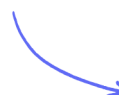


Practice Paper 6 (Probability & Statistics 2)

Scan here to return to the course
or visit [savemyexams.com](https://www.savemyexams.com)



Total Marks

/50

1 (a) The times, T seconds, of a random sample of 8 TV adverts on *Channel π* are as follows.

35 61 42 40 37 35 53 49

Calculate unbiased estimates of the population mean and variance of T .

(3 marks)

(b) It is known that the population of T follows a Normal distribution with a variance of 81.

Using the sample from part (a), find a 95% confidence interval for the population mean of T .

(3 marks)

(c) Write down the probability that the confidence interval contains the true population mean.

(1 mark)

2 (a) The average length, L , of a unicorn's horn is 91 cm with a variance of 5 cm². Luna researches unicorns and believes that unicorns that were born beneath a rainbow have longer horns. To test her belief, Luna takes a random sample of 12 unicorns that were born beneath a rainbow and measures the length of their horns.

- (i) Write suitable null and alternative hypotheses to test Luna's claim.
- (ii) What assumption do we need to make about the length of unicorn horns so that a normal distribution can be used for the mean of the sample, $\bar{L}$.

(2 marks)

(b) Given that the critical value for the hypothesis test is 92.1 cm, calculate the probability of a Type I error.

(3 marks)

(c) For unicorns born under a rainbow, the population mean is subsequently found to be 91.5 cm while the variance remains unchanged. Using the critical value from part (b), find the probability that Luna's test would have produced a Type II error.

(3 marks)

- 3 (a)** A shop manager buys a box of 100 small chocolate bars. The mass, in grams, of a chocolate bar can be modelled using $N(85, 1.5^2)$.

Find the probability that a randomly selected chocolate bar is less than 81 g.

(2 marks)

- (b)** Explain why a Poisson distribution can be used as an approximation for the number of chocolate bars in the box of 100 that weigh less than 81 g.

(2 marks)

- (c)** Using a suitable approximation, find the probability that more than one chocolate bar in the box weighs less than 81 g.

(3 marks)

- 4 (a)** Zien and her younger sister Ying are playing a game of 'rock, paper, scissors'. Zien claims that she can predict which out of the three options - rock, paper or scissors - Ying will choose every time they play the game. Ying thinks Zien is just guessing so she decides to conduct a hypothesis test to test Zien's claim. They play the game 10 times and if Zien is correct in 8 or more games, Ying agrees to accept Zien's claim.

State suitable null and alternative hypotheses for Ying's test.

(1 mark)

- (b)** Calculate the probability of a Type I error.

(3 marks)

- (c)** Find the range of values that Ying could have been using for the significance level of her test, and hence write down the most likely significance level she used.

(2 marks)

- 5 (a)** Whilst writing an essay, Gamu notices that she makes spelling mistakes at a rate of 7 for every 150 words. Gamu models the number of spelling mistakes she makes using a Poisson distribution.

Find the maximum number of words Gamu can write before the probability of her making a spelling mistake exceeds 0.75.

(4 marks)

- (b)** Gamu is asked to write three short essays by her lecturer. She writes one containing 100 words, one containing 200 words and one containing 250 words. An essay is returned by Gamu's lecturer if more than 1% of its words contain spelling mistakes.

Find the probability that all three short essays are returned.

(7 marks)

6 (a) The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{k}{x^3} & 2 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = \frac{8a^2}{a^2 - 4}$.

(3 marks)

(b) Given that $E(X) = \frac{20}{7}$ show that $a = 5$ and hence find the value of k .

(5 marks)

(c) Find the exact value of $\text{Var}(X)$.

(3 marks)