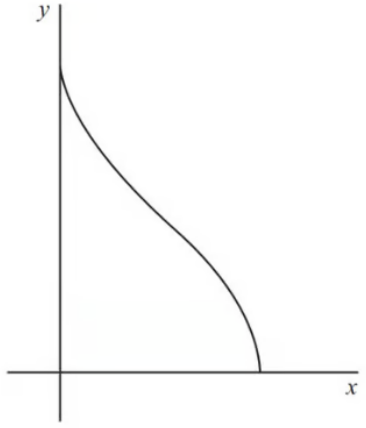
Circle your answer.

17	13	5	12
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(1 mark)

3 Determine which one of these graphs represents a function whose inverse function exists.

Tick (✓) **one** box.

A  ☐	B  ☐
C  ☐	D  ☐

(1 mark)

4 (a) $f(x) = 4x^3 + 4x^2 - 23x - 30$

Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2 marks)

(b) Factorise $f(x)$ completely.

(4 marks)

(c) Write down all the real roots of the equation $f(x) = 0$.

(2 marks)

- 5 (a)** In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.

The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

Write down the number of birds the scientists released into the nature reserve.

(1 mark)

- (b)** According to this model, how many birds will be in the reserve after 3 years?

(2 marks)

- (c)** How long will it take for the population of birds within the reserve to reach 500?

(2 marks)

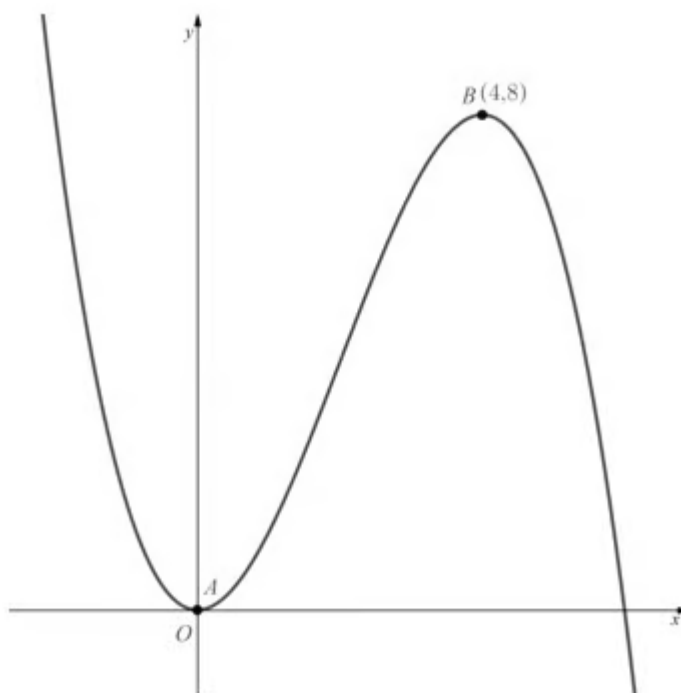
6 (a) Write $\left(\frac{3}{5}\right)^x$ in the form e^{kx} , giving the value of k to three significant figures.

(2 marks)

(b) Write $\left(\frac{4}{7}\right)^{3t}$ in the form e^{kt} , giving the value of k to three significant figures.
State, and justify, whether this would represent exponential growth or decay.

(2 marks)

- 7 (a)** The diagram below shows the graph of $y = f(x)$. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A .



In separate diagrams, sketch the curves with equation

- (i) $y = -f(x)$
- (ii) $y = f(4x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation.

(4 marks)

- (b) On the graph of $y = af(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a .

(2 marks)

- 8 Show that, for all values of k ,

$${}^kC_1 = k$$

(2 marks)

- 9 In the expansion of $(a - x)^4$, the coefficient of the x^2 term is 96.
Given that $a > 0$, find the value of a .

(2 marks)

- 10 Given that ${}^nC_3 = 35$ find the value of n .

(3 marks)

- 11 (a)** A geometric sequence has first term 900 and a common ratio r where $r > 0$. The 18th term of the sequence is 18.

Show that r satisfies the equation $17 \log r + \log 50 = 0$.

(3 marks)

- (b)** Hence or otherwise find the value of r correct to 3 significant figures.

(1 mark)

- 12 (a)** A geometric series has first term 9, and the sum of the first three terms of the series is 19. The common ratio of the series is r .

Show that $9r^2 + 9r - 10 = 0$.

(3 marks)

- (b)** Find the two possible values of r .

(2 marks)

- (c)** Given that the series converges, find the sum to infinity of the series.

