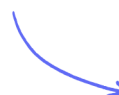


Practice Paper 3 (Pure & Statistics)

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Total Marks /100

1 $f(x) = \arctan x$

State the maximum possible domain of f .

Tick (✓) **one** box.

$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$	☐
$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$	☐
$\{x \in \mathbb{R} : -\pi \leq x \leq \pi\}$	☐
$x \in \mathbb{R}$	☐

(1 mark)

2 Find the value of the below expression.

$$\frac{120!}{118! \times 4!}$$

Circle your answer.

$\frac{15}{59}$	Undefined	595	70210
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(1 mark)

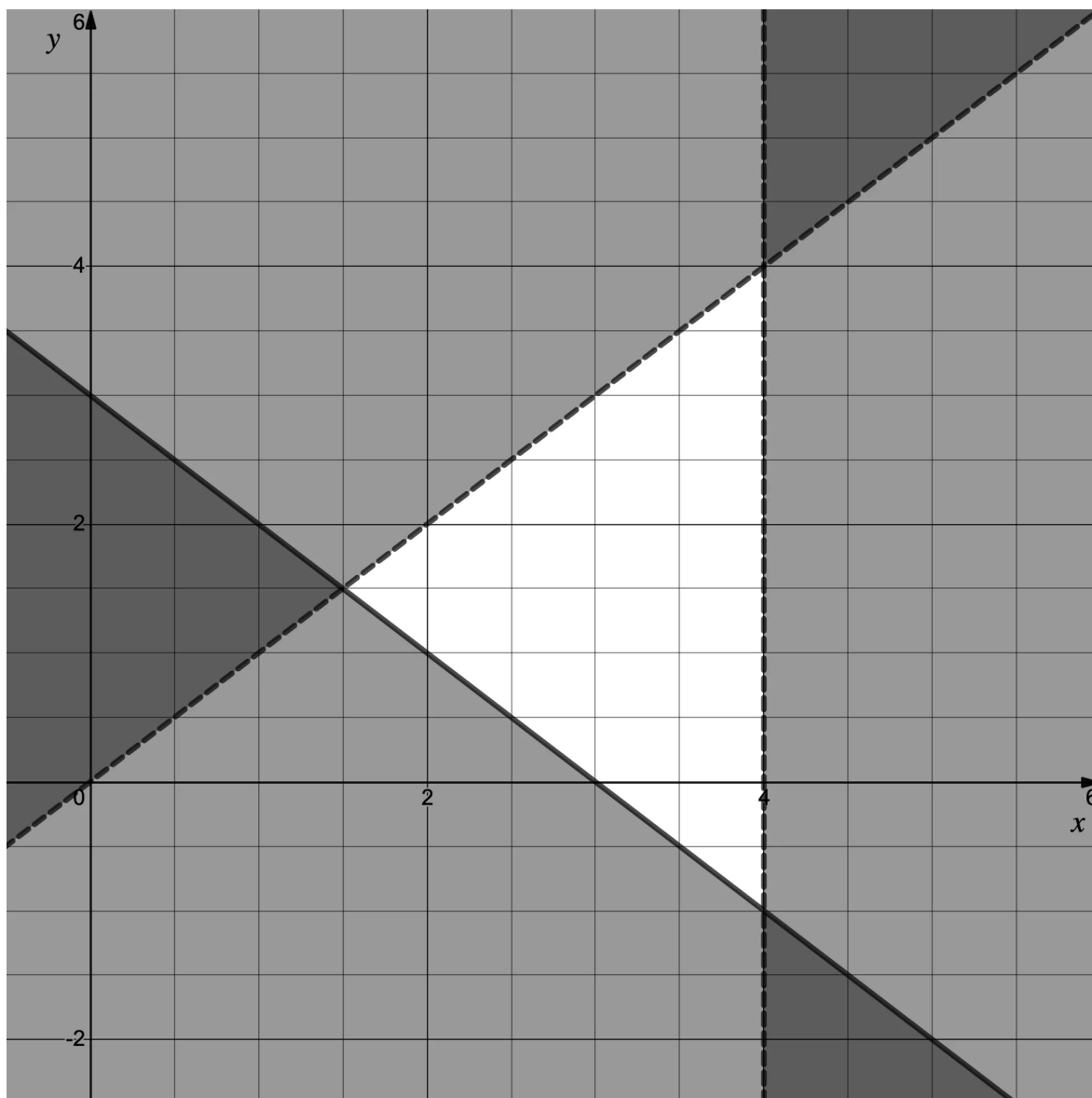
3 Determine which one of the formulae below defines an increasing sequence for $n \geq 1$.

