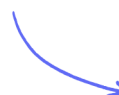


Practice Paper 3 (Pure & Statistics)

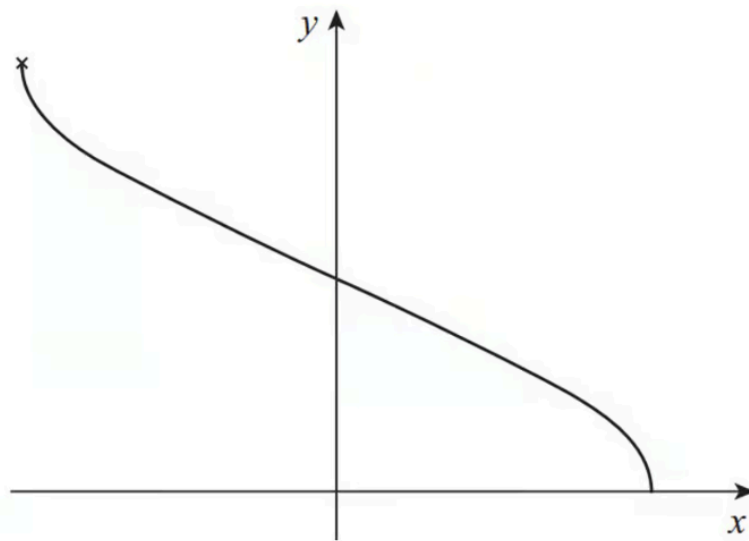
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Total Marks

/100

1 The graph of $y = \arccos x$ is shown.



State the maximum value of y on the graph.

Circle your answer.

1	π	2	2π
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(1 mark)

2 Simplify fully

$$\frac{(3x-15)(x+5)}{(5+x)(5-x)} \text{ for } x \neq \pm 5$$

Circle your answer.

-3	3	$\frac{3x-15}{5-x}$	$\frac{15-3x}{5-x}$
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(1 mark)

3 Given $f(x) = 4x^3$ obtain

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Circle your answer.

$\frac{4h^3}{h}$	$12x^2$	x^4	$\frac{4(x+h)^3 - 4x^3}{h}$
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(1 mark)

4 (a) Find the first three terms, in ascending powers of x , in the expansion of $(5 - 2x)^4$.

(3 marks)

(b) Use your answer to part (a) to estimate $(4.5)^4$.

(2 marks)

5 A sector of a circle, OPQ , is such that it has radius 3.4 cm and the angle at its centre, O , is $\frac{3\pi}{4}$ radians.

- (i) Find the length of the arc PQ .
- (ii) Find the area of the sector OPQ .

(4 marks)

6 Given that θ is small, write an approximation in terms of θ for

$$2\sin \theta + \cos \theta - \tan^2 \theta$$

(3 marks)

- 7 (a)** A company manufactures food tins in the shape of cylinders which must have a constant volume of $150\pi \text{ cm}^3$. To lessen material costs the company would like to minimise the surface area of the tins.

By first expressing the height h of the tin in terms of its radius r , show that the surface area of the cylinder is given by $S = 2\pi r^2 + \frac{300\pi}{r}$.

(2 marks)

- (b)** Use calculus to find the minimum value for the surface area of the tins. Give your answer correct to 2 decimal places.

(4 marks)

- 8** The two parallel sides of a trapezium have lengths of $(4 + \sqrt{3}) \text{ cm}$ and $(6 - 3\sqrt{3}) \text{ cm}$, and the area of the trapezium is $(10\sqrt{3} - 6) \text{ cm}^2$. Showing clear algebraic working, determine the perpendicular distance between the two parallel sides of the trapezium. Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4 marks)

- 9 (a)** Frankie opens a savings account with £400.

Compound interest is paid at an annual rate of 3%.

Show that at the end of the first year Frankie has £412 in the savings account.

(1 mark)

- (b)** At the start of the second year, and each subsequent year, Frankie adds another £400 to the savings account.

Write down the amount of interest the £400 invested at the start of year 2 will earn by the start of year 3.

