A Level · CIE · Maths

50 mins 7 questions

Practice Paper 6 (Probability & Statistics 2)

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Total Marks	/50

1 (a) The standard normal distribution is given by $Z \sim N(0,1^2)$.

Given that $P(-a<Z<a)=0.99$,

(i) Explain why $P(Z<a)=0.995$,

(ii) Hence find the value of a.

(2 marks)

(b) The probability of success in a trial is p. A random sample of 64 trials are carried out and there were 24 successes.

Calculate the proportion of the trials in the sample, p_s, that were successful.

(1 mark)

(c) The formula for the end-points of the 99% confidence interval for the probability of success, p is given by:

$$p_s \pm z \times \sqrt{\frac{p_s(1-p_s)}{n}} \qquad where\ P(-z<Z<z)=0.99$$

Calculate an approximate 99% confidence interval for the probability of success in a trial, p.

(2 marks)

2 (a) Remy has a biased coin with the probability of it landing on tails being 0.05.

Using a Poisson approximation, find the probability that the coin lands on tails more than 4 times when Remy flips it 60 times.

(3 marks)

(b) Remy flips the coin 2000 times.

Using a normal approximation, find the probability that the coin lands on tails fewer than 120 times when Remy flips it 2000 times.

(5 marks)

3 (a) Roger has a keen fascination with rabbits that live on Easter Island, in particular the lengths of their ears. The mean length of a rabbit's ear on Easter Island is μ cm. He takes a random sample of 40 rabbits and finds that a 90% confidence interval for μ is 8.3 to 9.9 cm.

Using the same sample, find a 99% confidence interval for μ. Give the end-points correct to 1 decimal place.

(6 marks)

(b) Roger wants to find a confidence interval for μ with a width of 1.28 cm. Find the confidence level that Roger should use.

(4 marks)

(c) Roger takes 100 random samples and calculates a 95% confidence interval for each one. How many of the confidence intervals would be expected to contain the true population mean, μ.

(1 mark)

4 (a) The continuous random variable, X, has probability density function

$$f(x) = \begin{cases} k(3x^4 - 2x) & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

Show that $k = \dfrac{5}{78}$.

(2 marks)

(b) (i) Find $P(1.2 < X < 1.8)$

(ii) Find $P(X < 1.5)$

(4 marks)

5 (a) The random variable $X \sim B(60, 0.08)$ is approximated by $Y \sim Po(\lambda)$.

Explain why this approximation is valid and state the value of λ.

(1 mark)

(b) (i) Find $P(X = 4)$, giving your answer to 6 decimal places.

(ii) Find $P(Y = 4)$, giving your answer to 6 decimal places.

(4 marks)

(c) Hence find the percentage error when a Poisson distribution is used to approximate $P(X = 4)$.

(2 marks)

6 (a) The distributions of the independent random variables X and Y are $N(8,2)$ and $N(9,3)$ respectively.

Find the probability that

(i) $2X + 3Y > 44$

(ii) The sum of two observations from X and three observations from Y are greater than 44.

(5 marks)

(b) Explain why your answers to (a) part (i) and (ii) are different.

(1 mark)

7 (a) The standard normal distribution is denoted by $Z \sim N(0,1^2)$.

(i) Write down a formula that links the standard normal distribution, Z , to the distribution $\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$.

(ii) Find the value of d such that $P(Z < d) = 0.05$, correct to 3 decimal places.

(3 marks)

(b) The population mean of the random variable $X \sim N(\mu, 10^2)$ is being tested using a null hypothesis $H_0 : \mu = 30$ against the alternative hypothesis $H_1 : \mu < 30$. A random sample of observations is taken from the population and the sample mean is calculated as 28.

Using a 5% level of significance, there is not enough evidence to reject the null hypothesis.

(i) Show that $\sqrt{n} < 8.2245$

(ii) Hence find the greatest possible value for the sample size, n .

(4 marks)