Tick (✓) **one** box.

$u_n = 2^{n-1}$	☐
$u_n = \cos(180n^\circ)$	☐
$u_1 = 20 \quad u_{n+1} = u_n - 5$	☐
$u_n = 20 - 3n$	☐

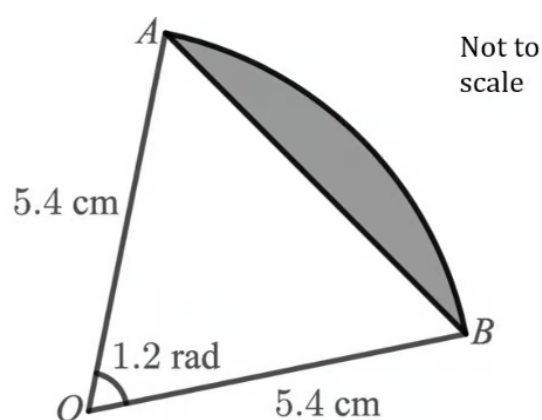
(1 mark)

4 Write down the inequalities that define the region R shown in the diagram below.



(4 marks)

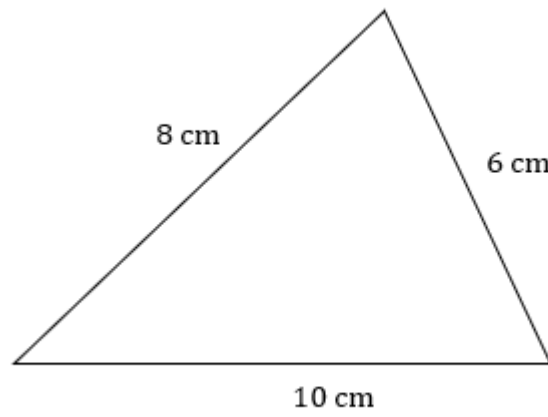
5 The diagram below shows the sector of a circle OAB .



- (i) Find the area of the sector OAB , giving your answer to 3 significant figures.
- (ii) Find the area of the triangle OAB , giving your answer to 3 significant figures.
- (iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

(5 marks)

6 Prove that a triangle with side lengths of 8 cm, 6 cm and 10 cm must contain a right-angle. You may use the diagram below to help.



Not to
scale

(4 marks)

7 (a) Show that

$$\frac{11}{(2x-3)(x+4)}$$

can be written in the form

$$\frac{A}{2x-3} + \frac{B}{x+4}$$

(3 marks)

(b) Hence find

$$\int \frac{11}{(2x-3)(x+4)} \, dx$$

writing your answer as a single logarithm.

(4 marks)

- 8 (a)** Scientists introduced a small number of rare breed deer to a large wildlife sanctuary.

The population of deer, within the sanctuary, is modelled by

$$D = 20e^{0.1t}$$

D is the number of deer after t years of first being introduced to the sanctuary.

Write down the number of deer the scientists introduced to the sanctuary.

(1 mark)

- (b)** How many years does it take for the deer population to double?

(2 marks)

- (c)** Give one criticism of the model for population growth.

(1 mark)

- (d)** The scientists suggest that the population of deer are separated after either 25 years or when their population exceeds 400.

Find the earliest time the deer should be separated.

(2 marks)

- 9 (a)** Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years

A model for the mass of carbon-14, y g, in an object originally containing 100 g, at time t years is

$$y = 100e^{-kt}$$

where k is a constant.

Find the value of k , giving your answer to three significant figures.

(2 marks)

- (b)** The object is considered as having no radioactivity once the mass of carbon-14 it contains falls below 0.5 g. Find out how old the object would have to be, to be considered non-radioactive.

(2 marks)

- (c)** A different object currently contains 25g of carbon-14.
In 500 years' time how much carbon-14 will remain in the object?

(2 marks)

10 (a) The curve C has equation

$$\frac{1}{5}x^2e^y = 5$$

Show that C intersects the x -axis at the points $(-5, 0)$ and $(5, 0)$.

(2 marks)

(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y .

(2 marks)

(c) Hence find the gradients of C at the two points where C intercepts the x -axis.