(1 mark)

- (c)** Explain why the amount of money in the savings account, in pounds, at the end of year 2 can be written as

$$(400 \times 1.03) \times 1.03 + 400 \times 1.03.$$

(2 marks)

- (d)** Hence show that after n years, the amount in pounds in Frankie's savings account will be

$$400(1.03 + 1.03^2 + 1.03^3 + \dots + 1.03^n).$$

(2 marks)

- (e)** Show that the sum of the geometric series $1.03 + 1.03^2 + 1.03^3 + \dots + 1.03^n$ is given by

$$\frac{103}{3}(1.03^n - 1)$$

(2 marks)

- (f)** Hence find the amount of money in Frankie's savings account at the end of 12 years.

(2 marks)

- 10** Use integration by parts to find, in terms of e , the exact value of

$$\int_0^1 (5x - 4)e^{3x} \, dx$$

(6 marks)

- 11** A curve has the equation $y = x(x + 6)^2 + 4(3x + 11)$.

The point $P(x, y)$ is the stationary point of the curve.

Find the coordinates of P and determine its nature.

(5 marks)

12 (a) Given $y = x^3 - 6x^2$ show that $\frac{d^2y}{dx^2} = 6x - 12$.

(2 marks)

(b) Solve the inequality $\frac{d^2y}{dx^2} < 0$ and hence determine the set of values for which the graph of $y = x^3 - 6x^2$ is concave.

(2 marks)

13 Aurora has collected data from 40 similar-sized beaches to investigate any correlation between the slope of the beach and the frequency of waves.

She calculates the correlation coefficient.

Which of the following statements best describes her answer of $\frac{13}{7}$?

Tick (✓) **one** box.

Definitely incorrect	☐
Probably incorrect	☐
Probably correct	☐
Definitely correct	☐

(1 mark)

14 The random variable X is such that $X \sim B(n, p)$.

The mean value of X is 48.

The variance of X is 36.

Find p .

Circle your answer.

0.875	0.125	0.75	0.25
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(1 mark)

Sample 1				Sample 2			
Propulsion Type ID	Body Type ID	CO ₂ (g/km)	CO (g/km)	Propulsion Type ID	Body Type ID	CO ₂ (g/km)	CO (g/km)
1	2	222	0.698	1	2	187	0.375
2	3	165	0.518	1	3	165	0.518
1	6	240	0.174	1	6	163	0.518
2	6	200	0.518	2	6	159	0.09
1	13	157	0.395	1	13	178	0.656
2	13	178	0.656	2	13	145	0.347
1	14	192	0.284	1	14	165	0.518
2	14	153	0.562	1	14	192	0.427
1	14	165	0.518	2	14	145	0.109
2	96	264	0.341	1	96	264	0.341

- 15 The large data set contains data on 407 Ford cars from 2002. A selection of data relating to the CO₂ and CO emissions (g/km) of Ford cars from 2002 is given above.

The large data set includes data on a variety of variables including propulsion and body type. Roughly three quarters of the Ford cars in the data set are petrol cars.

- State which of the two samples above is more representative and why.
- If a stratified sample by body type had been used to select the data in the two samples above, roughly how many Ford cars of each type would you expect there to have been in the complete data set?

(3 marks)

- 16 (a)** As part of an experiment, 15 maths teachers are asked to solve a riddle and their times, in minutes, are recorded:

8	12	19	20	20
21	22	23	23	23
25	26	27	37	39

An outlier is an observation which lies more than ± 2 standard deviations away from the mean.

Show that there is exactly one outlier.

(4 marks)

- (b)** State, with a reason, whether the mean or the median would be the most suitable measure of central tendency for these data.

(2 marks)

17 (a) A and B are two events with $P(A) = x$ and $P(B) = y$, where $x \neq 0$. Given that A and B are independent, find the following probabilities in terms of x and y :

(i) $P(B|A')$

(ii) $P(A \cap B|A)$

(3 marks)

(b) A group of 18- to 25-year-olds, and a group of people over 65 years old, were asked whether they would prefer to holiday in Ibiza or Skegness. The following two-way table shows part of the results of the survey:

	Ibiza	Skegness	total
18-25			99
over 65			45
total	64	80	144

(i) Find $P(\text{over 65})$ and $P(\text{Ibiza})$

(ii) Given that for the people in the sample the events 'is over 65' and 'prefers to holiday in Ibiza' are independent, find the missing values and complete the table of survey results.