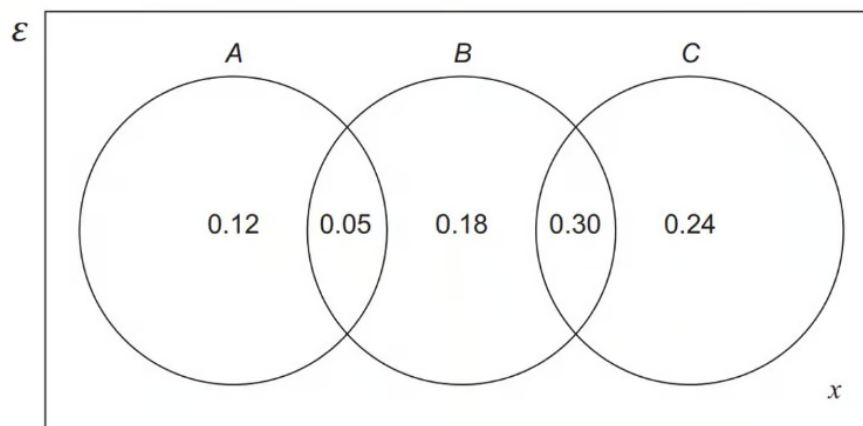
(3 marks)

- 13** Show that

$$\sin 3\theta + \sin \theta \equiv 4 \sin \theta - 4 \sin^3 \theta$$

(5 marks)

- 14 The probabilities of events A , B and C are related, as shown in the Venn diagram below.



Find $P(A \cup C)$.

Circle your answer.

0	0.36	0.71	0.35
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(1 mark)

- 15 The table below shows the heights of sunflowers at the start of each week, for 9 weeks.

Week	1	2	3	4	5	6	7	8	9
Height (cm)	2	8	15	20	26	34	44	48	53

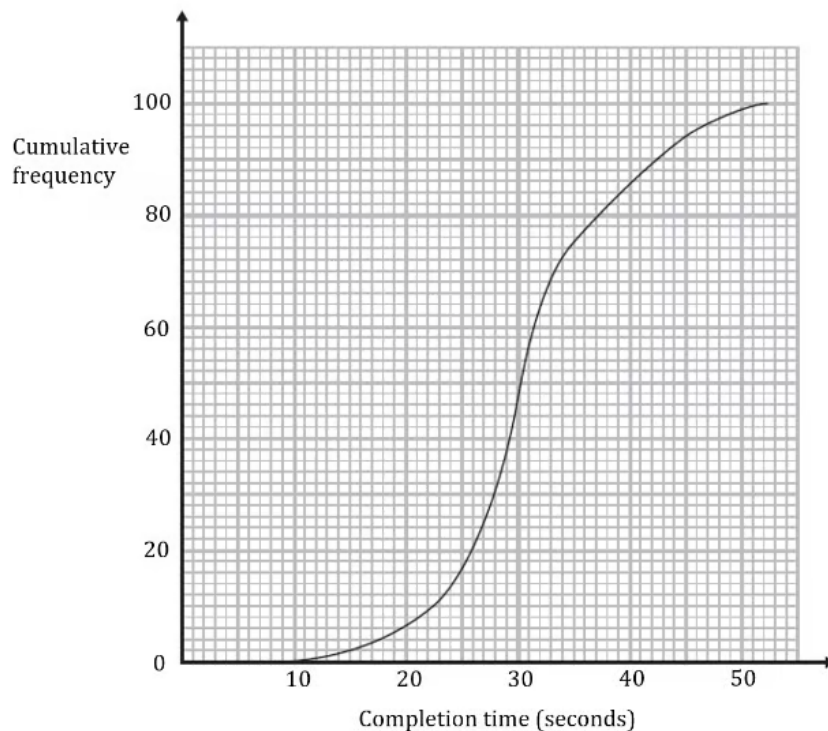
Calculate the standard deviation of these heights, correct to 1 decimal place.

Circle your answer.

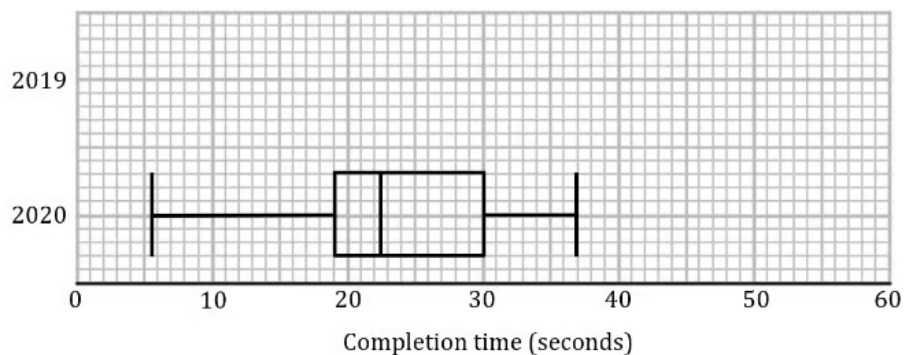
17.1	292.2	27.8	14.7
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(1 mark)

- 16 (a)** The cumulative frequency diagram below shows completion times for 100 competitors at the 2019 Rubik's cube championships. The quickest completion time was 9.8 seconds and the slowest time was 52.4 seconds.