(2 marks)

11 (a) The curve C is defined by the equation $\ln y = 1 - xy$.

The point $P(1, 1)$ lies on C .

Show that

$$\frac{dy}{dx} = -\frac{y^2}{1 + xy}$$

(4 marks)

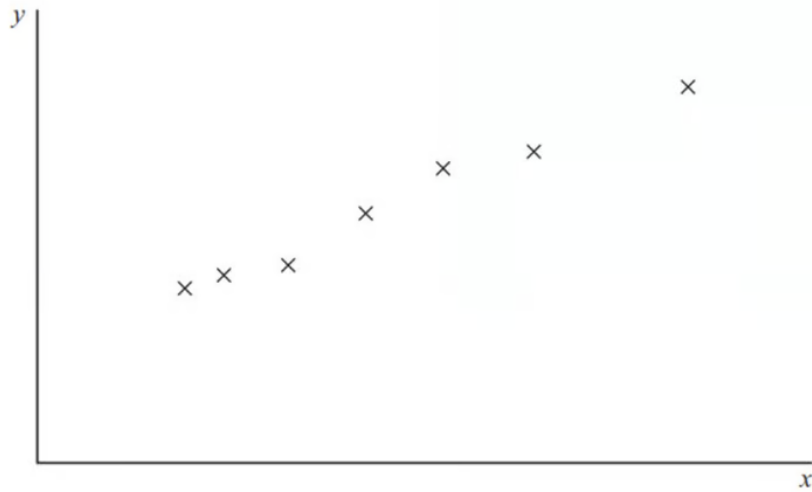
(b) Find the gradient of the tangent to C at point P , and hence find the gradient of the normal to C at point P .

(3 marks)

(c) Find the equation of the normal to C at point P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(2 marks)

12 Which of the options below best describes the correlation shown in the diagram below?



Circle your answer.

Moderate Positive	Strong Positive	Moderate Negative	Strong Negative
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(1 mark)

13 Mark is one of a team of people interviewing hockey fans as they exit a stadium.

He is asked to survey every 20th person who exits.

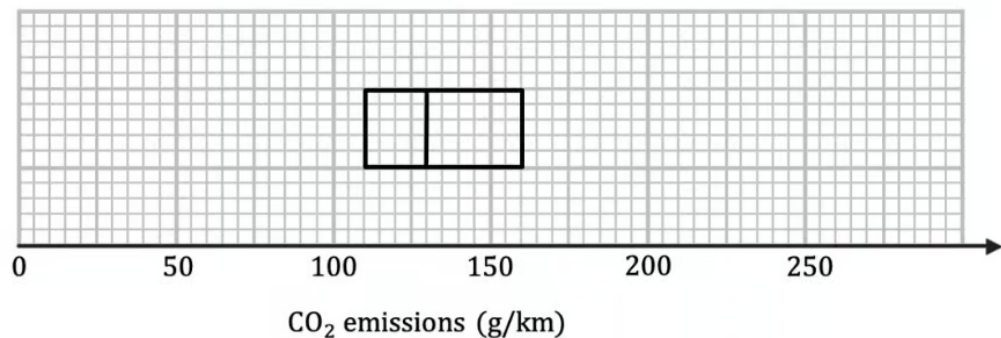
Identify the name of this type of sampling.

Circle your answer.

Simple Random	Quota	Stratified	Systematic
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(1 mark)

- 14 (a)** The incomplete box plot below shows the carbon dioxide (CO₂) emissions, measured in grams per kilometre, of a random sample of 34 vehicles taken from the large data set.



Find the interquartile range.

(1 mark)

- (b)** An outlier is defined as any data value that falls either more than

$1.5 \times$ (interquartile range) above the upper quartile or less than

$1.5 \times$ (interquartile range) below the lower quartile.

Find the boundaries (fences) at which outliers are defined.

(2 marks)

- (c)** The random sample of 34 cars has one outlier, a CO₂ reading of 250 g/km.

Complete the box plot given that the maximum and minimum values should be located at the boundaries (fences) at which outliers are defined.

Any outliers should be indicated with a cross (×).

(3 marks)

15 (a) A snowboarder is trying to perform the Poptart trick.

The snowboarder has a success rate of 25% of completing the trick.

The snowboarder will model the number of times they can expect to successfully complete the Poptart trick, out of their next 12 attempts, using the random variable $X \sim B(12, 0.25)$.