(4 marks)

- 18 (a)** The IQ of a student at Calculus High can be modelled as a random variable with the distribution $N(126, 50)$. The headteacher decides to play classical music during lunchtimes and suspects that this has caused a change in the average IQ of the students.

Write suitable null and alternative hypotheses to test the headteacher's suspicion.

(1 mark)

- (b)** The headteacher selects 10 students and asks them to complete an IQ test. Their scores are:

127, 127, 129, 130, 130, 132, 132, 132, 133, 138

Test, at the 5% level of significance, whether there is evidence to support the headteacher's suspicion.

(5 marks)

- (c)** It was later discovered that the 10 students used in the sample were all in the same advanced classes.

Comment on the validity of the conclusion of the test based on this information.

(1 mark)

19 (a) The random variable X has the probability function

$$P(X=x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k .

(2 marks)

(b) Find $P(X > 3)$.

(2 marks)

20 The random variable $X \sim B(50, 0.75)$. Find:

- (i) $P(40 > X \geq 30)$
- (ii) $P(X < 29 \text{ or } X > 38)$
- (iii) $P(X \leq 52 \text{ and } X > 31)$

(3 marks)

- 21 (a)** It is known that customers have an 83% success rate when attempting to order bundles of 5 or more festival tickets from a particular website. A second website claims that its customers have a greater probability of success.

State suitable null and alternative hypotheses to test the second website's claim.

(2 marks)

- (b)** In one day, 1159 out of 1358 bundles of 5 or more tickets are successfully ordered from the second website.

Test the second website's claim using a 1% significance level.

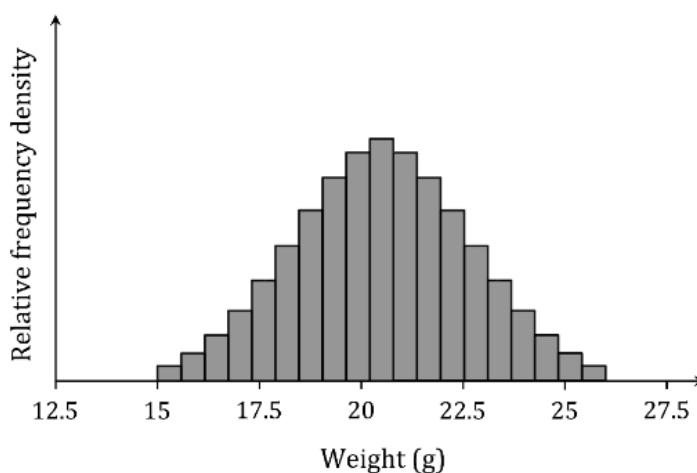
(3 marks)

- (c)** It is suggested that instead of using a 1% significance level for the test, the critical region $X \geq 1150$ could have been used for testing the second website's claim against observed data for the number of successes out of 1358 attempted orders.

Find the probability of incorrectly rejecting the null hypothesis if that critical region is used.

(3 marks)

- 22 (a)** The following histogram shows the distribution of weights, in grams, of a population of dormice in the UK:



- (i) Explain why it would be appropriate to use a normal distribution to model the distribution of weights of the dormouse population.
- (ii) Explain how the histogram could be altered so as to better approximate the smooth curve of the normal distribution.

(3 marks)

- (b)** The mean and standard deviation for the weights of the dormouse population are calculated to be 20.5 g and 2.6 g respectively. A normal curve is drawn corresponding to these values.

Write down the values of the weight that correspond to the line of symmetry and the points of inflection of the normal curve.

(3 marks)

(c) Use the properties of the normal distribution to determine whether each of the following statements is likely to be true. In each case give a reason for your answer.

- (i) 84% of the dormice have a weight that is less than 23.1 g.
- (ii) More than 99% of the dormice have a weight that is between 15.3 g and 28.3 g.
- (iii) 18.5% of the dormice have a weight that is either less than 17.9 g or greater than 25.7 g.

(4 marks)