The grid below shows a box plot of the 2020 championship data. Draw a box plot on the grid to represent the 2019 championship data.



(4 marks)

- (b)** (i) Compare the distribution of completion times for the 2019 and 2020 championships.
- (ii) Given that the 2020 championships happened after the global pandemic, during which many competitors spent months at home, interpret your findings from part (b)(i).

(3 marks)

- 17 (a)** 240 students are surveyed regarding their belief in supernatural creatures. 144 say they believe in unicorns (U). 75 say they believe in vampires (V). Of those who believe in vampires, 27 also believe in unicorns.

Draw a two-way table to show this information.

(2 marks)

- (b)** One student is chosen at random. Find:

(i) $P(U')$

(ii) $P(U' \cap V')$

(iii) $P(U|V)$

(iv) $P(V|U)$

(4 marks)

- 18 (a)** *Adrenaline* is a new rollercoaster at a theme park. It is known that the time a customer spends in the queue follows a normal distribution with a variance of 52 minutes². The mean time spent in a queue for other rollercoasters is 41 minutes. The manager of the theme park wants to use a hypothesis test to investigate whether the mean time in the queue for *Adrenaline* is different to the mean time for the other rollercoasters. She takes a sample of 10 customers over a period of several days and records their times spent in the queue for *Adrenaline*.

Find the critical region for the test at the 10% level of significance. State your hypotheses clearly.

(4 marks)

- (b)** The queuing times for the 10 people in the sample are:

38	49	40	39	49
39	59	32	55	41

State the conclusion of the test in context.

(3 marks)

- 19** A company wants to survey 15% of its staff to find out whether employees would like to continue working from home after the Covid-19 pandemic. The company's 580 members of staff are grouped by job as follows: 295 engineers, 11 managers, 154 office staff and 120 apprentices.

Suggest a suitable sampling method and explain how the company can use this method to obtain its sample.

(5 marks)

- 20 (a)** A teacher, Ms Pearman, claims that there is a positive correlation between the number of hours spent studying for a test and the percentage scored on it.

Write down suitable null and alternative hypotheses to test Ms Pearman's claim.

(1 mark)

- (b)** Ms Pearman takes a random sample of 25 students and gives them a week to prepare for a test. She records the percentage they score in the test, s %, and the amount of revision they did, h hours.

Ms Pearson calculates the product moment correlation coefficient for these data as $r = 0.874$.

Given that the p-value for the test statistic $r = 0.874$ is 0.0217, test at the 5% level of significance whether Ms Pearman's claim is justified.

(2 marks)

- 21 (a)** A machine is used to fill bags of potatoes for a supermarket chain. The weight, W kg, of potatoes in the bags is normally distributed with mean 3 kg and standard deviation σ kg.

Given that 7% of the bags contain a weight of potatoes that is at least 50 g more than the mean, find:

$$P(2.9 \leq W \leq 3.1).$$

(3 marks)

- (b)** Twelve of the bags of potatoes are chosen at random.

Find the probability that not more than one of the bags will contain less than 2.96 kg of potatoes.

(4 marks)

- 22 (a)** In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

Suggest a suitable distribution for X and comment on any necessary assumptions.

(2 marks)

- (b)** Find the probability that

- (i) 90% or more of the residents chosen prefer Woostershire brand sauce
- (ii) none of the residents chosen prefer the other well-known brand.

(3 marks)

- (c)** The pollster knows that there is a greater than 97% chance of at least k of the 30 residents preferring Woostershire brand sauce, where k is the largest possible value that makes that statement true.

Find the value of k .

(2 marks)

- 23 (a)** Vauxhall state that 8.5% of their cars produce less than 100 g/km of carbon dioxide emissions.

Elaine would like to investigate this claim using the large data set by testing the following hypotheses:

$$H_0 : p = 0.085 \qquad H_1 : p \neq 0.085$$

There are 1069 Vauxhall cars in the large data set. The critical regions are stated to be $X \leq 75$ and $X \geq 107$.

Calculate the actual level of significance based on these critical regions.

(3 marks)

- (b)** 73 Vauxhall cars out 1069 in the large data set have less than 100 g/km of carbon dioxide emissions.

State a conclusion to the hypothesis test for this value, giving a reason for your answer.

(2 marks)

- (c)** With reference to the large data set, state one limitation of your conclusion.

(1 mark)