- (i) Give a reason why the model is suitable in this case.
- (ii) Suggest a reason why the model may not be suitable in this case.

(2 marks)

(b) Using the model, find the probability that the snowboarder

- (i) successfully completes the Poptart trick more than 3 times in their next 12 attempts
- (ii) fails to successfully complete the trick on any of their next 12 attempts.

(2 marks)

16 The random variable $X \sim B(50, 0.85)$. Find:

- (i) the largest value of q such that $P(X < q) < 0.16$
- (ii) the largest value of r such that $P(X \geq r) > 0.977$
- (iii) the smallest value of s such that $P(X > s) < 0.025$.

(6 marks)

17 (a) A and B are two events with $P(A) = 0.47$ and $P(B) = 0.31$. Given that A and B are independent, write down

(i) $P(A|B)$

(ii) $P(B|A')$

(2 marks)

(b) A group of middle and senior school students were asked whether they preferred vinegar or ketchup as a topping on their chips. The following two-way table shows the results of the survey:

	vinegar	ketchup	total
middle	49	21	70
senior	63	27	90
total	112	48	160

(i) Find and $P(\text{ketchup} | \text{middle})$ and $P(\text{middle} | \text{ketchup})$.

(ii) Use your results from part (b)(i) to show that for the students in the sample 'is in middle school' and 'prefers ketchup on chips' are independent events.

(4 marks)

- 18 (a)** The table below shows data from the United States regarding annual per capita chicken consumption (in pounds) and the unemployment rate (% of population) between the years 2005 and 2014:

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Chicken consumption (pounds)	86.4	86.9	85.5	83.8	80.0	82.8	83.3	80.8	82.3	83.8
Unemployment rate (%)	5.08	4.62	4.62	5.78	9.25	9.63	8.95	8.07	7.38	6.17

The product moment correlation coefficient for these data is $r = -0.821$.

The critical values for a 10% two-tailed test are ± 0.5495 .

State what is measured by the product moment correlation coefficient.

(1 mark)

- (b)** (i) Write down suitable null and alternative hypotheses for a two-tailed test of the correlation coefficient.
- (ii) Show that, at the 10% level of significance, there is evidence that the correlation coefficient is different from zero.

(3 marks)

- (c)** A newspaper's headline states:

"Eating chicken is the secret to reducing the unemployment rate in the US!"

Explain whether this headline is fully justified.

(1 mark)

- 19 (a)** Pizza Prince is a fast-food restaurant which is known for their Crown pizza. The weights of Crown pizza are normally distributed with standard deviation 42 g. It is thought that the mean weight, μ , is 350 g.

A restaurant inspector believes that the mean weight of the Crown pizza is less than 350 g. She visits the restaurant over the period of a week, and samples and weighs five randomly selected Crown pizzas. She uses the data to carry out a hypothesis test at the 5% level of significance.

She tests $H_0 : \mu = 350$ against $H_1 : \mu < 350$.

When the inspector writes up her report, she can only find the values for four of the weights, these are shown below:

325.2 356.1 319.7 300.5

Given that the result of the hypothesis test is that there is insufficient evidence to reject H_0 at the 5% level of significance, calculate the minimum possible value for the missing weight, w . Give your answer correct to 1 decimal place.

(4 marks)

- (b)** The inspector remembers her assistant claiming that if she had used a 10% level of significance then the outcome to the hypothesis test would have been different.

Using this information, write down an inequality for w .

(4 marks)

20 (a) For the standard normal distribution $Z \sim N(0, 1^2)$, find:

(i) $P(Z < 1.5)$

(ii) $P(Z > -0.8)$

(iii) $P(-2.1 < Z < -0.3)$

(4 marks)

(b) The random variable $X \sim N(2, 0.1^2)$.

By using the coding relationship between X and Z , re-express the probabilities from parts (a) (i), (ii) and (iii) in the forms $P(X < a)$, $P(X > b)$ and $P(c < X < d)$ respectively, where a , b , c and d are constants to be found.

(3 marks)

- 21 (a)** The random variable $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 34.451) = 0.001$ and $P(X < 14.792) = 0.2$

Use the relationship between X and the standard normal variable Z to show that the following simultaneous equations must be true:

$$\mu + 3.0902\sigma = 34.451$$

$$\mu - 0.8416\sigma = 14.792$$

(4 marks)

- (b)** By solving the simultaneous equations in (a), determine the values of μ and σ .

(2 marks)